# OPTIMUM GUIDANCE STRATEGY FOR RENDEZVOUS MISSION IN EARTH-MOON L2 HALO ORBIT

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Abstract: This study addresses relative dynamic motion of two spacecrafts in Earth-Moon L2 Halo orbit and rendezvous trajectory design strategy in terms of safety. This study is related to design architecture of future space station in cislunar orbits which is being discussed in Global Exploration Roadmap. It is supposed that safety requirements for new cislunar station are similar to those of the International Space Station (ISS). Therefore, approach trajectory must be designed such that the targeted maneuvers and related free drift trajectories do not intercept the space station directly. In low Earth orbits (LEO), this type of requirements can be achieved by offset targeting scheme considering Hill's equation of motion. In EML2 Halo orbit, orbital motion is totally different from that in LEO because effect of three body problem is dominant. Therefore, it is essential to clarify relative orbital motion to establish safe trajectory design strategy. First, stability of orbital motion in EML2 Halo orbit is discussed. Invariant manifolds associated to the halo orbits are evaluated to identify safe injection point of visiting vehicle. Second, characteristics of relative orbital motion are assessed to establish safe approach strategy. It is identified that specific safe region can be determined considering natural free drift motion of two spacecrafts. Third, established strategies are evaluated by means of montecarlo simulation.

Keywords: Earth-Moon L2, Halo Orbit, Rendezvous, Guidance, Optimum Trajectory.

## 1. Introduction

This study is motivated by international discussion regarding design architecture of future space station in cislunar orbits. According to Global Exploration Roadmap, a concept of new space station in the lunar vicinity is proposed as the intermediate step to expand the capabilities needed for future Mars missions [1]. An Earth-Moon L2 (EML2) halo orbit is one of the candidates in which new station will be located.

Rendezvous technology is indispensable to develop and sustain future space station. To develop rendezvous strategy, we must consider unique conditions in cislunar orbits. Because orbital motion is totally different from that in low Earth orbits (LEO), a guidance law must be developed considering dynamics dominated by the features of multibody problem.

Many past studies related to cislunar orbits are available in literature. Those studies are classified to several categories. First category is to analyze a characteristic of orbital mechanics in terms of multibody problem. Discussion related to stability is a major topic in this kind of research. Invariant manifolds associated to Halo orbits are also discussed because it is useful to have spacecrafts transferred between two or more different systems. Second category is trajectory design considering specific mission. Trajectory optimization problem such as transfer orbit

design is a major topic because space missions require minimum fuel consumption or minimum time of flight. On the other hand, rendezvous problem in cislunar orbits are not sufficiently studied as of today. However, heritage and lessons learned of rendezvous mission in LEO can be utilized for cislunar rendezvous missions. Trajectory design concept and associated requirements and constraints can be reused because the essence of rendezvous mission is the same regardless of the orbit. Especially, accumulated experiences of rendezvous mission to the International Space Station (ISS) can be transferred directly to ciclunar rendezvous mission design including its operation concepts.

In this paper, circular restricted three body problem (CRTBP) is utilized to describe orbital motion in cislunar orbit. First, stability of orbital motion in EML2 Halo orbit are discussed. Second, characteristics of relative orbital motion are assessed to establish safe approach strategy. Third, established strategies are evaluated by means of montecarlo simulation.

## 2. Equations of motion

# 2.1. Circular restricted three body problem

This study aims to establish trajectory design strategy, therefore CRTBP is suitable to simulate basic dynamic motion. The CRTBP describes the dynamics of a body with negligible mass under the gravitational influence of two massive bodies. Massive bodies are called the primaries, where the primaries move in circular orbits about their barycenter. Geometry of CRTBP is depicted in Figure 1, where  $m_1, m_2$  are mass of the primaries.



Figure 1. Geometry of CRTBP

Mass parameter and equation of motion are described as follows:

$$\mu = \frac{m_2}{m_1 + m_2} \tag{1}$$

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x \\ \ddot{y} + 2\dot{x} &= \Omega_y \\ \ddot{z} &= \Omega_z \end{aligned} \tag{2}$$

where  $\Omega_x$ ,  $\Omega_y$ ,  $\Omega_z$  denote partial derivatives of the function as follows:

$$\Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1 - \mu)$$

$$r_1^2 = (x + \mu)^2 + y^2 + z^2$$

$$r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2$$
(3)

For fixed  $\boldsymbol{\mu},$  the Jacobi integral is defined as follows:

$$J(x, y, z, \dot{x}, \dot{y}, \dot{z}) = 2\Omega(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$
(4)

