RAPID GEOSYNCHRONOUS TRANSFER ORBIT ASCENT PLAN GENERATION

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Abstract: *Geosynchronous Transfer Orbits require several maneuvers conducted at selected subsequent apogees in a manner that bring spacecraft to final target longitude and drift rate relative to a geosynchronous orbit. These maneuvers are designed to occur at longitudes free from radio frequency interference with operational spacecraft. In addition they are optimized to minimize propellant consumption. Finally, one must also account for cancellation of any single burn. Therefore one must also design backup plans also clear of radio frequency interference. This paper presents a methodology to rapidly construct a wide range of available alternatives addressing these constraints. The methodology uses several kinds of optimizing targeting algorithms, combining analytic equations to minimize propellant consumption for each maneuver while at the same time using techniques of differential correction to target relevant orbital products such as burn longitude and argument of latitude.*

Keywords: *Geosynchronous Transfer Orbit, Maneuver Design*

1. Introduction

Geosynchronous transfer orbit (GTO) starts with launch vehicle injection into an orbit with low perigee and apogee near geosynchronous orbit altitude. The GTO maneuver sequence is comprised of several perigee-raising maneuvers that bring the spacecraft into near geosynchronous final orbit. While the location of these burns occurs in same approximate location in inertial space, the location within the belt of spacecraft in geosynchronous orbit changes from burn to burn. The sequence of maneuvers is chosen such that the final GTO maneuver takes place a few degrees away from target geo belt longitude, with an orbit at the conclusion of that last maneuver that is nearly geosynchronous, of near zero inclination, but with a small longitude drift rate that carries the spacecraft toward the target longitude.

Typically, injection orbit apogee is safely below geosynchronous altitude, as a result collision is not a driving concern in mission design. Radio frequency interference however is a principal consideration. Open zones are ranges of longitudes within the geo belt where the spacecraft do not share the same uplink and down link frequencies as beacons your spacecraft uses during GTO.

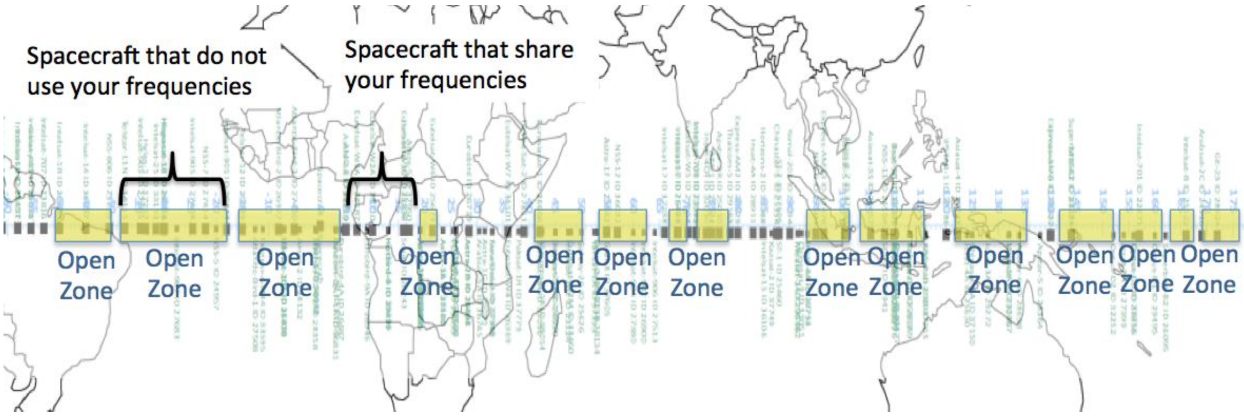

Figure 1. Open longitude zones among spacecraft that do not share your frequencies

Figure 1 illustrates the typical situation. The geosynchronous belt is populated with more than 400 operational spacecraft, shown here as black boxes labeled in green with the spacecraft name. There are usually 5 to 10 good size zones, marked here in yellow, around the geo belt where communications with your spacecraft uses frequencies different than the geosynchronous spacecraft parked there. Conducting maneuvers in these zones means that one can communicate with the spacecraft during preparation and monitoring without concern of signals interfering with other operational spacecraft.

This paper presents the means to quickly access the widest possible range of ascent strategies that will meet the constraints of: low propellant consumption; radio frequency interference avoidance and; availability of reasonable backup plans.

3. Analytic Targeting of the Next Maneuver Longitude

GTO injection orbits are elliptical, with apogee located near the ascending or descending node and perhaps 50 or 100 km below the geosynchronous belt. Perigee is a few hundred kilometers altitude. The inclination suited the launch site, and can be from a few degrees to perhaps 50 degrees.

