STUDY OF SMALL FORCES FOR MISSION ANALYSIS

Alain Lamy[(1)], **Albert Alcarraz Garcia**[(1)]
[(1)] *CNES, 18 avenue Edouard Belin 31401 Toulouse Cedex 9, FRANCE,*
Alain.Lamy@cnes.fr

Keywords: *Orbit perturbations, small forces, mission design, long-term propagation*

Abstract: *The study of the effects of orbit perturbations is central in spaceflight dynamics mission design, for Earth orbits in particular, as it is key to mission feasibility: the perturbations impact the station keeping cost (and the station keeping window size) and the lifetime of the disposal orbit for instance.*
The orbital perturbations that are usually considered are limited to gravity (zonal and tesseral harmonics), solar radiation pressure, drag and third body. The other ones are not supposed to have significant effects.
This paper focusses on those forces whose effects are often neglected in early design phases: solid tides, apparent acceleration and albedo are examples of such forces and are dealt with in this paper. The objective of the study is multiple. One aspect is the illustration how the effects of perturbations can be evaluated in an efficient way (through analytical equations giving the averaged effects whenever possible). Some interesting results are given, regarding the drift rate of inclination under the influence of solid tides for circular Sun-synchronous orbits or the effect of apparent acceleration on the orbit's angular momentum. Some of the results given in this paper have also been implemented in STELA, CNES software used for orbit long-term propagation.

Keywords: *Orbit perturbations, small forces, mission design, long-term propagation*

Acronyms and notations:

CNES	*Centre National d'Etudes Spatiales* - French Space Agency
LEO	Low Earth Orbit - apogee altitude less than 2000 km (IADC definition)
MLTAN	Mean Local Time of the Ascending Node
RAAN	Right Ascension of the Ascending Node
SRP	Solar Radiation Pressure
STELA	Semi-analytic Tool for End of Life Analysis (CNES tool for orbit long-term propagation)
a, sma	Semi-major axis
e, ecc	Eccentricity
i, inc	Inclination
Ω	Right ascension of the ascending node
ω	Argument of perigee
M	Mean anomaly
n	Mean motion
μ_p	Gravitational constant of perturbing body

1. Introduction

Mission design activities related to flight dynamics are varied but some major aspects are:

- The definition of an orbit meeting the mission's objectives,
- The study of the effect of perturbations and their impacts on the mission (station keeping cost, maneuvers…),
- The long-term evolution of the orbit after the end of mission,
- etc…

The perturbations that are considered for the design of Earth orbits are usually limited to:

- Gravitational force exerted by the Earth (constant potential),
- Gravitational perturbation of third bodies, mainly the Sun and the Moon,
- Atmospheric drag force,
- Force caused by solar radiation pressure.

Of course, many other perturbation sources exist and are taken into account in precise orbit calculations for instance. Regarding mission design, these often neglected perturbations may still have some impacts on the orbit's long-term evolution in particular, so it is useful to have some ways to easily evaluate their effects.

We'll focus on the following perturbations: solid (Earth) tides, apparent acceleration and albedo with the objective to evaluate the effects on usual orbits in a way adapted to mission analysis, and derive useful results.

The paper is organized in 3 main parts:

- Brief description of the averaging process and how it is used,
- Some (often analytical) results about the 3 forces listed above,
- A few applications to frequently considered orbits.

2. Perturbation averaged effects

2.1 Computation of averaged effects (over one orbit period)

The perturbation effects can be sorted in 3 categories depending on their frequencies: short-term (less than 1 orbital period), long-term (several days or more) and secular.

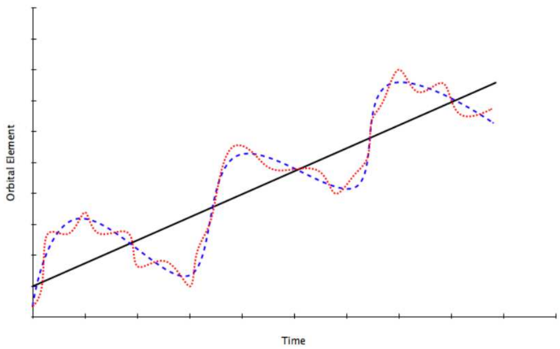

Figure 1: Short-term, long-term and secular effects

The process used to extract the long-term effects consists in averaging the time derivatives over one orbit period:

$$\bar{\dot{\alpha}} = \frac{1}{T}\int_0^T f(x(t),t)dt$$

where α is any orbital element, x the set of all variables (including orbital elements) that affect the orbit, and T is the orbit period.

