A STUDY OF ORBIT ESTIMATION FOR A SPACECRAFT BY USING DIFFERENCED RADIOMETRIC DATA

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Abstract: *An approximate orbital elements (state vector) analytic model for Earth-based range measurements is presented and is used to derive a representative analytic approximation for differenced Doppler measurements. The analytical models are tasked to investigate the ability of these data types to estimate spacecraft geocentric angular motion, the station's clock and frequency offsets, and signal-path calibration errors over a period of a few days, in the presence of systematic station location and transmission media calibration errors. Sensitivity analysis suggest that a few delay calibration errors are the dominant systematic error source in most of the tracking scenarios investigated; as expected, the differenced Doppler data were found to be much more sensitive to some calibration errors than difference range. In this paper, it is described sensitibity analysis for orbit estimation by the analytical model.*

Keywords:*Orbit estimation, Sensitivity analysis, Delta-DOD, Estimator*

1. Introduction

The mathematical models for approximating the differenced range and Doppler measurements were based on the assumption that spacecraft geocentric angular coordinates remained constant over time – a reasonable assumption given that the performance characteristics of these types were investigated for a single tracking pass alone.

In this analysis, the information content of the tracking passes is investigated, with the spacecraft angular coordinates assumed to vary linearly with time. What follows is a detailed derivation of a six-parameter differenced range and Doppler observable model, which is used to assess the performance data types under a variety of tracking scenarios. Despite the fact that realistic navigation operations scenarios are not investigated here, due to the relatively short data arc lengths assumed, the station combination, and the absence of line of sight data such as two-way Doppler or range, the resulting analysis does provide some useful insight into the merit and potential of the differenced data types for navigation purposes. Namely, "VLBI" techniques have some operational advantages over the Delta-VLBI techniques of delta differenced one-way range (ΔDOR) and delta differenced one-way Doppler (ΔDOD) in that differenced data can be acquired without interruption of spacecraft command and telemetry activities – a characteristic that may prove invaluable during periods of the approach of the approach phase preceding planetary encounters or spacecraft maneuvers. Despite the operational shortcomings of ΔDOR and ΔDOD, it must be acknowledged that they are, for the most part, self-calibratioting data types and are therefore less dependent upon accurate externally supplied calibrations of various potential error sources.

2. Observation model

The mathematical models presented here account for effects due to observing platform and transmission media errors on the differenced data types. The angular motion of an interplanetary spacecraft is nearly linear, hence, the angular rate coordinates of the spacecraft are assumed to be constants for this analysis. The differenced range do impact the differential Doppler model; consequently, the observation partial derivatives required for information content and sensitivity analysis become more involved computationally.

2.1. The expressions for the observables

The approximate differenced range observable model is taken to be

$$\Delta\rho \approx \Delta\rho_b + \tau_{ion} + \tau_{tro} + \tau_{clock} \tag{1}$$

where

$\Delta\rho_b$: differenced range term
τ_{ion} : delay due to static ionosphere caribration errors
τ_{tro} : delay due to static troposphere caribration errors
τ_{clock} : delay due to station clock and frequency offset errors

All delay terms are assumed to be in distance units. From this formulation, an approximate differenced range rate observable, proportional to the differenced Doppler observable, follows directly via a time derivative of Eq. (1) yielding

$$\Delta\dot{\rho} \approx \Delta\dot{\rho}_b + \dot{\tau}_{ion} + \dot{\tau}_{tro} + \dot{\tau}_{clock} \tag{2}$$

$\Delta\dot{\rho}_b$: differenced Doppler geometric term
$\dot{\tau}_{ion}$: delay - rate due to static ionosphere caribration errors
$\dot{\tau}_{tro}$: delay - rate due to static troposphere caribration errors
$\dot{\tau}_{clock}$: delay - rate due to station clock and frequency offset errors

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2.2. Differenced range and Doppler models

The differenced range term can be expressed

$$\Delta\rho_b = \mathbf{B} \cdot \left(\frac{\mathbf{r}}{r} \right) = r_B \cos\delta \cos H_B + z_B \sin\delta \tag{3}$$

where

$\mathbf{B}$: Baseline vector
$\mathbf{r}$: Spacecraft geocentric position vector
r_B : Baseline component normal to the spin axis of Earth
z_B : Baseline component parallel to the spin axis of Earth
H_B : Baseline hour angle
α : Spacecraft right ascention
β : Spacecraft declination

The spacecraft angular coordinates are approximated by a Taylor series expansion about some reference epoch t_0

$$\alpha = \alpha_0 + \dot{\alpha}_0 (t - t_0) + \cdots$$

$$\delta = \delta_0 + \dot{\delta}_0 (t - t_0) + \cdots$$

where

Higher order terms are not modeled.