For a given energy C, it defines a five-dimensional manifold:

$$F(C) = \{ (x, y, z, \dot{x}, \dot{y}, \dot{z}) \in \mathbf{R}^{6} | J(x, y, z, \dot{x}, \dot{y}, \dot{z}) - C = 0 \}$$
(5)

### 2.2. Relative Motion

Geometry of relative motion in CRTBP is depicted in Figure 2. Relative state vector of chaser and target is defined as follows:

$$\begin{bmatrix} r_{rel} \\ v_{rel} \end{bmatrix} = \begin{bmatrix} r_c \\ v_c \end{bmatrix} - \begin{bmatrix} r_t \\ v_t \end{bmatrix}$$
(6)

where the subscript rel represents relative state, c represents chaser state and t represents target state.



Figure 2. Geometry of relative motion in CRTBP

# 3. Rendezvous trajectory of visiting vehicle to cislunar station

### 3.1. Trajectory design: LEO to EML2 Halo orbit

This study assumes rendezvous mission to cislunar station, for example resupply by unmanned vehicle. The cislunar station is assumed to be located at EML2 Halo orbit whose magnitude Az is 12,000 km. The visiting vehicle departs LEO and it is injected into lunar transfer orbit [2]. Its trajectory is controlled by powered lunar swing-by and small correction maneuvers. Finally, the visiting vehicle is injected into EML2 Halo orbit whose magnitude Az is 10,000 km. The injection point is located at apogee of Halo orbit. The time of flight of this phase is about 8 days. Transfer trajectory is illustrated in Figure 3. Cislunar station orbit and visiting vehicle injection orbit are illustrated in Figure 5 is expressed in the coordinate frame whose origin is target and each axis is parallel to that of Earth-Moon rotating frame.



Figure 3. Transfer trajectory LEO to EML2 Halo orbit



Halo Orbit (Chaser: Az=10,000[km], Target: Az=12,000[km])

Figure 4. EML2 Halo orbits for cislunar station and visiting vehicle injection



Figure 5. Chaser relative state to target

#### 3.2. Trajectory design: HOI to proximity of the station

After injected into EML2 Halo orbit, the visiting vehicle performs relative approach sequence. First, injection error of HOI maneuver is eliminated by correction maneuver. Second, it is injected into approach trajectory which targets proximity communication range. Third, when the vehicle arrived at proximity injection point, it performs maneuver to cancel residual relative velocities. After this sequence, relative navigation using proximity communication comes to be available. Final approach phase follows relative navigation establishment. An example of maneuver plan is described in Table 1. Actually, length of period between two maneuvers need to be determined based on orbit determination performance. In this study, minimum arc of 6 hours is assumed to obtain effective orbit determination solution. A relative trajectory design example is shown in Figure 6.



Figure 6. Relative trajectory design example after HOI

#	1	2	3	4
Maneuver	HOI	TCM4	P_inj	P_stop
Objective	Halo orbit injection	HOI maneuver error	Transit to proximity	Cancel residual
		elimination	injection point	relative velocities
Epoch	HOI (Initial epoch)	HOI + 6 hr	HOI + 12 hr	HOI + 48hr
Target	HOI Point	Reference trajectory	Proximity injection	Proximity injection
			point	point

### Table 1. Maneuver plan after HOI

# 4. Safety Optimum Guidance Strategy

# 4.1. Safe free drift trajectory requirement

A relative trajectory design result described in section 3 is according to safety optimum guidance strategy. For visiting vehicles to the ISS, requirements regarding "safe free drift trajectory" are applied. An example of a requirement for the HTV is "*Prior to AI, the HTV shall target trajectories such that the targeted maneuvers and related free drift trajectories stay outside of the AE for a minimum of 24 hours.*", where *AE (Approach Ellipsoid)* is a region around the ISS for which all pre-AI trajectories must remain outside and *AI (Approach Initiation)* is the start of the maneuver to penetrate the AE by executing a burn with resulting dispersed (3-sigma) trajectory. For this requirement, montecarlo analysis results show that the HTV targeted maneuvers and related free drift trajectories at the start of the AE for a minimum of 24 hours.



