Flight Dynamics Challenges for the GRACE Follow-On Mission

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Abstract: *The GRACE Follow-On (GRACE-FO) mission is a partnership between NASA and the German Research Centre for Geosciences (GFZ) and will be operated by the German Space Operations Centre (GSOC) / DLR. The baseline launch date is in August, 2017. It is a follow on mission for the successful GRACE (Gravity Recovery and Climate Experiment) mission. Two twin satellites are flying approximately 220 kilometers apart in a polar, nearly circular orbit starting at an altitude of 500 kilometers, which slowly decreases during the mission. For a nominal mission lifetime of 5 years detailed measurements of Earth's gravity field and atmosphere are collected and provided to the science community to support an understanding of the distribution and flow of mass on and within the Earth. Key tasks are getting into and maintaining the formation of the two satellites and operation of the experimental Laser Ranging Interferometer (LRI) payload allowing highly accurate inter-satellite range rate determination. This paper gives an overview of the GRACE-FO mission and its main Flight Dynamics challenges. A deeper look into the formation acquisition strategy and the operations support of the LRI instrument is provided.*

Keywords: *GRACE Follow-On Mission, Flight Dynamics Operations, Formation Flying, Laser Ranging Interferometer.*

1. Introduction

The objective of the GRACE mission, launched in 2002, was to provide a new model of the Earth's gravity field every 15 to 30 days [1]. After 14 years in orbit, the mission, originally planned for 5 years, is coming close to the end. Therefore the GRACE-FO mission is designed by NASA and GFZ to continue this exceptional task by consistently and uninterruptedly providing the same kind of data to the science community beyond the GRACE mission [2]. Funded by GFZ, GSOC is responsible for the operations of the satellites. To reach the mission goals the along-track distance between the two spacecraft must be kept within 220 km ± 50 km and range changes must be determined very accurately. As GRACE did, GRACE-FO is carrying a Microwave Ranging Instrument (MWI) for satellite-to-satellite tracking at K-band frequencies. Additionally, to provide even more accurate inter-satellite range change measurements a Laser Ranging Interferometer (LRI) is mounted on each satellite for technology demonstration purposes. Both instruments are being provided by NASA's Jet Propulsion Laboratory (JPL).

The GRACE-FO satellites—GRACE-FO1 & GRACE-FO2—are being manufactured for JPL by Airbus Defence and Space, a division of Airbus Group SE, and will be placed into a polar, nearly circular, 500 km altitude orbit using a single launch of a Dnepr rocket from Baikonur, Kazakhstan in August, 2017. After separation from the rocket the two satellites are in slightly differently sized orbits that cause a drift apart with a nominal drift rate of about 200 km/day for several days. This drift has to be stopped and reversed to bring them into the required formation of 220 km apart. The maneuver strategy for target formation acquisition, rationale, and boundary conditions are described in detail in chapter 2.

The operations of the inter-satellite ranging instruments—MWI and LRI—require pointing information based on two-line elements (TLEs) of both spacecraft, which are provided via command from the ground. To improve this TLE-based pointing information onboard each satellite a GPS-based frame correction is applied for K-band pointing. For LRI operations this corrected pointing still is not accurate enough. Thus, the offset between GPS-based and TLE-based pointing is computed and modelled on the ground using a parametric fit method. The computed fit parameters are uploaded to the satellites by ground command together with each TLE set to compute a more accurate pointing using the TLEs and fit data. The parametric fit model and the implementation of the parameters computation is presented in chapter 3.

2. Acquisition of Target Formation

GRACE-FO2 is separated from the Dnepr rocket a quarter second later than GRACE-FO1. Because the upper stage of the Dnepr rocket is accelerating during these separations, this time difference results in a difference between the magnitudes of the velocities of the two spacecraft. In addition, the upper stage has a 20 deg pitch angle at the times of separation, so the separation impulses themselves increase the velocity difference somewhat. The nominal velocity difference amounts to about 0.75 m/s and results in an offset of the semi-major axis between the two satellites causing an along-track drift of about 200 km per day. Therefore after roughly one day the desired distance of 220 km is already reached. Unfortunately the spacecraft are not ready so soon after launch to perform a maneuver and stop the drift. Thus they continue separating. A maneuver strategy has been developed to reverse the drift and acquire the target formation within the Launch and Early Operations Phase (LEOP). This strategy also includes a removal of the differences in eccentricity and in inclination. The timing and boundary conditions and the derived maneuver strategy are further described in the following subchapters.

2.1 Formation Control Concept

The orbit of a spacecraft can be described using the six mean Keplerian elements: semi-major axis a, eccentricity e, argument of perigee ω, inclination i, right ascension of the ascending node Ω, and argument of latitude u, which is the sum of argument of perigee and mean anomaly, $u=\omega+M$. The elements e and ω are taken to be the polar coordinates of an eccentricity vector $\vec{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix} = e \cdot \begin{pmatrix} \cos\omega \\ \sin\omega \end{pmatrix}$: the elements e_x and e_y are sometimes known as the semi-equinoctial elements designated k' and h'.