A time derivative of Eq. (3) yields the analytic differenced range rate value

$$\Delta \rho_b = -r_B (\dot{H}_B \cos\delta \cos H_B + \dot{\delta} \sin\delta \cos H_B) + z_B \dot{\delta} \cos\delta \tag{4}$$

where

H_B : time rate of change of the baseline hour angle

2.3. Ionosphere delay and delay rate

A simple model does existing, however, which approximates the behavior of ionosphere delay as a function of elevation for an "average" homogeneous ionosphere.

The ionosphere delay model associated with the differenced range data type is taken to be

$$\tau_{ion} = \left(\frac{C \tau_{z\,ion}}{D + \sin\gamma} \right)_{station1} - \left(\frac{C \tau_{z\,ion}}{D + \sin\gamma} \right)_{station2} \tag{5}$$

where

$\tau_{z\ \ ion}$: Zenith ionosphere delay

γ: Station − spacecraft elevation angle

The ionosphere delay-rate model for the differenced Doppler is arrived at by differentiating Eq. (5) with respect to time, which gives

$$\dot{\tau}_{ion} = \left(\frac{\partial \tau_{ion}}{\partial \gamma} \dot{\gamma} \right)_{station1} - \left(\frac{\partial \tau_{ion}}{\partial \gamma} \dot{\gamma} \right)_{station2} \tag{6a}$$

in which

$$\frac{\partial \tau_{ion}}{\partial \gamma} = \frac{-C \tau_{z\ \ ion} \cos\gamma}{(D + \sin\gamma)^2} \tag{6b}$$

2.4. Troposphere delay and delay rate

A simple troposphere delay model which yields results commensurate with the more complicated empirical models, for elevation angles in excess of about few degrees, is given by

$$\tau_{tro} = \frac{\tau_{z\ \ tro}}{\sin\gamma} \tag{7}$$

where

$\tau_{z\ \ tro}$: Zenith troposphere delay

In Eq. (6), the zenith delay term , is assumed to represent the total troposphere delay. The model used for differenced range measurements, using Eq. (7), is then given by

$$\tau_{tro} = \left(\frac{\tau_{z\ tro}}{\sin\gamma}\right)_{station1} - \left(\frac{\tau_{z\ tro}}{\sin\gamma}\right)_{station2} \tag{8}$$

A useful modification of Eq. (7) is to express sin γ for each station as a function of spacecraft declination and individual station hour angle, which is accomplished by the following relation

$$\sin\gamma = \frac{1}{r_{station}}(r_{sp}\cos\delta\cos H + z_h \sin\delta) \tag{9}$$

where

r_{sp} : Station spin radius

z_h : Station z−hight

H : Station hour angle

Deriving the troposphere delay-rate model simply requires a time derivative of Eq. (9), thereby yielding

$$\dot{\tau}_{tro} = \left(\frac{\partial\tau_{tro}}{\partial\gamma}\dot{\gamma}\right)_{station1} - \left(\frac{\partial\tau_{tro}}{\partial\gamma}\dot{\gamma}\right)_{station2} \tag{10a}$$

in which

$$\frac{\partial\tau_{tro}}{\partial\gamma} = \frac{-\tau_{z\ tro}\cos\gamma}{\sin^2\gamma} \tag{10b}$$

2.5. Clock offset and rate

The station clock offset is modeled as a random ramp, which consists of a random bias term to account for clock offset calibration errors in the ground instrumentation together with station signal path calibration errors, and a rate term representing the frequency offset calibration error between the two tracking stations participating in the three-way link.

$$\tau_{clock} = b_T + f_0(t - t_0) \tag{11}$$

where

b_T : total clock bias

f_0 : frequency offset

Second order effects, such as frequency drift are neglected. It is easy to derive the clock delay rate model virtue of a time derivative of Eq. (11), which yields

$$\dot{\tau}_{clock} = f_0 \tag{12}$$

3. Information content analysis

The partial derivatives of any data type represent, to first order, the ability of that data type to sense changes in a spacecraft trajectory.