If f does not explicitly depend on time, the averaged effect on the Keplerian orbital element α (semi major axis, eccentricity, inclination…) can be written:

$$\bar{\dot{\alpha}} = \frac{1}{2\pi}\int_0^{2\pi} \dot{\alpha}(a,e,i\,...)\,dM$$

The integral can sometimes be computed analytically using either Lagrange equations (if the force derives from a potential) or Gauss equations. Note that other types of orbital elements could also be used for particular orbit types.

If a potential function exists, it is first averaged (over the mean anomaly), then the usual Lagrange equations can be used. Otherwise, the time derivatives of the orbital elements can be averaged using the Gauss equations.

To make the integral computable, it is often assumed that the force has practically no impact over one orbit period, so that all the terms that appear in $\dot{\alpha}(a,e,i\,...)$ except the anomaly can be considered as constant, which is often a good approximation.

If no simple expression of the force exists, or if the integral cannot be computed analytically, one may have to resort to numerical averaging. A certain number of points are then evenly spaced in the orbit (in mean, true, eccentric anomaly as desired), and the average value can be computed by quadrature. This numerical process can also be used to check the analytical expressions.

2.2 Evaluation of long term evolution

If the perturbation effects are small enough, and if the actual orbit remains close to a "reference orbit", the long-term effects of the perturbation under study can be evaluated as follows:

Let $X(t)$ be the (mean) nominal trajectory, that is, the set of mean orbital elements computed without taking the perturbation under study into account.

The (averaged) perturbation effects $\Delta\dot{X}_p(t)$ can then be computed "around" this reference trajectory by using the averaging process described above.

Integrating $\Delta\dot{X}_p(t)$ yields the effect of the perturbation ΔX_p and an estimate of the new trajectory affected by the perturbation: $X_{total} = X + \Delta X_p$. One may iterate a few times, each time setting the newly obtained trajectory as the "reference" trajectory.

One should not forget indirect effects though. For instance if the perturbation affects inclination, the change in inclination will in turn affect other orbital elements (RAAN, …) due to the effect of J2 (mainly). The indirect effect on RAAN can then be written:

$$\Delta\dot{\Omega}_{ind} = \frac{\partial\dot{\Omega}}{\partial a}\Delta \mathrm{a}_p + \frac{\partial\dot{\Omega}}{\partial e}\Delta \mathrm{e}_p + \cdots$$

where Δa_p, Δe_p, … are the integrated effects on the semi major axis, eccentricity, … respectively, and the partial derivatives are computed only considering J2.
Taking these indirect effects is important as their amplitude can be bigger than the direct effects as it will be seen for the solid tides.

But they are cases for which some caution is necessary, for instance when the direct and indirect effects are strongly coupled.
Let's consider as an example the effect of SRP on the mean eccentricity vector of the "SWOT" orbit (circular, with an altitude of 890 km, and an inclination of 77.6 deg). The nominal orbit is computed considering the central and zonal terms, and is frozen. The process used to evaluate the effect on $e_x = e\ cos\omega$ and $e_y = e\ sin\omega$ can be the following:

- Compute the effects of the perturbation ($\dot{e}_{x,p}$ and $\dot{e}_{y,p}$) on the (frozen) reference orbit,
- Include the indirect effects due to J2/J3 by integrating a simplified model:
  $\Delta\dot{e}_x = -K\ \Delta e_y + \dot{e}_{x,p}(t)$,
  $\Delta\dot{e}_y = K\ \Delta e_x + \dot{e}_{y,p}(t)$
  where K is the same as $\dot{\omega}$ computed using J2 only. Δe_x and Δe_y are the increments that should be added to the nominal trajectory to take SRP into account.

The result is given in Figure 2. On the left, the perturbation effects that result from the calculation just described. The SRP coefficient is 1.5e-2 m^2/kg. On the right the comparison with the "real" trajectory for which the perturbations are integrated all together as done classically. The evaluation error is small (around 1%) and is mainly due to the fact that the reference ("exact") solution was computed using zonal harmonics up to degree 7.