The relative motion of a target satellite (index T) with respect to a reference main spacecraft (index M) can be parameterized by the corresponding set of relative mean orbital elements [3]:

$$\begin{pmatrix} \Delta a \\ a\Delta e_x \\ a\Delta e_y \\ a\Delta i_x \\ a\Delta i_y \\ a\Delta u \end{pmatrix} = \begin{pmatrix} a_T - a_M \\ a_M\left(e_T \cos\omega_T - e_M \cos\omega_M\right) \\ a_M\left(e_T \sin\omega_T - e_M \sin\omega_M\right) \\ a_M\left(i_T - i_M\right) \\ a_M\left(\Omega_T - \Omega_M\right)\sin i_M \\ a_M\left(u_T - u_M\right) \end{pmatrix}, \tag{1}$$

where angular elements are expressed in radians. The relative eccentricity vector and relative inclination vector are often rewritten according to Equations 2 and 3, and can be reflected as radial and out-of-plane offsets, respectively.

$$a\Delta\vec{e} = a \cdot \begin{pmatrix} \Delta e_x \\ \Delta e_y \end{pmatrix} = a \cdot \delta e \cdot \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}, \tag{2}$$

$$a\Delta\vec{i} = a \cdot \begin{pmatrix} \Delta i_x \\ \Delta i_y \end{pmatrix} = a \cdot \delta i \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{3}$$

Here $a\delta e$ and $a\delta i$ are the magnitudes of these vectors and φ and θ represent their phases termed relative argument of perigee and relative ascending node, respectively. They characterize the geometry of the relative orbit and determine the location where the maximum vertical and horizontal distances are reached.

These relative parameters are not stable due to natural perturbations. The magnitude of the relative eccentricity vector $a\Delta e$ is stable but its direction φ changes over time. $a\Delta i_x$ is quite stable, but since $a\Delta i_y$ depends on $\Delta\Omega$ and a relative inclination $a\Delta i_x$ causes a drift of $\Delta\Omega$, $a\Delta i_y$ changes directly depending on $a\Delta i_x$ [4]. Since the GRACE-FO spacecraft are twin satellites the relative semi-major axis Δa is rather stable as long as the satellites' masses and attitudes are similar. The drift of the relative argument of latitude $a\delta\Delta u$ depends on Δa according to

$$a\delta\Delta u = -\frac{3}{2} n \cdot \Delta a \cdot \Delta t, \tag{4}$$

with n being the mean motion $n \approx \left(\mu/a^3\right)^{0.5}$, where μ = 398600.4415 km³/s², and Δt being the duration of the drift being considered.

The relative orbital elements can be controlled by customized maneuvers. The direction and size of control maneuvers can be computed using the simplified Gauss' Eq. 5 [5].

$$\begin{pmatrix} \delta\Delta a \\ a\delta\Delta e_x \\ a\delta\Delta e_y \\ a\delta\Delta i_x \\ a\delta\Delta i_y \\ a\delta\Delta u \end{pmatrix} = \frac{1}{n} \begin{bmatrix} 0 & 2 & 0 \\ \sin u & 2\cos u & 0 \\ -\cos u & 2\sin u & 0 \\ 0 & 0 & \cos u \\ 0 & 0 & \sin u \\ -2 & -3n(t - t_{\Delta v}) & -\sin u / \tan i \end{bmatrix} \begin{pmatrix} \Delta v_R \\ \Delta v_T \\ \Delta v_N \end{pmatrix}, \tag{5}$$

where u is the mean argument of latitude at the time of the maneuver, and n as above being the mean motion. The maneuver is given in the Hill's orbital frame, which is often also referred to as the radial-transverse-normal frame (RTN) [6]. Note that Eq. 5 is generally valid for circular orbits but it has been shown [7] that the delta-e equations work for an elliptical reference orbit as well as for a circular one (to first order in eccentricity), with a variation on the RTN coordinate frame that can be ignored for nearly circular orbits. Since GFO is flying in a nearly circular orbit the error is negligible and Eq. 5 is applicable for operations.

2.2 Formation Parameters after Separation and Target Formation Geometry

Since the two spacecraft are separated from the upper stage of the rocket at slightly different times while it is accelerating and the separation impulses themselves include along-track and cross-track components, a relative geometry is built up from the beginning. The resulting nominal formation parameters after separation are listed in Table 1.

The motivation for the target formation parameters, also given in Table 1, is shortly described here. A main requirement for the mission is an along-track separation $a\Delta u$ of 220 km (± 50 km) which shall be reached within LEOP (15 days after launch) and shall be kept rather stable implying a nominal relative semi-major axis Δa of about 0 m. There are no explicit mission requirements for the other parameters but some implicit requirements can be derived from requirements on the orbits. For example, the inclinations are required to be within ±0.05 deg of 89 deg, so $a\Delta i_x$ must be less than 12000 m; similarly, the range rate between the spacecraft is required to be less than 3 m/s, so $a\Delta e$ must be less than 1355 m. Beyond these requirements, the following requests shall be considered.

Table 1. Formation Parameters after Nominal Separation and Target Formation

	After Separation [m]& [deg]	**Target Formation [m]& [deg]**
Δa	1,400	0
$a\Delta e$	1,400	0
φ	n/a (~170.0)	n/a
$a\Delta i_x$	500	0
$a\Delta i_y$	0	0
$a\Delta u$	0	220,000 ± 50,000

A difference between the eccentricities of the orbits results in varying radial offset between the two spacecraft and therefore in a pitch variation of the pointing vectors over the orbit. A consequence is that the pitch angles for a precise pointing of the instruments towards each other are different which in turn causes a differential drag between the two satellites. Therefore a relative semi-major axis is built up, triggering an along-track drift and requiring more maneuvers for formation maintenance. Thus, the relative eccentricity vector $a\Delta e$ shall be kept as small as possible throughout the mission.

An out-of-plane offset—or relative inclination vector $a\Delta i$—results in a yaw angle of both spacecraft, which shall be kept small, too for the same reasons. Since the initial offset is mainly an x-component offset, it also causes a drift of the relative Right Ascension of Ascending Node (RAAN) $\Delta\Omega$, which results in a change of $a\Delta i_y$ and therefore an increasing $a\Delta i$. Consequently, the initial out-of-plane offset shall be removed as soon as possible after launch.

Another advantage of removed $a\Delta e$ and $a\Delta i$ is to be able to perform satellite swaps, i.e. interchanging leading and trailing satellites, whenever requested and being independent from the times when the relative eccentricity / inclination vector separation provides a safe fly-by (cf. [8]).