The information content of a particular data type is effectively described by the characteristics and behavior of its partial derivatives, and refers to the ability of a data type to determine the various elements that constitute a spacecraft trajectory model.

3.1. Differenced range and Doppler partial derivatives and error analysis

A linear model is assumed for the regression equation expressed by

$$\mathbf{z} = H_x \mathbf{x} + n \tag{13}$$

where

$\mathbf{z} = [z_1, z_2, \cdots, z_N]^T$: vector of N observations

$\mathbf{x} = [\delta_0, \alpha_0, a_{T_1}, a_{T_2}, \cdots a_{T_n(Pass)}, \dot{\delta}_0, \dot{\alpha}_0, f_0]^T$:
vector of parameters to be estimated

$n = [n_1, n_2, \cdots, n_N]^T$: vector of N
independent Gaussian measurement noise

and H_x is the matrix of vector partial derivatives or partials of the observable, at the time of observation, with respect to the estimated parameter set:

$$H_x = \begin{bmatrix} \partial z_1 / \partial \mathbf{x} \\ \partial z_2 / \partial \mathbf{x} \\ \cdots \\ \partial z_N / \partial \mathbf{x} \end{bmatrix}$$

In this analysis, the observation set **z** contains differenced range and Doppler measurements.
For a weighted least-squares estimator, the statistics associated with the estimation error can be readily computed by using the partial derivative matrix, H_x. A weighted least-squares estimate is one that minimizes the weighted sum of squares of the deviations between the actual and predicted measurements expressed by the scalar, quadratic cost function Q, written as

$$Q = \frac{1}{2}[\mathbf{z} - H_x \hat{\mathbf{x}}]^T W [\mathbf{z} - H_x \hat{\mathbf{x}}] \tag{14}$$

in which $\hat{\mathbf{x}}$ is the optimal estimate of the unknown parameter vector $\mathbf{x}$ and W is taken to be a symmetric, positive definite weighting matrix. The case in which $W = \Gamma_n^{-1}$, where Γ_n is the covariance matrix associated with data noise vector n, the estimate $\hat{\mathbf{x}}$ that minimizes Q is the unbiased, minimum-variance estimate of $\mathbf{x}$, and is given by

$$\hat{\mathbf{x}} = \left[H_x^T \Gamma_n^{-1} H_x\right]^{-1} H_x^T \Gamma_n^{-1} \mathbf{z} \tag{15}$$

A priori statistics and regression equation can be combined to derive a modified form of the weighted least-squares estimator, expressed as

$$\hat{\mathbf{x}} = \left[\tilde{J}_x + H_x^T \Gamma_n^{-1} H_x\right]^{-1} H_x^T \Gamma_n^{-1} \mathbf{z} \tag{16}$$

The term $\tilde{J}_x$ denotes the a priori information array and is usually taken to be equal to inverse of $\hat{\Gamma}_x$, the initial covariance matrix for $\mathbf{x}$.

3.1. Error covariance

Recall that the baseline hour angle varies linearly with time and can be expressed as

$$C_B = C_{B_0} + \omega(t - t_0) \tag{17}$$

where

C_{B_0} : epoch baseline hour angle

$\omega t_0 = \alpha_0$

For this study, a symmetric tracking pass was assumed about C_{B_0} from which lower and upper limits on the baseline hour angle were used to accumulate the differenced range and Doppler information array .

The lower and upper baseline hour angle limits, C_{B_t} and C_{B_u} , respectively, were taken to be

$$C_{B_t}, C_{B_u} = C_{B_0} - \Psi, C_{B_0} + \Psi \tag{17}$$

where

Ψ : tracking pass halh − width

No a priori statistics were assumed for the spacecraft angular coordinate parameters to be estimated by the filter. Conversely, a priori information was assumed to be available for the clock bias and frequency offset parameters, based on extrapolations of current the ranging and calibration system capabilities. The a priori information array $\tilde{J}_X$ was thus taken to be

$$J_X = diag\left[0,0,\left(\frac{1}{\sigma_{bT1}}\right),\left(\frac{1}{\sigma_{bT2}}\right),......,\left(\frac{1}{\sigma_{bTn(pass)}}\right),0,0,\left(\frac{1}{\sigma_{f_0}}\right)^2\right] \tag{18}$$

where

σ_{bT1} : one-sigma a priori clock offset uncertainty for the i th tracking pass

σ_{f_0} : one-sigma a priori frequency offset uncertainty

It is well known that unmodeled delays due to clock offset and station signal path calibration error can be a major factor preventing differenced range data from yielding angular precisions comparable to those of ΔDOR data, and the addition of differenced Doppler data will not necessarily help, as they are nearly insensitive to clock offsets. Because the epoch declination estimate is most affected by the uncertainty in the station clock offset, only the improvement in declination precision is shown (see Fig.1).