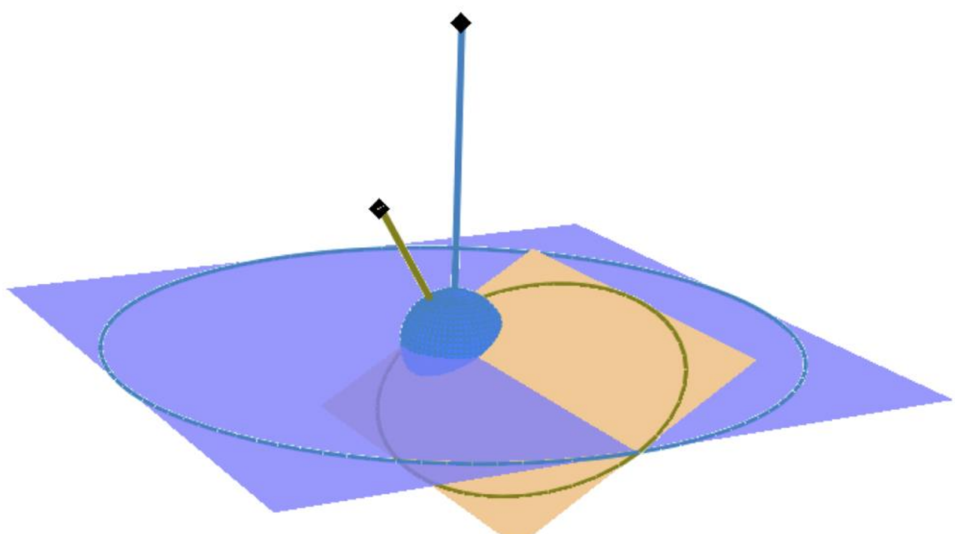

Figure 2. GTO Injection Orbit

These factors simplify GTO maneuver design. First, maneuvers are designed to keep line of apsides near the line of nodes, but raises perigee. The choice in post-maneuver semi-major axis must be made so that the Earth longitude of the next maneuver apogee will fall into an open zone in the belt. Given some initial longitude of the present apogee, the geometry required to arrive at a given longitude at a subsequent apogee is that illustrated in Fig. 3.

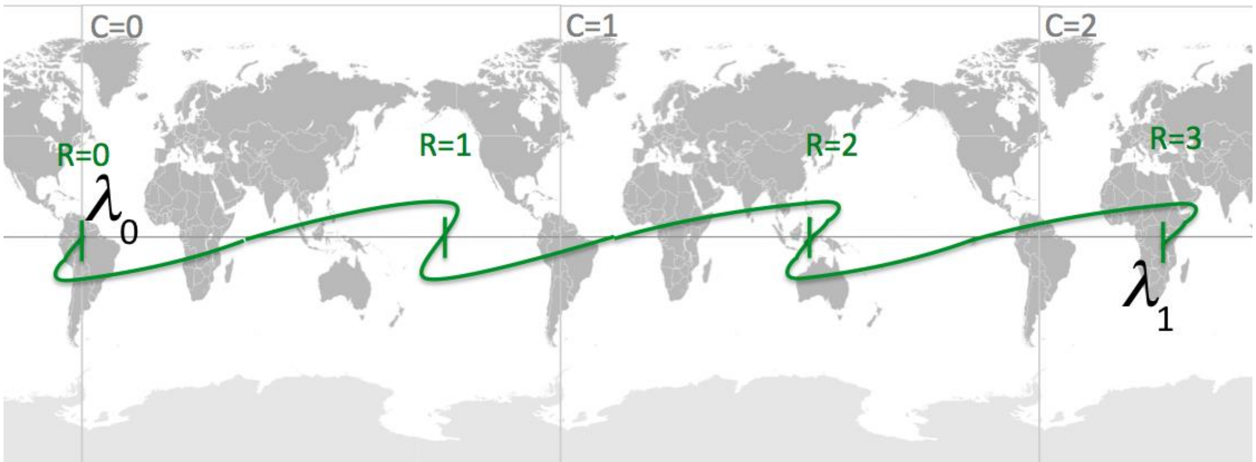

Figure 3. Arriving at a target longitude from a given starting longitude

Where

λ_0 = Earth longitude of present maneuver

λ_1 = Intended Earth longitude for the next maneuver

R = Intended number of orbit revolutions until the next maneuver

C = Intended number of complete circuits of the Earth until the next maneuver

Equation 1 presents an analytic estimate of the required longitude change needed to travel from λ_0 to λ_1

$$\Delta\lambda_{rev} = \frac{(\lambda_1 - \lambda_0) + 2\pi C}{R} \tag{1}$$

Equation 2 is the semi-major axis of the corresponding orbit

$$a = a_{geo}\left(1 - \frac{\Delta\lambda_{rev}}{2\pi}\right)^{2/3} \tag{2}$$

Where

a_{geo} = Semi-major axis of an ideal geosynchronous orbit

2. Analytic Minimal Prop Consumption

Orbit inclination at separation is one suited to the launch site and launch vehicle. While the inclination at the conclusion of ascent will be near zero degrees, the inclination change in each burn must be tailored to match the chosen semi-major axis. When the line of nodes and line of apsides are collinear, the location of each maneuver will be perpendicular to the plane containing initial angular momentum L_{inj} and final orbit angular momentum L_{final}. Furthermore, each maneuver rotates the orientation of the angular momentum vector to a new value L_{mid} by some angle Dr_{mid} in this plane. This is illustrated in Fig. 4.