2.3 Maneuver Strategy to Acquire the Target Formation

The initial relative semi-major axis Δa of 1,400 m, results in an along-track drift $a\delta\Delta u$ of roughly 200 km/day—according to Eq. 4. The conservative assumption that the spacecraft will be ready for maneuvering on day 5 after launch, results in an along-track distance of about 1,000 kilometers at the time of the first maneuver. The target distance of 220 km shall be acquired by the end of LEOP, i.e. within 15 days. To keep the relative semi-major axis stable after target formation acquisition the effects of drag on the two spacecraft need to be the same, so the propellant consumption of the two satellites shall be well-balanced. This also helps maximize the potential lifetime of the mission, which must end when either of the spacecraft runs low on propellant since the propellant is used for attitude control as well as for translational maneuvers.

At this time the basic idea for getting the satellites into formation is to reduce the size of the higher satellite's orbit and raise the orbit of the lower satellite above the other, so that the drift is reversed and the two spacecraft drift towards each other. As soon as the desired distance of about 220 kilometers is achieved, the lower satellite (which was originally higher) will raise its orbit to match the other satellite's altitude and thereby the drift is stopped. This basic idea has an implicit bias toward raising the orbits within the constraint of balancing the propellant usage between the two spacecraft: again, this maximizes the lifetime of the mission by reducing the drag on the spacecraft.

Depending on the initial formation and the timing constraints the following impulsive maneuver strategy has been derived. On day 5 a turn-around maneuver (TAM) has to be performed to reverse the drift. Taking into account some time margins it was decided to use 4 days for the drift-back phase. Using Eq. 4 and solving for Δa, the needed relative semi-major axis of roughly -1,360 m can be computed:

$$\Delta a = -\frac{2}{3}\frac{a\delta\Delta u}{n\cdot\Delta t} = -\frac{2}{3}\frac{(1000-220)}{n\cdot 345600} \approx -1.360 \text{ km}. \tag{6}$$

According to Eq. 5 a maneuver size of about 1.53 m/s can be derived to change Δa from 1,400 m to –1,360 m:

$$\Delta v_T = \frac{n}{2}\delta\Delta a = \frac{n}{2}\left(-1360 - 1400\right) \approx -1.53\,\text{m/s.} \tag{7}$$

To finally stop the reverse drift after 4 days the current relative semi-major axis needs to be removed, i.e. a separation-arrest maneuver (SAM) of the size of about 0.75 m/s is necessary:

$$\Delta v_T = \frac{n}{2}\delta\Delta a \approx \frac{n}{2}1360 = 0.75\,\text{m/s.} \tag{8}$$

As stated above the fuel consumption of both spacecraft shall be similar, thus the total Δv of 2.28 m/s must be split by two, i.e. each spacecraft must provide a Δv of 1.14 m/s. This results in a TAM of GRACE-FO1 of the complete 1.14 m/s to raise its orbit. GRACE-FO2 has to lower its orbit using the remaining part of the TAM's Δv, which is 1.53 – 1.14 = 0.39 m/s. Finally, GRACE-FO2 performs the SAM of 0.75 m/s. This impulsive maneuver strategy is summarized in Table 2.

Table 2. TAM & SAM Δv Summary

dV [m/s]	GRACE-FO1	GRACE-FO2
TAM	1.14	-0.39
SAM	-	0.75
total	1.14	1.14

2.4 Implementation of the Maneuver Strategy

The implementation of the basic idea as described above is constrained by the limited acceleration available from the propulsion subsystem—about 0.00017 m/s^2.

According to Eq. 5 the relative eccentricity vector $a\Delta e$ and relative inclination vector $a\Delta i$ can be removed only if the maneuvers are executed at specific arguments of latitude u. Deviations from those locations reduces the effectiveness of the maneuvers. A separate analysis showed that an offset of about ±10 minutes is acceptable, allowing us to maneuver for 20 minutes centered at the optimal latitude u. A maneuver duration of 20 minutes is equivalent to a Δv of roughly 0.18 m/s for the GRACE-FO satellites.

To perform the maneuvers as given in Table 2 while also including an effective reduction of the relative eccentricity vector and relative inclination vector as requested, the TAMs and SAMs need to be split into series of maneuvers lasting no longer than 20 minutes each. The relative eccentricity vector is changed by along-track maneuvers and can therefore be realized by the nominal TAM and SAM series without spending extra fuel. On the other hand, the reduction of the relative inclination vector requires out-of-plane maneuvers. Therefore, it is advised to perform combined in-plane/out-of-plane maneuvers with a yaw angle of ±45 deg to save some fuel, keeping in mind that the out-of-plane offset should be removed as soon as possible. To remove the initial offset, which is nominally about 500 m, according to Eq. 5 an out-of-plane maneuver of about 0.55 m/s is necessary:

$$\Delta v_N = n \cdot a\delta\Delta i = n \cdot 500 \approx 0.55 \text{ m/s.} \quad (9)$$

This means that each spacecraft has to spend roughly 0.115 m/s extra when the inclination corrections are combined with the TAMs or SAMs. To keep the attitudes of both spacecraft similar (so as to equalize the drag on them) the out-of-plane maneuvers should be executed close to each other, e.g. alternating by one orbit, keeping the yaw turn for the same period.

The maneuver burn durations are planned in decreasing order to minimize execution errors, to allow for counteraction of previous execution errors, and to permit a smooth stepwise and very accurate drift stop. A possible approach for a maneuver strategy to reach the target formation within the considered times fulfilling the mentioned boundary conditions is described in the following.

Figure 1 depicts the TAM maneuver sequence. Green vertical lines represent in-plane maneuvers, blue vertical lines represent combined in-plane/out-of-plane maneuvers. An ‘M’ at the top of a maneuver line indicates that the maneuver is executed by the ‘Master’ satellite, here GRACE-FO1. An ‘S’ at the bottom stands for ‘Slave’, here GRACE-FO2.