The results shown in Fig.1 indicate that ability of the differenced range data to determine the clock bias parameters is relatively weak in the near-zero declination regime; this is reflected in the more dramatic improvement seen for the clock synchronization value, as is evident in the same figure, and the lesser improvement seen for the higher declination magnitudes. The inability of the filter to reduce the uncertainty in frequency offset is reflection of its current highly precise calibration value.

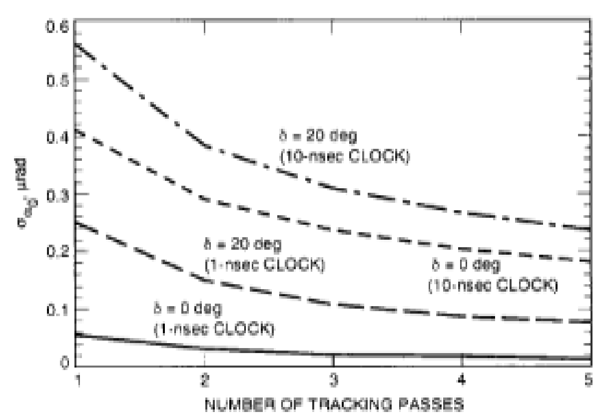

Figure 1. Comparison of USUDA -CANBERRA baseline declination for varying clock bias values

4. Sensitivity analysis

A useful analysis is the sensitivity matrix method, which is frequently used in orbit determination error analyses and provides a means to distinguish among the effects of several different unmodeled systematic error sources on the parameter estimates.
Knowledge of the sensitivity matrix enables one to compute the full-consider error covariance matrix, which accounts for the computed uncertainty due purely to random measurement noise plus the uncertainty induced by unmodeled consider parameters.

5. Results

Estimation statistics for the two stations baseline are summarized in Fig.2, in which identical assumptions on data sampling rate and measurement accuracy characteristics are made as for the USUDA-CANBERRA study, as well as on a priori statistics.
Results suggest that the differenced data types can together deliver about 0.2 to 0.5 μrad precision for the geocentric angular coordinates and about 3×10^{-12} to 40×10^{-12} rad/s precision for the angular rates, at the conclusion of five successive tracking passes.

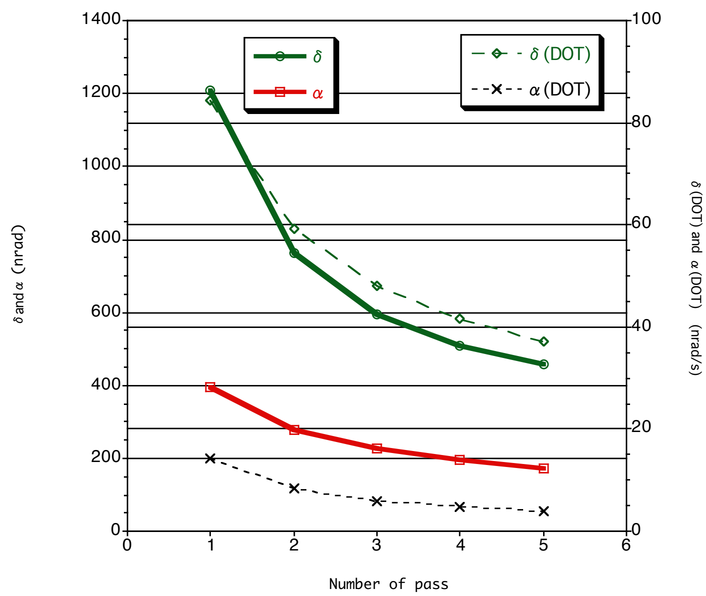

Figure. 2 Differenced Dopp.+Range angular

Clearly, the results in this case are superior for the δ=10 (deg.) case by about factor of three in which the same tracking pass half-width value was assumed. For the 20 (deg.) case, on the other hand, better performance is seen for the USUDA-CANBERRA baseline in terms of being able to determine the epoch declination and measurement biases.
Typical case was run to determine the effects of unmodeled systematic errors on the differenced Doppler and range data.
These results reflect the total error in which the uncertainty due to the consider parameters is

combined with the estimated parameter uncertainty due to measurement noise to better reflect "real world" results.