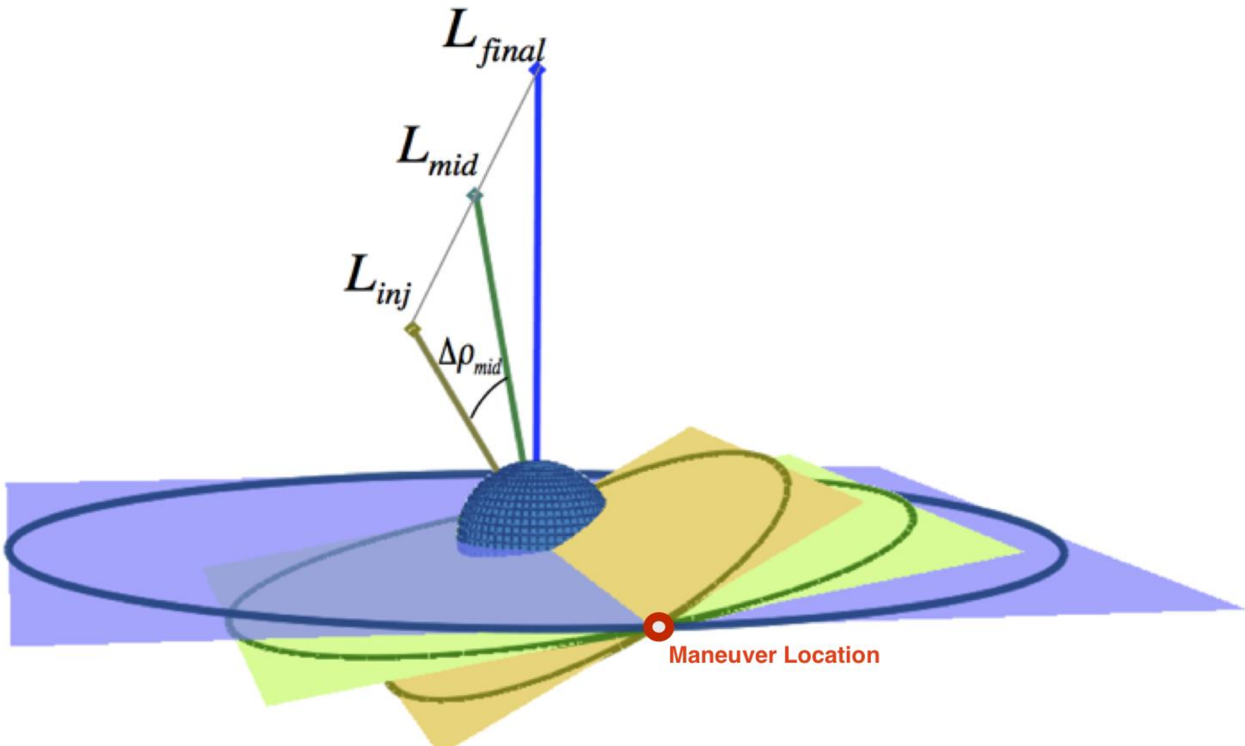

Figure 4. Orbital Angular Momentum Vector Evolution is Perpendicular to the Maneuver Location

In this case, Fig. 4 shows how propellant consumption can be minimized. The orbital angular momentum, L_{mid}, of the intermediate orbits must fall along the line between the initial and final orbital angular momentum.

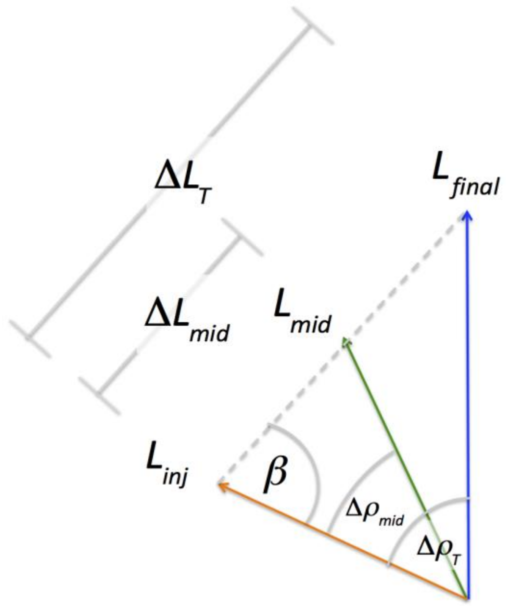

Figure 5. Intermediate Orbit Angular Momentum Vector Planning

Where

L_{inj} = Orbital angular momentum vector at injection
L_{final} = Intended Angular momentum vector at the conclusion of GTO
L_{mid} = Angular momentum vector during an orbit midway from injection to final
ΔL_T = Total required change in the angular momentum
$\Delta\rho_T$ = Total required rotation angle of the angular momentum vector
$\Delta\rho_{mid}$ = Angular change in the angular momentum between initial and intermediate orbit
β = Angle between initial angular momentum vector and total required change

In the analytic approximation, the size of the angle β in Fig. 4 is given by

$$\cos\beta = \frac{L_{final}^2 - L_{inj}^2 + \Delta L_T^2}{2L_{inj}\Delta L_T} \tag{3}$$

The magnitude of the change in angular momentum from initial to intermediate orbit has two roots:

$$\Delta L_{mid} = L_{inj}\cos B \pm \frac{1}{2}\sqrt{\left(2L_{inj}\cos\beta\right)^2 - 4\left(L_{inj}^2 - L_{mid}^2\right)} \tag{4}$$

The required change in the orientation of the angular momentum vector is:

$$\cos\Delta\rho_{mid} = \frac{-\Delta L_{mid}^2 + L_{inj}^2 + L_{mid}^2}{2L_{inj}L_{mid}} \tag{5}$$

2. Finite Burn Targeting Algorithms

The analytic approximations in the previous sections have the advantage of estimating a maneuver close to the needed duration. However integration of acceleration over finite arc introduces inefficiencies, and subsequent coast in the presence of small orbit perturbations are such that next target longitude will not be quite that intended. The approximation is close enough however that a simple differential will arrive at the exact maneuver size to arrive at the next target longitude.

Furthermore, the analytic equations assume that the line of nodes is collinear with the line of apsides. In practice they differ by a few degrees around the orbit arc. Maneuvers are typically executed with the mid-point centered between node and apogee, so the actual process must provide the user some freedom in selection of exact midpoint location.