Figure 7. HTV's safe free drift trajectory verification

# 4.2. Safe free drift trajectory assessment using invariant manifolds

It is preferable to design safe trajectories such that natural orbital motion has an effect to separate chaser and target. For HTV case, trajectories for far rendezvous phase (e.g. phase adjustment phase) are designed such that altitude of apogee is not higher than ISS orbit. In case of rendezvous trajectories in Halo orbit, orbital motion is more complicated than that in LEO. Invariant manifold is introduced to discuss relative free drift motion of two space crafts. Eq.2 can be transformed into linearized first order equation:

$$\dot{X} = AX, \qquad X = (x, y, z, \dot{x}, \dot{y}, \dot{z}) \in \mathbb{R}^6$$
(7)

where A represents the Jacobian matrix. Let  $\lambda_s$  and  $\lambda_u$  be eigenvalue of A.  $\lambda_s$  is negative real number and  $\lambda_u$  is positive real number. Let  $v_s$  and  $v_u$  be corresponding eigen vector. Stable and unstable manifold associated to specific point can be obtained by propagating perturbation given in the direction of the stable or the unstable eigenvector [3,4].

On the other hand, invariant manifold associated to periodic orbit is obtained by propagating perturbation given in the direction of eigenvector of monodromy matrix. Monodromy matrix is a state transition matrix for periodic orbit. Monodromy matrix is computed by integrating the system as follows:

$$\dot{X} = f(X), \qquad X(0) = X_{ini}, \qquad X = (x, y, z, \dot{x}, \dot{y}, \dot{z}) \in \mathbb{R}^{6}$$
$$\dot{M} = \left[\frac{\partial f}{\partial x}\right] M, \quad M(0) = I, \quad M \in \mathbb{R}^{6 \times 6}, \quad I \in \mathbb{R}^{6 \times 6}$$
(8)

where M is a monodromy matrix, I is a unit matrix and  $X_{ini}$  is a state at which monodromy matrix is computed. State of manifolds is computed by integrating the system as follows:

$$\dot{X}_{m,c} = f(X_{m,c}), \qquad X_{m,c}(0) = X_{c,ini} \pm d\nu_c$$
$$\dot{X}_{m,t} = f(X_{m,t}), \qquad X_{m,t}(0) = X_{t,ini} \pm d\nu_t$$
$$X = (x, y, z, \dot{x}, \dot{y}, \dot{z}) \in \mathbb{R}^6$$
(9)

where the subscript *m* represents manifold state, *c* represents chaser state, *t* represents target state and *ini* represents initial condition at which the state starts propagation. *d* represents the size of the perturbation and v represents eigenvector. Invariant manifolds related to EML2 Halo orbits defined in Figure 4 are shown in Figure 8. At a point in Halo orbit, two stable and two unstable manifolds are generated according to eigenvectors of monodromy matrix.



Figure 8. Invariant manifold related to EML2 Halo Orbit Left: Chaser Halo orbit, Right: Target Halo orbit Purple: Stable manifold, Orange: Unstable manifold

State of manifolds is transformed into relative state as follows:

$$X_{m,c,rel} = X_{m,c} - X_t$$

$$X_{m,t,rel} = X_{m,t} - X_t$$

$$\mathbf{X} = (x, y, z, \dot{x}, \dot{y}, \dot{z}) \in \mathbf{R}^6$$
(10)

where the subscript *rel* represents relative state, *m* represents manifold state, *c* represents target state and *t* represents target state. Relative state of invariant manifolds are shown in Figure 9. Relative state of invariant manifolds is effective to evaluate safety of free drift trajectories. The superposition of unstable manifolds of both chaser and target is shown in Figure 10. Target unstable manifolds. In this case, if size of perturbation is sufficiently small, safety can be achieved because the intersection does not occur immediately after propagation.



Figure 9. Relative state of invariant manifolds Left: Chaser manifolds relative state , Right: Target manifolds relative state Purple: Stable manifold, Orange: Unstable manifold, Blue: Relative chaser state



Figure 10. Superposition of unstable manifolds Light blue: Chaser unstable manifolds, Light green: Target unstable manifolds

#### 4.3. Safe free drift trajectory assessment for proximity injection point

Assessment using invariant manifolds is effective to clarify long term orbital motion. Proximity injection point is close to the station, therefore analysis to clarify short term orbital motion is important as well. Because of complicated orbital mechanics it is expected that relative orbital motion has variation according to relative position and location in Halo orbit. Now let define specific location. Location (A) is apogee side of Halo orbit, (B) and (D) are located where magnitude of y is maximum, and (C) is perigee side of Halo orbit. Orbital propagation is performed with initial condition such that chaser is located at specific position and its relative velocity to target is zero. Initial relative position is defined inside a square in X-Y plane whose size is 400km and center of which is origin of the frame. Analysis result is shown in Figure 11. It is identified that free drift trajectories are sensitive to both initial relative position and location in Halo orbit. This is because potential described in Eq.3 is different according to location that leads to variation of eigenvalues and direction of eigenvectors. Some trajectories directly hit the target and some trajectories initiated in specific region never approach the target. Therefore, safe region can be identified by this analysis in terms of safe free drift trajectory.