Here, it is seen that the consider parameter effects can be quite substantial, even after several passes of data have been acquired. Although the estimated clock delay terms are only marginally affected by the unmodeled error sources, the parameters constituting the spacecraft angular motion are more severally impacted.

To provide a reference point for comparison with the differenced range and Doppler results, angular precision and angular rate precision estimates were computed for both ΔDOR and Δ DOD data acquired from single baseline over a period of a few days.

In these calculation, it was assumed that one ΔDOR measurement and Δ DOD measurement acquired simultaneously each day from the USUDA-CANBERRA baseline for five successive days.

The measurement accuracies assumed for these data were 20 cm for ΔDOR and 0.05 mm/s for Δ DOD; these measurement accuracies representative of the performance that can be achieved at X-band frequencies.

The results for five different declination values ranging from -20 (deg.) to 20 (deg.) are given in Fig. 3

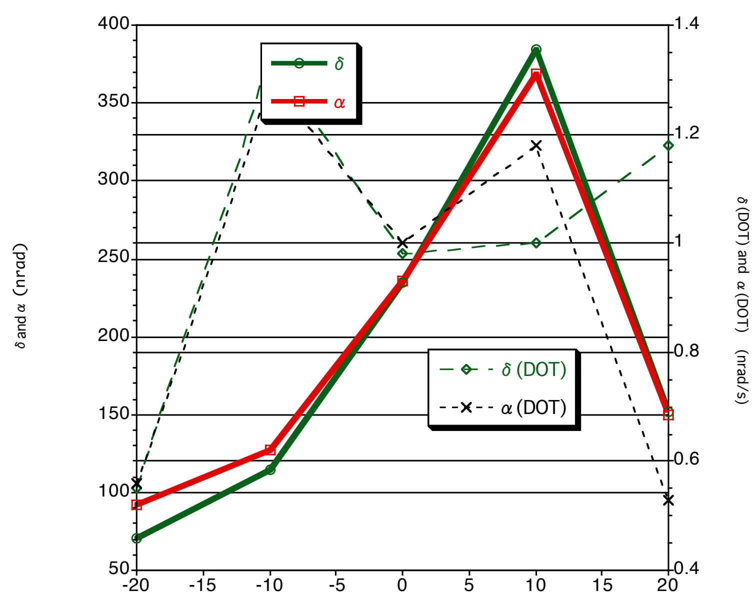

Figure.3 ΔDOR/ΔDOD angular estimation

The baseline hour angle for each pair od ΔDOR/ΔDOD measurements was chosen so that a spacecraft at the specified declination angle would be observed at or near the maximum elevation angle from two stations complexes.

Small departures of up to 10 (deg.) in the baseline hour angle away from this configuration were intentionally made so as to vary the observing geometry some, although no attempt was made to choose the baseline hour angles for each day in such a way as to optimize the results.

6. Conclusions

Error covariance calculations suggested that a few differenced Doppler plus ranging passes were capable of yielding angular position estimates with a precision on the order of 0.1 to 0.4 μrad, and angular rate precision on the order of 3 to 25×10^{-12} rad/s – this in the absence of any a priori statical information on the coordinate parameters.

Results based on sensitivity analysis calculations suggested that the most dominant systematic error source in most the tracking scenarios that were investigated was troposphere zenith delay calibration error.

As expected, the differenced Doppler data were found to be more sensitive to troposphere calibration error than the differenced range data. However, it was also discovered that by raising the elevation cutoff to 15 (deg.) at both stations constituting the baseline, the effect due to troposphere calibration errors were significantly reduced.

For comparison purpose, error covariance calculations were also performed using ΔDOR and ΔDOD data which yielded angular precisions on the order of 0.07 to 0.4μrad, and angular rate precisions on the order of 0.5 to 1.0×10^{-12} rad/s.

7. References

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