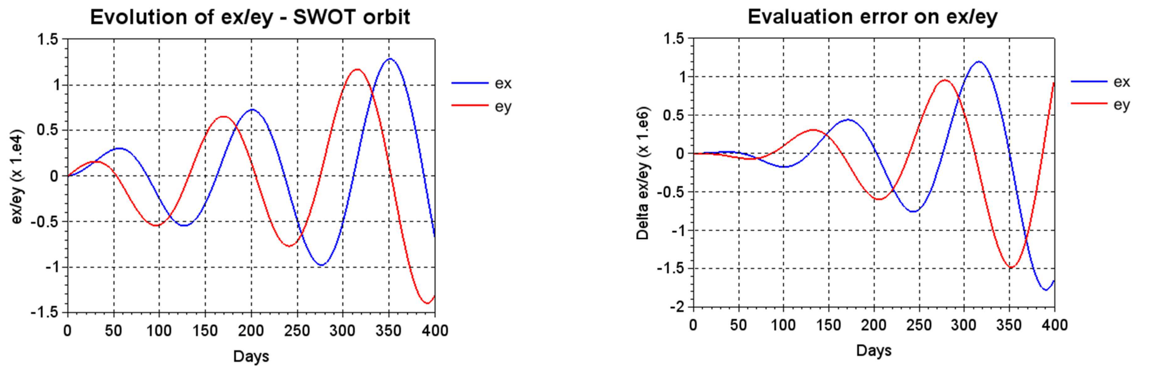

Figure 2: Effect on SRP on eccentricity vector (SWOT orbit)

3. Study of a few perturbations

In this part, we'll detail the effects of a few perturbations, including analytical results whenever possible.

3.1 Solid tides

Expression of the force

We use the Love model that states that the potential associated with the force can be written:

$$\Delta U = \sum_{n=2}^{\infty} k_n \left(\frac{R_E}{r}\right)^{n+1} \frac{\mu_p}{r_p} \left(\frac{R_E}{r_p}\right)^{n} P_n(cos\psi)$$

With P_n : Legendre polynomial of degree n, and μ_p : gravitational constant of the perturbing body.

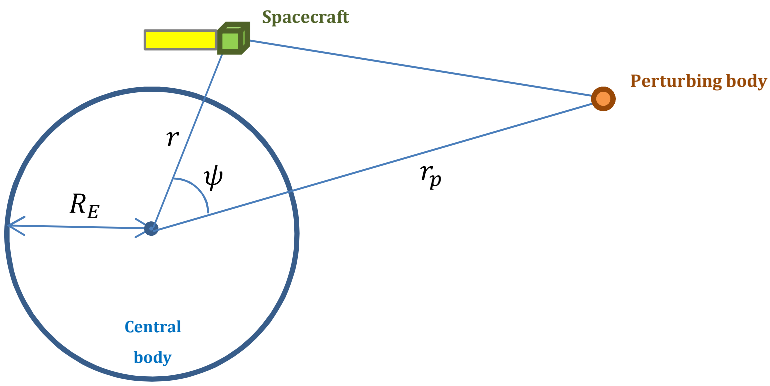

Figure 3: Solid tides geometry

The k_n coefficients are the Love numbers. k_2 is around 0.3 for the Earth.

If truncated to degree 2, the expression becomes:

$$\Delta U_2 = \frac{k_2 R_E^5}{r^3} \left(\frac{\mu_p}{r_p^3} \frac{3\cos^2\psi - 1}{2}\right)$$

And the corresponding acceleration is:

$$\vec{\gamma}_2 = \overrightarrow{grad}\Delta U_2 = \frac{1.5\, k_2 R_E^5}{r^4} \frac{\mu_p}{r_P^3} [\,(1 - 5cos^2\psi)\,\vec{u}_s + 2cos\psi\,\vec{u}_P\,]$$

With $\vec{u}_s$: unit vector from central body to spacecraft, and $\vec{u}_P$: unit vector from central body to perturbing body.

Averaged effects

Using the methods described in section 2 (use of Lagrange equations), averaged effects on the orbital elements can be derived.