Putting these ideas together, there are several versions of maneuver targeting to consider:

1. Drift Longitude Targeting
    a. After separation, coast a selected range of orbit revolutions to a chosen true anomaly or argument of latitude
    b. Keep only those permutations of orbit revolutions that land within an open zone

2. Longitude Targeting Maneuver
    a. Starting at a given longitude
    b. Specify a selection of longitude targets, acceptable permutations of Earth circuits and orbit revolutions and target true anomaly or argument of latitude
    c. For each permutation of longitude, Earth circuit and orbit revolution, compute first the analytic and then the differentially corrected semi-major axis.
    d. In each case compute the corresponding value of Δr_{mid} that optimizes the angular momentum evolution
    e. Keep only those solutions for which the maneuver size is suitable

3. Drift Rate Targeting Maneuver
    a. Starting at a given longitude, usually the intended longitude at the conclusion of GTO.
    b. Specify the target longitude drift rate at the conclusion of the maneuver
    c. Compute first the analytic and then differentially corrected semi-major axis that achieves the target drift rate
    d. Compute the corresponding value of Δr_{mid} that optimizes the angular momentum evolution.

4. Post GTO-Drift Target Longitude Maneuver
    a. Similar to the Drift Rate Targeting case, but specifying the intended longitude at the conclusion of post-GTO drift and the number of orbit revolutions allotted to arrive there.

3. Establishing a Range of Maneuver Sizes Using Dual Path Targeting

Rather than targeting single longitudes, it is better to determine the range of maneuver sizes required to travel from the extremes of starting and ending zones. This has the advantage of bracketing the full range of smallest to largest burn sizes needed to travel to any point in that zone.

The smallest burn will be that needed to travel from the western edge of the present zone and arrive at the eastern edge of the next zone. The largest maneuver travels from the present eastern to the next western edge. This is illustrated in Fig. 6.

Figure 6. Maneuver size vs. Zone Targeting

4. Maneuver Strategies to Travel from Zone to Zone

Taken together, one can devise several useful strategies for transfer from one open zone to another.

1. Coast and check if arrived in an open zone
    a. Start in a given initial longitude zone
    b. Specify the allowable set of target open zones
    c. Specify the allowable range of orbit revs
    d. Apply <u>Drift Longitude Targeting</u> to western and eastern edge of initial zone
    e. Keep those results where evolution of both edges land in a target zones

2. Maneuver and coast to next target open zone
    a. Start in a given initial longitude zone
    b. Specify engine performance characteristics and restrictions on burn duration
    c. Specify the allowable set of target open zones
    d. Specify the allowable range of orbit revs and Earth circuits
    e. Establish minimum burn duration by applying the <u>Longitude Targeting Maneuver</u> using each permutation of revs, circuits and target zone, starting from the western edge of the initial zone and travelling to the eastern edge of the target zone
    f. Establish maximum burn duration using the initial east edge, travelling to the target west edge.

g. Keep those cases whose maneuver sizes are suitable

3. Maneuver and target drift rate
    a. Start in a given initial longitude zone
    b. Specify engine performance characteristics and restrictions on burn duration
    c. Specify target post-maneuver drift rate
    d. Use the <u>Drift Rate Targeting Maneuver</u> to maneuver size needed for the starting conditions of either edge of the initial longitude zone. Because previous maneuvers were different, the initial conditions at each edge will be different, so one will arrive at minimum and maximum maneuver durations.

These strategies can be toggled together according to the specifics of the mission at hand, such as is shown in Fig. 7. When applied to an initial injection state, one obtains a tree of options as is illustrated in Fig. 8.