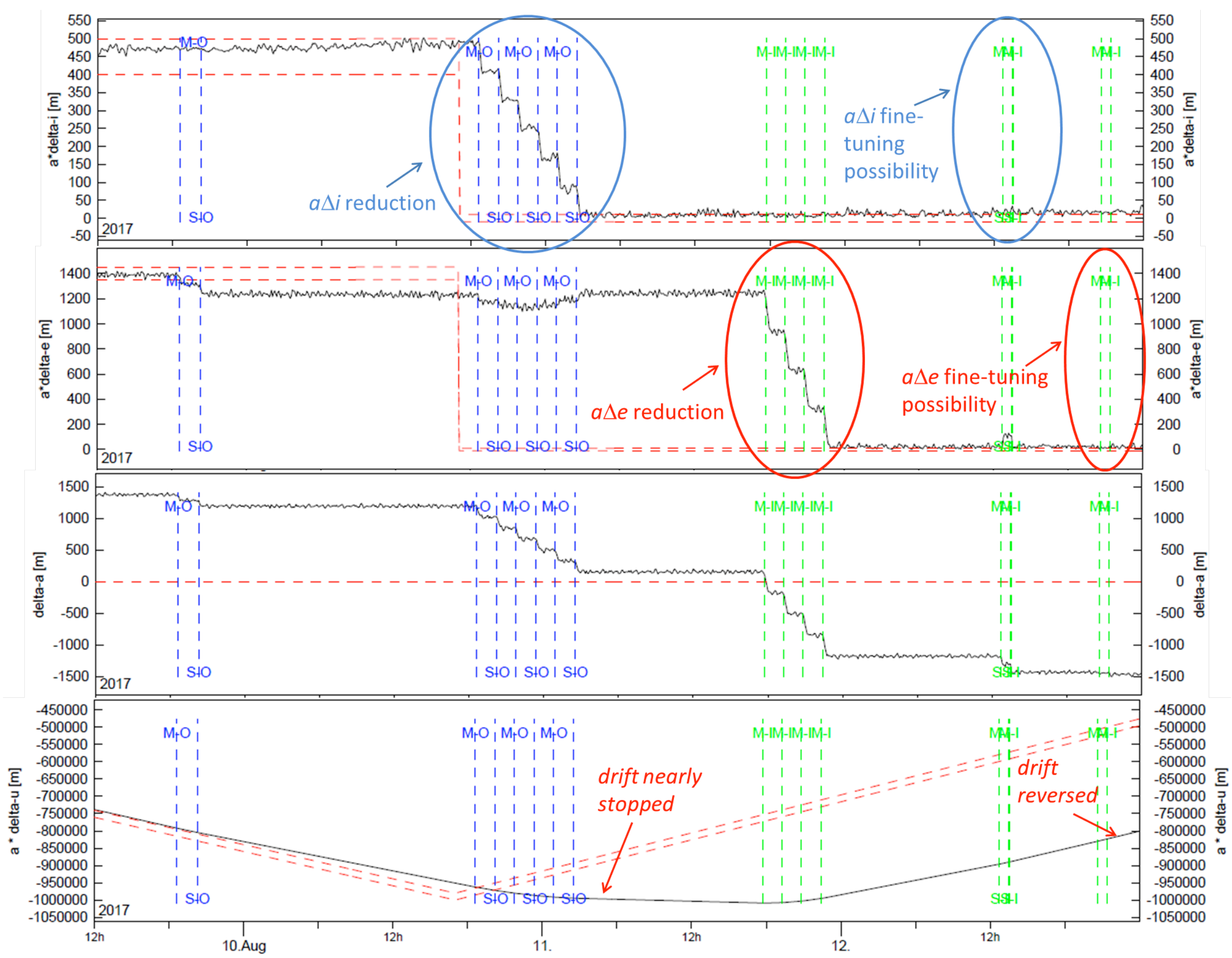

Figure 1. Turn-Around Maneuver Sequence (TAMs): from top to bottom: aΔi, aΔe, Δa, aΔu. Blue vertical lines stand for out-of-plane maneuvers, green vertical lines for in-plane maneuvers

At the beginning of day 5 after launch (2017/08/09 18:00 UTC) a small maneuver is planned for calibration purposes on each spacecraft, separated by one orbit. These calibration maneuvers hardly change the formation, but they should occur during a contact to a ground station to allow a real-time monitoring of the maneuvers. Including a yaw turn of 45 deg they are executed mainly for maneuver performance calibration and in-orbit procedure validation. One day later, on 2017/08/10 18:00 UTC, a sequence of combined in-plane/out-of-plane maneuvers starts (depicted by 6 blue vertical lines)—the first part of the TAMs. The top plot in Fig. 1 clearly shows how the relative inclination vector is stepwise removed by this sequence, where the maneuvers are separated by one orbit and executed by alternating satellites. Plot #3 depicts the relative semi-major axis and shows how it is nearly removed by this first sequence. In plot #4 it can be seen that the along-track drift has almost stopped now.

Another day later, on 2017/07/11 18:00,—with sufficient time for maneuver calibration and possible maneuver re-planning—a block of in-plane maneuvers removes the relative eccentricity vector, see plot #2. These maneuvers are each separated by one orbit and all executed by GRACE-FO1. The reverse drift has started now, see plot #4.

More than 12 hours later a pair of maneuvers on each spacecraft adjusts the drift rate. Here, also a fine-tuning of the relative inclination vector is possible. Finally, another pair of in-plane maneuvers on GRACE-FO1 establishes the final drift rate. Here a fine-tuning of the relative eccentricity vector is conceivable. The sizes of the final TAMs are reduced to have smaller absolute execution errors in order to establish the best possible drift rate. Note that once the relative eccentricity vector has been removed, each following along-track maneuver has to be split into two equally sized maneuvers separated by half an orbit to avoid building up aΔe again.