After some tedious calculations, the time derivatives of the Keplerian orbital elements are found to be:

$$\frac{da}{dt} = 0$$
$$\frac{de}{dt} = 0$$
$$\frac{di}{dt} = 6\,K\,X_R\,Z_R$$
$$\frac{d\omega}{dt} = 3\,K\,(1 - 3Z_R^2) - 6\,K\,\frac{\cos i}{\sin i} Y_R Z_R$$
$$\frac{d\Omega}{dt} = \frac{6K}{\sin i} Y_R Z_R$$
$$\frac{dM}{dt} = 3K\sqrt{1 - e^2}(1 - 3Z_R^2)$$

With:

$$K = \frac{k_2 \mu_P R_E^5}{4\,r_P^3\,a^5(1 - e^2)^2\,n}$$

X_R, Y_R and Z_R are the components of the unit vector from central body to perturbing body in a frame such that the x-axis is directed towards to the ascending node and the z-axis is parallel to the orbit's angular momentum.

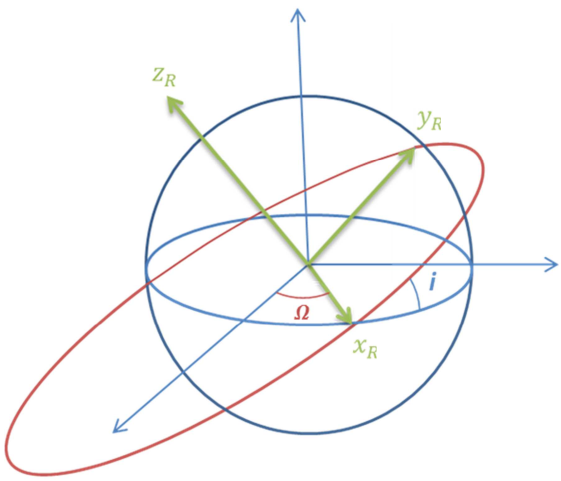

Figure 4: Orbit frame used for solid tides equations

Numerical application:

Since the product $X_R * Z_R$ is always smaller than $\sqrt{2}/2$, di/dt is smaller than $3K\sqrt{2}$. Here are some numerical values for the Sun, with $k_2 = 0.3$:

	Max value of di/dt (deg/year)
LEO (500x500)	1.4e-2
LEO (1400x1400)	9.2e-3
MEO (20000x20000)	1.3e-4
GTO (200x36000)	7.7e-4

Note that, for the Moon, the values would be larger by a factor of about 2.2.

Alternative (equivalent) equations for nearly circular orbits are the following:

$$\frac{da}{dt} = 0$$

$$\frac{de_x}{dt} = -K\, e_y \left[3(1-3Z_R^2) - \frac{6}{\tan i} Y_R Z_R\right]$$

$$\frac{de_y}{dt} = K\, e_x \left[3(1-3Z_R^2) - \frac{6}{\tan i} Y_R Z_R\right]$$

$$\frac{di}{dt} = 6\,K\,X_R Z_R$$

$$\frac{d\Omega}{dt} = \frac{6\,K}{\sin i} Y_R Z_R$$

$$\frac{d\alpha}{dt} = 3\,K\,\left(1+\sqrt{1-e^2}\right)(1-3Z_R^2) - \frac{6\,K}{\tan i} Y_R Z_R$$

Where $e_x = e\cos\omega$, $e_y = e\sin\omega$, and $\alpha = \omega + M$ (mean argument of latitude).

Averaged effects – second way:

The potential for the solid tides is very similar to the one for zonal harmonics:

$\Delta U_{zonal} = \frac{\mu}{r}\sum_{n=1}^{\infty} -J_n \left(\frac{R_E}{r}\right)^n P_n(sin\varphi)$, where φ is the latitude.

So the procedure is the following:

- Rotate the frame so that the z-axis becomes aligned with the direction of the perturbing body,
- In the formula that gives the potential for zonal harmonics:
  - replace R_E by: R_E^2/r_p
  - replace μ by: $(\mu_p\, R_E)/r_p$
  - replace J_n by $-k_n$

Then the potential that is computed is the potential for the solid tides.

Averaged solid tides effects can then be derived using (supposedly existing) analytical formulas used for zonal harmonics, taking care of multiplying the result by $\sqrt{\frac{\mu_p\, R_E}{\mu\, r_p}}$.

This method will be implemented in the next version of the STELA software (see [8]).

3.2 Apparent acceleration

Definition

The apparent acceleration comes into play when the reference frame in which the equations of the motion are written and integrated is considered as inertial whereas it is not exactly.