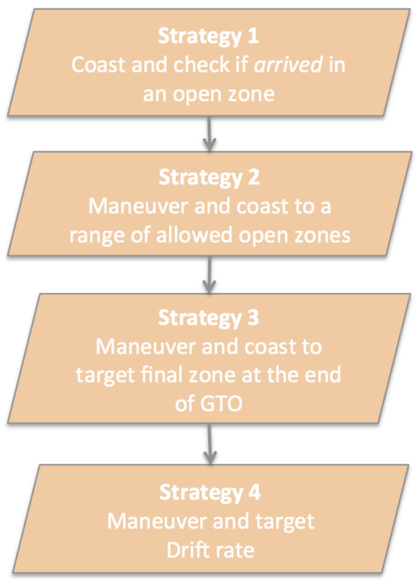

Figure 7. Sample Strategy Tree

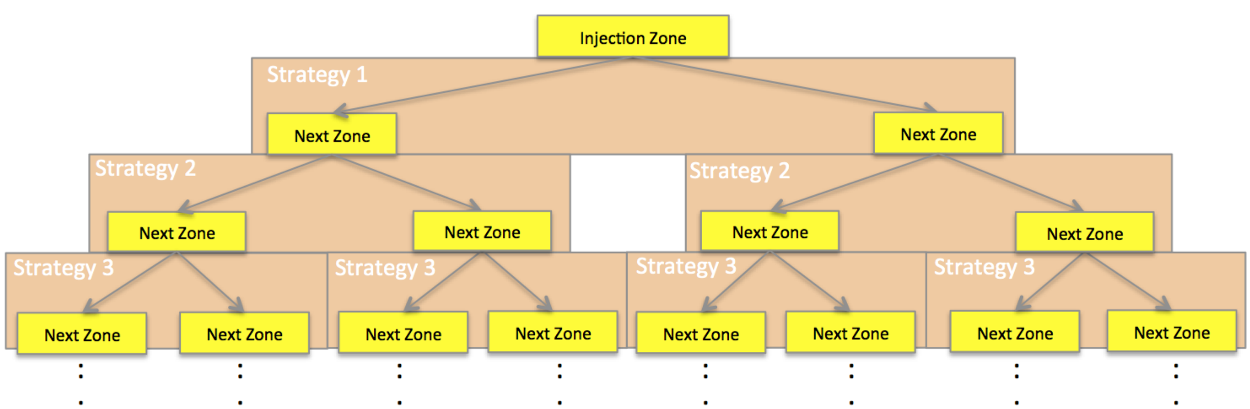

Figure 8. Applying a Strategy Tree to an Initial Injection State

5. Sample Case: Nominal Ascent

The overall intent is to provide the designer with tools that can be toggled together that achieve ascent tailor to the particular mission circumstances. Lets look at a concrete example to see how this works. Consider injection into the following state at injection:

Table 1. Sample Injection State

Parameter	West State	East State
Epoch	2015-04-01 22:30:00	2015-04-01 22:30:00
Orbit Number	0	0
Semi-Major Axis (km)	24468.63700	24468.63700
Eccentricity	00.7291170	00.7291170
Inclination (deg)	06.0000000	06.0000000
Right Ascension of the Ascending Node (deg)	-11.6394923	-11.6394923
Argument of Perigee (deg)	178.0000000	178.0000000
True Anomaly (deg)	-00.0000000	-00.0000000
Longitude (deg E)	358.9111063	358.9111063
Drift Rate (deg/rev)	200.8519138	200.8519138
Spacecraft Wet Mass (kg)	5947.0	5947.0

We will design a three-burn ascent sequence. Naturally one can also use these same tools to build a 4-burn sequence that results smaller maneuver sizes, but this well illustrates the technique.

1. INJECTION TO FIRST MANEUVER – NOMINAL PLAN
    a. Coast for 2 or 3 orbit revolutions.
    b. Stop at an argument of latitude = 0°
    c. Allow any range of longitudes.

2. FIRST MANEUVER AND COAST TO SECOND – NOMINAL PLAN
    a. Thrust = 492 N, I_{sp} = 321 s, min burn duration = 1000 s, max = 7000 s
    b. Coast for 2, 3 or 4 orbit revolutions.
    c. Stop at argument of latitude of 0°
    d. Allow for 1 or 2 complete circuits of the Earth during coast.
    e. Size the first burn to arrive at one of the following open zones.

Table 2. Sample Target Zones for the First Maneuver

Name	West Edge (deg E)	East Edge (deg E)
Zone 1	52.5	75.5
Zone 2	145.0	155.5
Zone 3	157.0	165.0
Zone 4	8.0	38.0
Zone 5	323.5	343.0
Zone 6	99.0	118.5
Zone 7	126.0	141.0
Zone 8	307.5	316.0
Zone 9	26.5	35.0
Zone 10	17.0	25.0
Zone 11	173.0	180.0
Zone 12	40.0	46.5
Zone 13	346.0	352.0

3. SECOND MANEUVER AND COAST TO FINAL – NOMINAL PLAN
    a. Thrust = 492 N, I_{sp} = 321 s, min burn duration = 1000 s, max = 7000 s
    b. Coast for 2 or 3 orbit revolutions.
    c. Stop at argument of latitude = 0°
    d. Allow for 1 or 2 complete circuits of the Earth during coast.
    e. Size the second burn to arrive at the following open zone.

Table 3. Sample Target Zone for the Second Maneuver

Name	West Edge (deg E)	East Edge (deg E)
Final Target Zone	355.0	5.0

4. THIRD MANEUVER – NOMINAL PLAN
    a. Thrust = 492, I_{sp} = 321, min burn duration = 1000 sec, max = 7000 sec.
    b. Size the maneuver so the subsequent coast arrives at 1.6° East after 4.5 revs.

The result is a tree of options to travel from initial injection to final GTO longitude is presented in Fig. 9. The corresponding details of one of the branches of this plan are presented in Tab. 4.