The separation arrest maneuver sequence (SAMs) is depicted in Fig. 2. These maneuvers are executed only in-plane by GRACE-FO2. The maneuvers are split pairwise to keep the relative eccentricity vector close to zero and they are in decreasing order of magnitude. The relative semi-major axis is removed stepwise, reducing the along-track drift until it is completely stopped by the last maneuver pair (aΔe and aΔi are not plotted here, since they are already set to zero by the TAMs). Using such a maneuver sequence a collision risk between the two satellites is minimized and the time for maneuver re-planning in case of execution errors or other problems is sufficient.

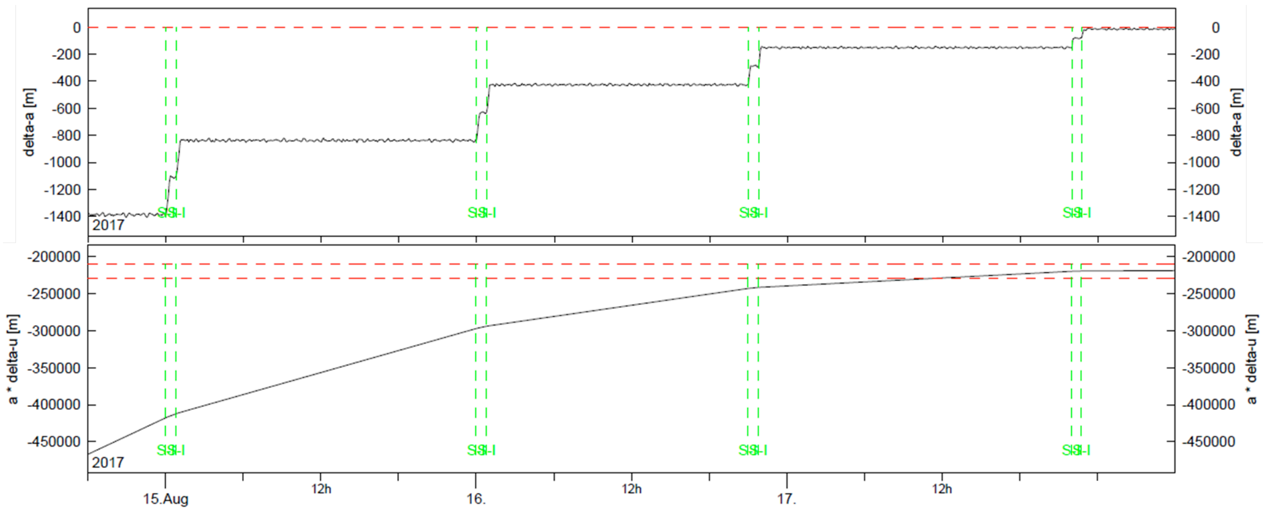

Figure 2. Separation-Arrest Maneuver Sequence (SAMs): top: Δ*a*, bottom: aΔ*u*

It shall be noted that currently a change of launch site from Baikonur to Yasny, Russia, is under discussion. Since the separation procedure and mechanism remain the same, also the nominal formation parameters are very similar. Only the angles of the relative eccentricity and relative inclination vectors are different since the argument of latitude at time of separation differs; their magnitudes don't change. Thus, a change of launch site mainly results in minor time shifting of the maneuvers, but the general strategy and maneuver sizes remain the same.

3. Laser-Ranging Interferometer Operations Support

The GRACE-FO satellites are very similar to their GRACE precursors, but carry a Laser-Ranging Interferometer (LRI) for technology demonstration purposes. Like the primary Microwave Ranging Instrument (MWI), the LRI measures fluctuations in the inter-spacecraft separation but with greater precision using laser interferometry. The LRI is designed to maintain pointing once the signal from the other spacecraft has been acquired; the initial acquisition, however, requires a pointing accuracy which cannot be achieved reliably enough by the originally implemented pointing direction computation on GRACE, which derives a pointing vector from the TLEs at the pointing spacecraft's argument of latitude as determined from GPS data [9].

Here, the two-line elements of both spacecraft are provided by ground command to both satellites, so that each of them can propagate the positions of both satellites and therefore compute a coarse pointing direction. Then the on-board available GPS data is used to estimate the local vertical, local horizontal coordinate frame in which the pointing direction has to be applied.

This corrected pointing as used for the MWI is still not accurate enough for LRI operations, though. Hence, a parametric model was developed at JPL [10] to estimate the differences between the pointing angles based on GPS-frame-corrected TLEs and the pointing angles derived from the high precision orbits based on GPS data of both spacecraft. The resulting fit parameters are computed on the ground for each spacecraft and sent to each spacecraft with both sets of TLEs, which allows a final pointing adjustment within the LRI to enable a more accurate over all pointing. The GPS frame correction, the parametric fit method, and its implementation at GSOC, as well as the analysis results will be described in the following subchapters.

3.1 GPS Frame Correction Theory

Using the position and velocity vectors *P* and *V* (in an Earth centered inertial coordinate frame (ECI)) of both spacecraft at a given time the relative pointing vector (*PV*) is just the difference of the positions divided by its norm $(P_2\text{-}P_1)/|P_2\text{-}P_1|$. Applying a transformation matrix $A_{ECI\text{->}RTN}$ (see Eq. 10), *PV* can be converted into the locale orbital frame—also referred to as radial-transverse-normal frame—to determine the line-of-sight vector (*LOS*) from one satellite to the other.

$$LOS = A_{ECI->RTN} \cdot PV, \quad A_{ECI->RTN} = \begin{pmatrix} \dfrac{P_1}{|P_1|} \\ \dfrac{P_1 \times V_1}{|P_1 \times V_1|} \times \dfrac{P_1}{|P_1|} \\ \dfrac{P_1 \times V_1}{|P_1 \times V_1|} \end{pmatrix}. \tag{10}$$

Thereof, in turn the pitch and yaw angles can be computed using Equations 11 and 12:

$$pitch = \arctan\left(LOS(1), \sqrt{LOS(2)^2 + LOS(3)^2} \right), \tag{11}$$

$$yaw = \arctan\left(-LOS(3), LOS(2)\right). \tag{12}$$

Note: A positive pitch angle is 'up' w.r.t. the TN-plane. A positive yaw angle is to the 'right' from the positive T-axis.