Figure 11. Relative free drift trajectories from target proximity

### 5. Verification by Montecarlo Simulation

## 5.1. Free drift trajectory after HOI maneuver

Montecarlo simulation is performed to assess trajectory safety. The simulation model includes CRTBP orbital propagation, navigation error, maneuver execution error and targeting logic. Model definition applied to this study is described in Table 2.

Orbit propagation	CRTBP	
Initial state error at HOI	Chaser: $5[\text{km}]$ , $5[\text{cm/s}]$ ( $3\sigma$ ) per axis	
	Target: 5[km], 5[cm/s] ( $3\sigma$ ) per axis	
Navigation Error	Chaser: $5[km]$ , $5[cm/s]$ ( $3\sigma$ ) per axis	
	Target: 5[km], 5[cm/s] ( $3\sigma$ ) per axis	
Guidance	Fixed time arrival by linearized state transition matrix	
Maneuver	Impulsive maneuver	
	Execution error: 1.0% (3 $\sigma$ ) in magnitude, 0.5° (3 $\sigma$ ) in pointing	
Simulation time	14 days after HOI	

Table 2. Model definition for montecarlo simu	lation
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Simulation result of free drift trajectory after HOI maneuver is shown in Figure 12. It is verified that minimum clearance between chaser distributed trajectory and target without distribution is about 300 km. It is identified that free drift trajectories are along chaser unstable manifolds which do not intersect the target. Based on this result, the strategy to have the chaser injected into a Halo orbit whose size is different from that of the target is effective to achieve safe free drift trajectory requirement. Since ditributed orbit is sensitive to error condition, size of Halo orbits for chaser and target must be chosen carefully.



Figure 12. Montcarlo simulation result (Free drift after HOI) Left: Relative range, Right: Orbit distribution (Light blue: Chaser trajectories, Light green: Target trajectories)

#### 5.2. Free drift trajectory after P\_stop maneuver

In this scenario, chaser performs all maneuvers defined in Table 1, then free drift orbit propagation is performed for remaining period. According to Table 1, time of flight from HOI to P\_stop maneuver is 2 days, therefore 12 days propagation is performed. First, safe region at P\_stop maneuver epoch is assessed. As discussed in section 4.3, chaser free drift trajectory map is obtained, and it is compared to target free drift trajectory map. Both free drift trajectories are shown in Figure 13. Safe region is identified around the point (x,y,z) = (0, 200, 0) [km]. On the other hand, it is identified that propagated trajectories from the point about (x,y,z) = (67, 100, 0) [km] directly approach the target. Montecarlo simulation result to which safe region is applied as target position of P\_inj maneuver is shown in Figure 14. Free drift trajectories of both chaser and target do not intersect each other, therefore the target position of P\_inj maneuver is appropriate in terms of safe free drift trajectory requirement.



Figure 13. Free drift trajectory map Left: Chaser free drift trajectory map, Right: Target trajectory distribution



Figure 14. Safe free drift trajectory after P\_stop maneuver Light blue: Chaser trajectories, Light green: Target trajectory

Unsafe condition is also tested to show this strategy is effective. For this simulation the point (x,y,z) = (67, 100, 0) [km] is selected as P\_inj maneuver target position. The result is shown in Figure 15. It is confirmed that free drift trajectories from "not safe" region may violate safety requirement.



Figure 15. Free drift trajectory after P\_stop maneuver (Unsafe case) Light blue: Chaser trajectories, Light green: Target trajectory

## 6. Conclusions

This study addresses relative dynamic motion of two spacecrafts in EML2 Halo orbit and rendezvous trajectory design strategy in terms of safety. First, trajectory design for rendezvous mission in EML2 Halo orbit is introduced. Second, safe free drift trajectory is assessed using a feature of invariant manifolds. It is concluded that relative state of invariant manifolds is effective to evaluate safety of free drift trajectories. Third, safe region for proximity injection point is discussed. It is identified that safety of free drift trajectories are sensitive to both initial relative position and location in Halo orbit. It is concluded that "free drift trajectory map" is effective to specify the safe region. For future work, it is planned to assess rendezvous trajectory in other EML orbits, such as Distant Retrograde Orbit (DRO) or Near Rectilinear Orbit (NRO) which are also candidates for future cislunar station. It is expected that optimum trajectories are identified by the methodology discussed in this paper.

### 7. References

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