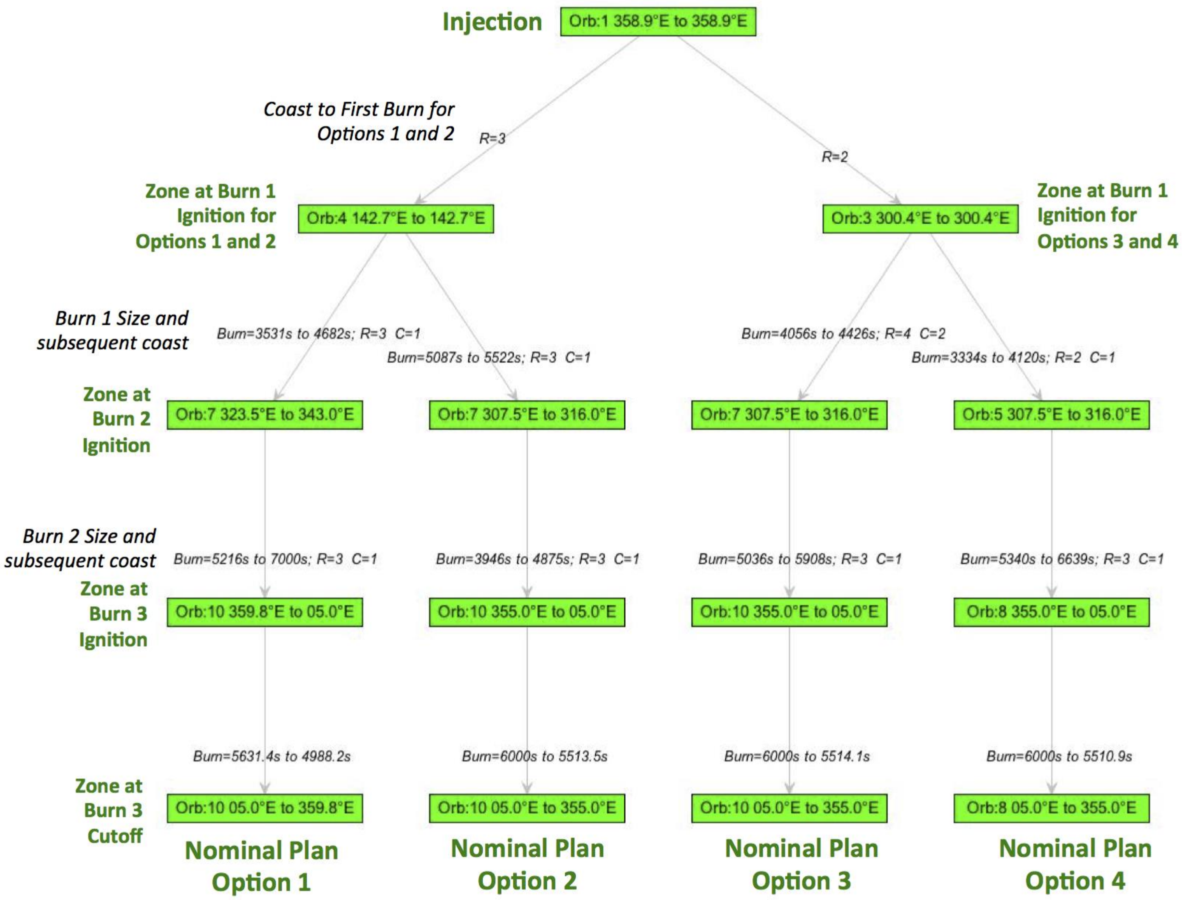

Figure 9. Sample Nominal Plan Options

Table 4. Sample Detailed Ascent Plan

Start Point

	West State	East State
Epoch	2015-04-01 22:30:00.000	2015-04-01 22:30:00.000
Orbit Number	1	1
SMA (km)	24468.6370000	24468.6370000
ECC	00.7291170	00.7291170
INC (deg)	06.0000000	06.0000000
RAAN (deg)	-11.6394923	-11.6394923
AOP (deg)	178.0000000	178.0000000
TA (deg)	-00.0000000	-00.0000000
LON (deg E)	358.9111063	358.9111063
Drift(deg/rev)	200.8519138	200.8519138

Ascent Plan

Start at Orbit Number=1 Longitude=358.9°E to 358.9°E
Propagate 2 orbits to Argument of Latitude=0°
Arrive at Orbit Number=3 Longitude=300.4°E to 300.4°E

Execute Burn: Duration=3333.7s to 4120s;
RA=82° to 82° DEC=-6.4° to -6.4° Thrust=456.9N Isp=321.8s
Propagate to 2 counts(s) of ArgLat=0°
Arrive at Orbit Number=5 Longitude=307.5°E to 316.0°E

Execute Burn: Duration=5340.4s to 6638.6s;
RA=81.3° to 81.1° DEC=-6.5° to -6.5° Thrust=456.9N Isp=321.8s
Propagate to 3 counts(s) of ArgLat=0°
Arrive at Orbit Number=8 Longitude=355.0°E to 05.0°E

Execute Burn: Duration=6000s to 5510.9s;
RA=80.4° to 80.8° DEC=-6.5° to -6.5° Thrust=456.9N Isp=321.8s
Arrive at Orbit Number=8 Longitude=05.0°E to 355.0°E

Finish Point

	West State	East State
Epoch	2015-04-05 10:46:41.114	2015-04-05 11:21:37.245
Orbit Number	8	8
SMA (km)	41931.2787950	41971.3746436
ECC	00.0596226	00.0564158
INC (deg)	00.0312628	00.0262570
RAAN (deg)	-32.3899289	-31.3581432
AOP (deg)	117.7429536	115.7959726
TA (deg)	-85.2225814	-85.5553648
LON (deg E)	05.0351629	355.0293944
Drift(deg/rev)	02.9788746	02.4666620

Note that many of the possible intermediate open zones contemplated in Tab. 2 for the second maneuver were thrown out. Suitable maneuver sizes were found for only two of the zones. However, we still have several good GTO options to choose from. Better still, in each option, there is a wide range of available maneuver durations at our disposal. This will be particularly handy when considering backup plans, each of which whittle away at the range of suitable maneuver durations.

6. Back up Planning

In the event that a maneuver in the nominal sequence must be cancelled, there should be suitable backup options. Think about it in terms of size of a given maneuver in the nominal plan. Of course the size must be right to reach the next nominal open zone. However, that same burn must also be suitable to arrive within a good backup open zone.

Figure 10. Nominal and Backup Zones

The method adopted here is to determine the range of burn sizes that arrive at the extremes of given nominal target zone, and then trim that range if necessary to arrive within the bounds of one of the selected backup zones.