Based on a TLE the rough position and velocity at any time can be computed using the SGP4 propagator model [11]. But the position and velocity information derived from GPS-based orbit determination is more accurate than the TLE-derived data. We shall use a subscript "RTN-TLE" to refer to a RTN frame with respect to a TLE-derived state, and "RTN-GPS" to a RTN frame with respect to a GPS state.

Onboard a satellite where only its own GPS data are available a pointing computation derived only from GPS data is not possible. Therefore, both TLE sets are provided to each satellite via ground command from which a pointing can be derived. To improve the TLE-based pointing the transposed of the GPS-based transformation matrix $\left(A_{ECI->RTN-GPS}\right)^T$ is applied to the TLE line-of-sight vector providing a corrected TLE pointing vector, $PV_{corrTLE}$, in ECI coordinates:

$$PV_{corrTLE} = \left(A_{ECI->RTN-GPS}\right)^T \cdot LOS_{RTN-TLE}. \tag{13}$$

Note that then the $LOS_{RTN\text{-}GPS}$ coordinates derived from $PV_{corrTLE}$ and the GPS state are numerically the same as the $LOS_{RTN\text{-}TLE}$ coordinates. The theoretical justification for this effective rotation of the pointing vector is that the perturbations that SGP4 ignores and thus are missing from the TLEs have basically the same effect on both spacecraft so the pointing between them in the RTN frame stays basically the same; the practical justification is demonstrated below. The result, how well the GPS frame correction improves the pointing, is depicted in Fig. 3 for one exemplary day. This GPS frame correction of the TLE pointing is performed on board to sufficiently improve the pointing information for the MWI. Nevertheless, this corrected pointing may still not be accurate enough for LRI operations.

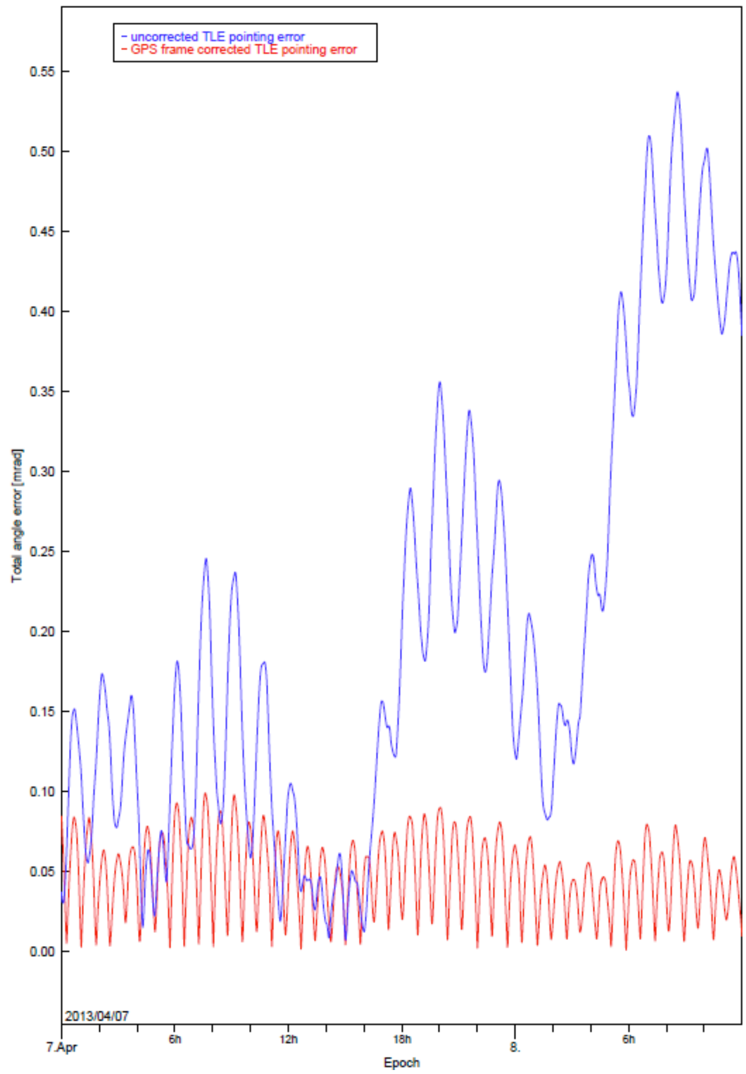

Figure 3. Pointing error of TLE w.r.t. GPS pointing in mrad; blue: uncorrected TLE pointing; red: GPS-frame corrected TLE pointing

3.2 Parametric Fit Model

According to Equations 11 and 12 the pitch and yaw angles are computed from the line-of-sight vector. When we take the differences between the angles computed from frame corrected TLEs (as described above) and those derived from GPS-based orbit determination, we get results such as those depicted in Fig. 4. The differences show a signature which obviously can be modelled by a parametric fit.

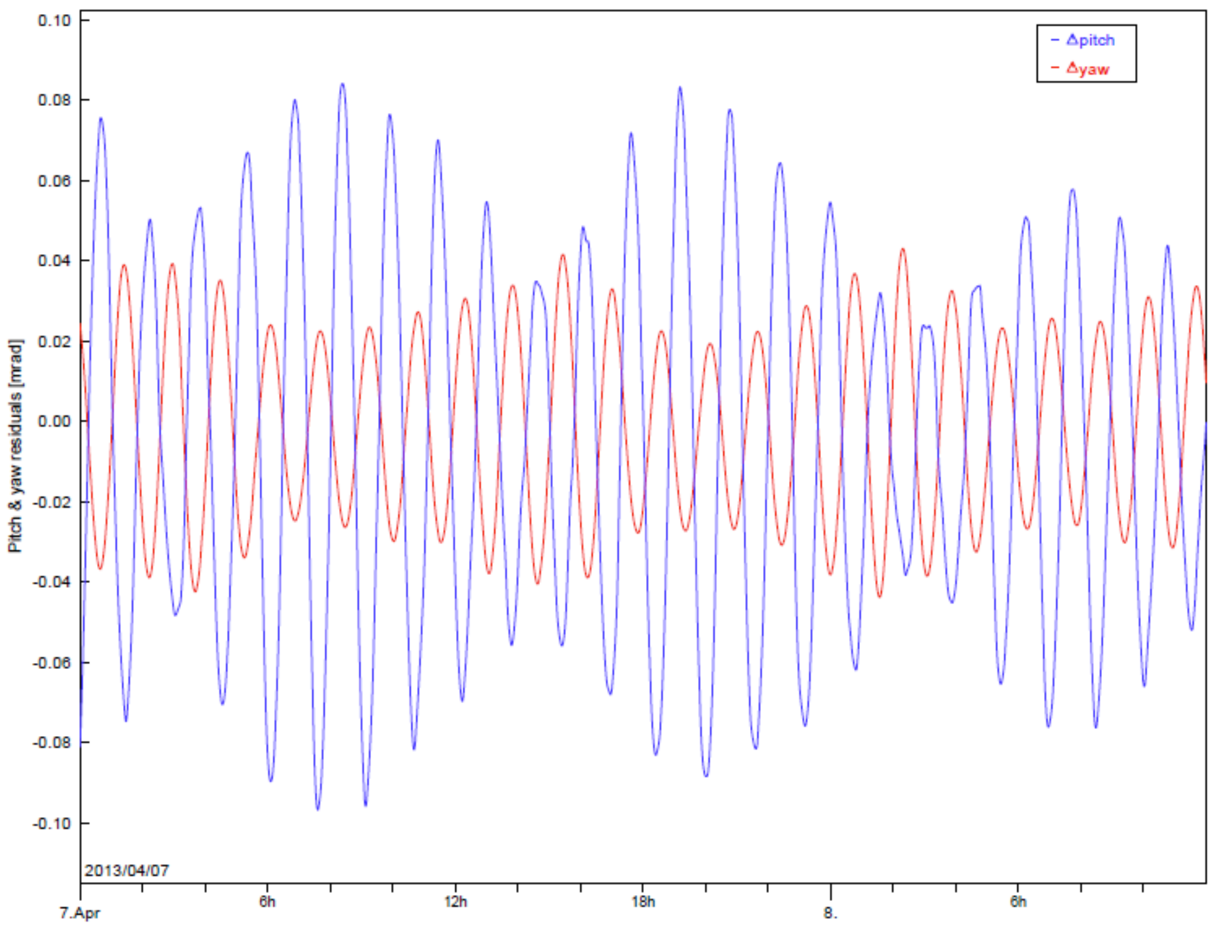

Figure 4. Pitch and yaw angle offsets between predicted pointing based on TLEs and GPS-based orbit determination data in mrad; blue: pitch offsets; red: yaw offsets

It was decided to apply 7-coefficient LRI parametric fits $F_p(t)$ and $F_y(t)$ of the following form for pitch and yaw offsets as shown in Fig. 4:

$$F(t) = \left[1 + a \cdot \sin\left(\frac{2\pi}{T_1}t + \phi_1\right)\right] A \cdot \sin\left(\frac{2\pi}{T_2}t + \phi_2\right) + B, \tag{14}$$

where we define the following LRI Correction Parameters.

- ➢ A –average amplitude of the sinusoid,
- ➢ a – modulation of the sinusoidal amplitude,
- ➢ T1–modulation period, ~ 12 hours,
- ➢ T2–"carrier" sinusoid period, ~ one orbit,
- ➢ ϕ1–modulation phase relative to start time,
- ➢ ϕ2–"carrier" sinusoid phase relative to start time,
- ➢ B –constant bias.

The quality of the LRI parametric fit is evaluated by calculating the residual between the offsets shown in Fig. 4 and the curve fits based on the seven LRI Correction Parameters for each angle. The resultant pointing residuals are computed by:

$$res = \sqrt{\Delta yaw^2 + \Delta pitch^2} \quad (15).$$

The task is to provide proper fit parameters to keep the error below the required threshold of 0.04 mrad with a 95% confidence for a period of 36 hours as depict in Fig. 5.