Furthermore, having arrived at the backup zone, one must be a replacement plan that starts at that point and tailored to arrive at target final longitude. Altogether, the backup planning is as follows:

1. Coast and check if arrived in an open zone – Cancellation Recovery Plan
    a. Specify the choice of acceptable backup open zones.
    b. Specify the range of additional orbit revs to coast to an open zone.
    c. Propagate trajectories from each starting point.
    d. Determine which permutation of orbit revs arrives at one of the specified backup zones
    e. Throw out the nominal plan if no suitable backup can be found

2. Maneuver and coast to next target open zone – Cancellation Recovery Plan
    a. Specify the choice of acceptable backup open zones.
    b. Specify the range of additional orbit revs and Earth circuits to coast to one of these zones.
    c. Start with the range of maneuvers sizes defined for the nominal burn.
    d. For each permutation of Earth circuit and orbit revs determine whether one can define a suitable range of maneuver sizes that fit within the nominal range that will allow the spacecraft to coast to one of the backup open zones.
    e. If necessary trim down the range of the nominal maneuver to fit within the longitude range of the backup zone.

3. Maneuver and target drift rate – Backup Plan
    a. Start in a given initial longitude zone.
    b. Specify engine performance characteristics and restrictions on burn duration.
    c. Specify target post-maneuver drift rate.

Having considered the recovery plan, then in each case we also need a subsequent ascent plan to carry the spacecraft from the backup open zone to the final GTO longitude.

7. Sample Backup Plan

Consider the same three-burn plan as before, but allow for cancellations of individual maneuvers as follows:

1. FIRST BURN CANCELLATION RECOVERY PLAN
    a. Coast an additional 2 or 3 revs and stop at the same argument of latitude.
    b. Allow any range of final longitudes.
    c. Use the First Maneuver Cancellation Recovery Sequence described below to complete ascent.

2. SECOND BURN CANCELATION RECOVERY PLAN
    a. Coast an additional 2 or 3 revs and stop at the same argument of latitude.
    b. Allow for 1 or 2 complete circuits of the Earth during coast.
    c. Set the allowable backup open zones to those in Tab. 2.
    d. Trim the range of allowed nominal maneuver sizes so as to also fit into the backup zone.
    e. Use the Second Maneuver Cancellation Recovery Sequence described below to complete ascent.

3. THIRD BURN CANCELATION RECOVERY PLAN
    a. Coast an additional 2 or 3 revs and stop at the same argument of latitude.
    b. Allow for 4 or 5 complete circuits of the Earth during coast.
    c. Set the allowable backup open zones to those in Tab. 2.
    d. Trim the range of allowed nominal maneuver sizes so as to also fit into the backup zone.
    e. Use the Third Maneuver Recovery Sequence described below to complete ascent.

For each of these cancellation cases, we also need a planning sequence to carry us to final GTO longitude. In this example, we can re-use most of the elements of the nominal plan. In practice, one can re-design as needed. In this example, we adopt a different recovery for third burn cancellation:

1. FIRST MANEUVER CANCELLATION RECOVERY SEQUENCE
    a. Maneuver 1: Same as the 1st maneuver of the nominal sequence.
    b. Maneuver 2: Same as the 2nd maneuver of the nominal sequence.
    c. Maneuver 3: Same as the 3rd maneuver of the nominal sequence.

2. SECOND MANEOUVER CANCELLATION RECOVERY SEQUENCE
    a. Maneuver 2: Same as the 2nd maneuver of the nominal sequence.
    b. Maneuver 3: Same as the 3rd maneuver of the nominal sequence.

3. THIRD MANEUVER CANCELLATION RECOVERY SEQUENCE
   This case requires an extra maneuver sequence than the nominal plan because we are already at the final longitude and the drift rate prior to the third maneuver will usually not be an integral fraction of 360 degrees per rev. The extra maneuver is needed to re-target the final longitude from the recovery open zone.
   a. Maneuver 3: Same as the 2nd maneuver of the nominal sequence
   b. Maneuver 4: Same as the 3rd maneuver of the nominal sequence

The result of this analysis will be several GTO trees, each with a nominal path as well as Backup paths in the event of cancellation of any one maneuver. A sample is presented in Fig. 11.

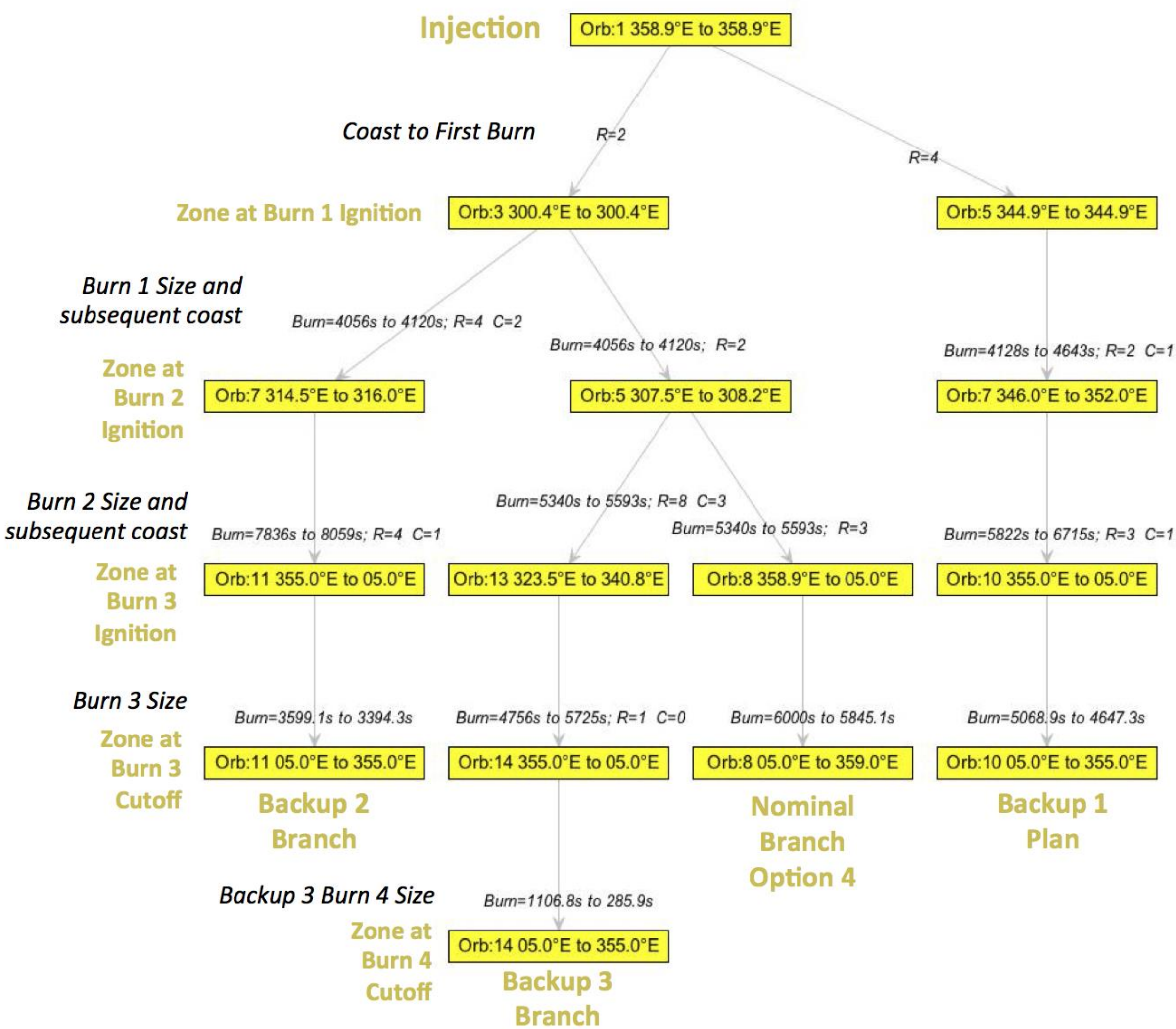

Figure 11. Sample Nominal Plan Tailored with Backup Plans

Note that the plan in Fig. 11 is based on just the fourth branch in Fig. 9. Looking more closely, one can see that the maneuver durations along the nominal branch of Fig. 11 are trimmed versions of those in Fig. 9. This is required because of the need to arrive within the bounds of the designated backup zone.

8. Exploiting the Maneuver Duration Range

The results present not just one single burn duration for each maneuver, but rather the range of duration that take subsequent apogees right to the edges of open zones. There are two key ways to exploit this range to find a result tailored to your mission needs.

First, at the edges of each zone, one is confronted with interference. Figure 12 illustrates that one can easily expect interference problems just before apogee, were it placed at the eastern extreme of an open zone.

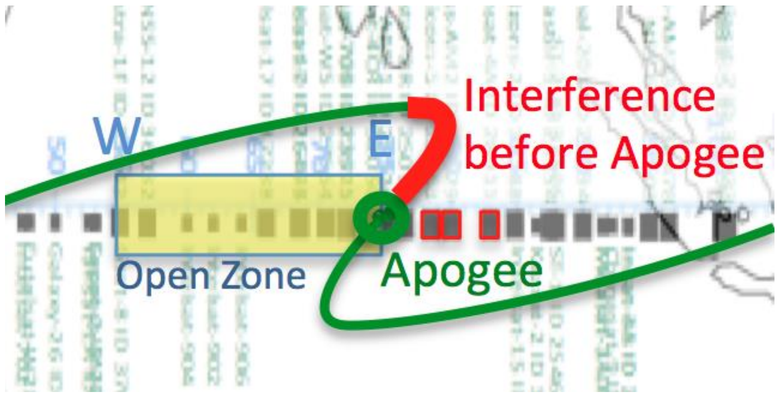

Figure 12. Interference at the Extrema

Naturally, in selection of the final ascent strategy, it would be better to choose maneuver durations that steer apogee more closely to the center of each open zone. For the example shown in Fig. 12, using a duration for the previous maneuver above the minimum pulls this apogee toward the center of the open zone. The best increase would depend on what extent of interference problems your mission can tolerate during pre-maneuver preparation.

A second way to exploit the maneuver duration range is when considering maneuver uncertainty. One typically expects 1 or 2 % over or under performance. In the example above, just a little underperformance and apogee would fall beyond the eastern edge of the zone. It would be better to stay closer to the mid-point of the maneuver duration range, so that this apogee is not on the edge of this open zone. The exact amount would of course depend on your mission uncertainties, the width of the zone and the maneuver size.

9. Conclusion

We have looked at how to examine a wide range of GTO ascent plans needed to travel from injection orbit to near geosynchronous drift orbit at a designated longitude. The approach seeks to contain all solutions that evade radio frequency interference during intermediate maneuvers. The resulting plans also minimize propellant consumption. Finally, these are designed to allow the construction of backup plans in the event that one of the nominal maneuvers must be cancelled.