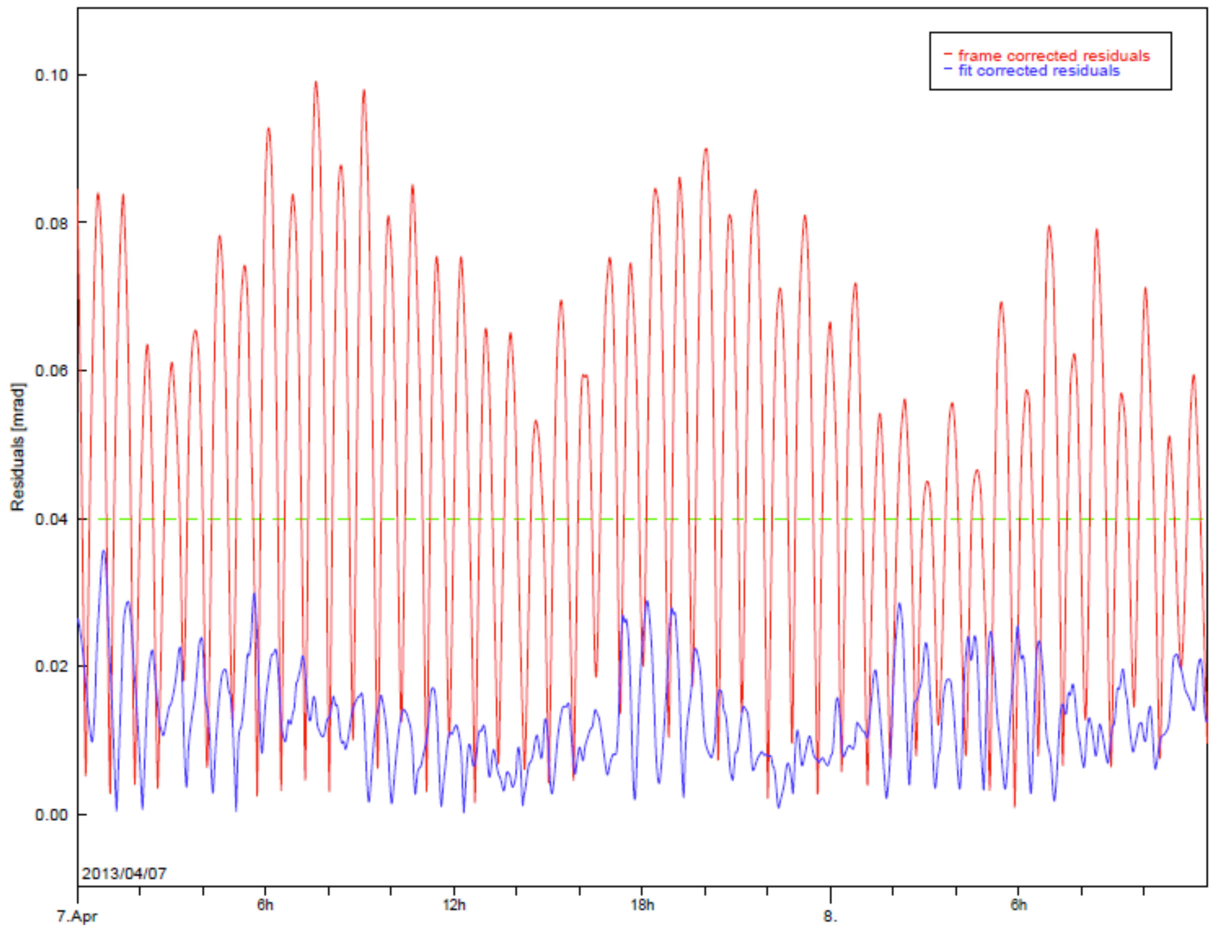

Figure 5. Overall Pointing Residuals in mrad; red: GPS-frame corrected TLE pointing offsets; blue: pointing offsets after LRI Parametric Fit correction; green horizontal line depicts the required limit for LRI operations

3.3 Implementation of Fit Parameter Computation and Analysis Results

The angle offsets (pitch and yaw can be treated separately) between frame-corrected TLE-based orbit and GPS-based orbit serve as references $\Delta pitch_{ref}$ and Δyaw_{ref}. The fit function $F(t)$ as given in Eq. 14 to model these reference offsets depends on 7 LRI Correction Parameters for each angle. The optimization of these Correction Parameters is formulated as a non-linear least-squares problem of minimizing the residuals of the function $F(t)$ and its reference for each of the angles:

$$\left\| F_y(t) - \Delta yaw_{ref} \right\| \overset{!}{=} \min$$
$$\left\| F_p(t) - \Delta pitch_{ref} \right\| \overset{!}{=} \min \quad (16).$$

The numerical solution of least-square problems is treated in various publications (e.g. [12]). The advanced modular Fortran Library for sequential least-squares estimation as described in [13] is taken to determine the Correction Parameters in practice at GSOC's Flight Dynamics department.

To evaluate the quality of the LRI Correction Parameters an independent tool computes the pitch and yaw angles based on frame-corrected TLEs and adds the offset derived from the fit parameters using Eq. 14. The resultant angles are compared to the angles computed from GPS-based orbit determination. The final overall pointing residuals as computed by Eq. 15 are visualized in a plot (cf. Fig. 5) and the statistics are logged.

This evaluation was performed with available GRACE data for a number of various independent and consecutive days, considering seasons with higher solar activities (2005 & 2014), lower solar activities (2008/09), and random days spread over the seasons of 2013. For each day the GRACE GPS data was used for orbit determination and the corresponding TLE was computed, too. Taking the GPS orbits and the TLEs of both GRACE satellites the LRI Correction Parameters were computed and validated as described above. Altogether the pointing errors of both satellites of about 95 'days' (i.e. 36 hour periods) have been analyzed. For many days (about 60 for GF1 and about 70 for GF2) the pointing offset could be kept below the required threshold of 0.04 mrad for the complete period, i.e. a confidence of 100% was reached here. The overall confidence of all days for each spacecraft was about 99.1790% and 99.6063%, respectively. The statistics are summarized in the following Table 3.

Table 3. Confidence Levels of Pointing Errors < 0.04 mrad (in%)

	GRACE-FO1	GRACE-FO2
2005 (16 days)	99.4250	99.1500
2008/09 (27 days)	99.7741	99.7815
2013 (29 days)	98.8759	99.5483
2014 (23 days)	98.6913	99.7913

Under the assumption that the orbit determination of GRACE-FO will be as accurate as for GRACE, the summarized results (based on GRACE data) successfully demonstrate that the approach of GPS-frame correction and parametric fit works very confident to fulfill the requirement on the total pointing accuracy (<0.04 mrad with 95% confidence over 36 hours) needed for LRI operations.

It shall be noted that the requirement refers to a pointing error between TLE-based and regular GPS-based orbit. But sample checks indicate that even against the orbit from precise orbit determination (POD), which is the close to the real orbit, the parametric fit works successfully.

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