When the dynamic equations are written in a non-inertial reference frame $\mathfrak{R}$ with a specific angular velocity $\vec{\Omega}$ with respect to an inertial frame ($\mathfrak{R}_0$), it is necessary to add an additional term to the acceleration $\overrightarrow{\gamma_C}$ called apparent acceleration and equal to:

$$\boxed{\overrightarrow{\gamma_C} = -2\vec{\Omega} \wedge \vec{v} - \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}) - \frac{d\vec{\Omega}}{dt} \wedge \vec{r}}$$

With:

- $\vec{r}$: spacecraft position vector,
- $\vec{v}$: spacecraft velocity vector relative to frame $\mathfrak{R}$.
- $\vec{\Omega} = \vec{\Omega}\,(\mathfrak{R}/\mathfrak{R}_0)$

Averaged effects

If applying the averaging process as describe previously, using the Gauss equations since the force does not derive from a potential, the averaged effects on the Keplerian orbital elements can be obtained:

$$\frac{da}{dt} = \frac{-2\sqrt{1-e^2}\,a\dot{\Omega}_W}{n}$$

$$\frac{de}{dt} = \frac{5e\sqrt{1-e^2}}{2n}\left(\Omega_P\Omega_Q + \dot{\Omega}_W\right)$$

$$\frac{di}{dt} = \frac{\sin\omega\sqrt{1-e^2}}{2n}\left(\dot{\Omega}_P + \Omega_Q\Omega_W\right) - \frac{(1+4e^2)\cos\omega}{2n\sqrt{1-e^2}}\left(\Omega_P\Omega_W - \dot{\Omega}_Q\right) + \left(\Omega_Q\sin\omega - \Omega_P\cos\omega\right)$$

$$\frac{d\omega}{dt} = \frac{1}{2n\sqrt{1-e^2}\sin i}\Big[(1-e^2)\big(\cos i\cos\omega\left(\dot{\Omega}_P + \Omega_Q\Omega_W\right) + \sin i\left(4\Omega_Q^2 + 3\Omega_W^2 - \Omega_P^2\right)\big)$$
$$+\ (1+4e^2)\cos i\sin\omega\left(\Omega_P\Omega_W - \dot{\Omega}_Q\right)\Big] + \frac{1}{\tan i}\left(\Omega_P\sin\omega + \Omega_Q\cos\omega\right) - \Omega_W$$

$$\frac{d\Omega}{dt} = \frac{1}{2n\sqrt{1-e^2}\sin i}\left[(e^2-1)\cos\omega\left(\dot{\Omega}_P + \Omega_Q\Omega_R\right) + (1+4e^2)\sin\omega\left(\dot{\Omega}_Q - \Omega_P\Omega_R\right)\right]$$
$$- \frac{1}{\sin i}\left(\Omega_P\sin\omega + \Omega_Q\cos\omega\right)$$

$$\frac{dM}{dt} = -3\Omega_R\sqrt{1-e^2} + \frac{1}{2n}\left[(e^2-1)\Omega_P^2 - (4e^2+6)\Omega_Q^2 - (3e^2+7)\Omega_R^2\right]$$

Where Ω_P, Ω_Q and Ω_W are the components of the angular velocity vector in the “PQW” frame. The “PQW” frame is such that the x-axis is directed towards the perigee and the z-axis has the same direction as the orbit’s angular momentum.

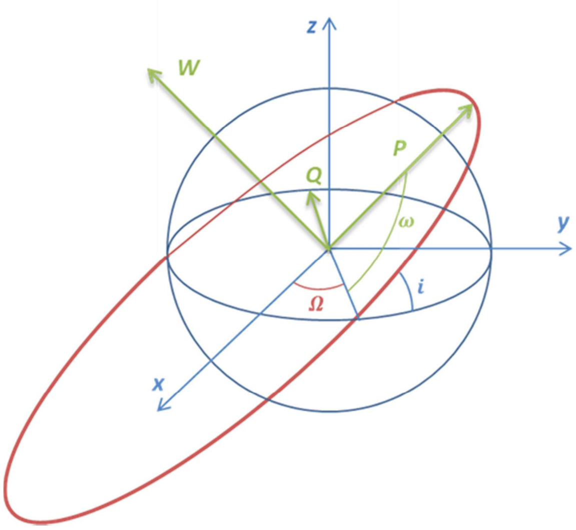

Figure 5: “PQW” orbit frame

One can immediately notice that:
- The force has no impact on the eccentricity of a circular orbit.
- The semi major axis is constant if the angular velocity vector and the angular momentum vector are perpendicular to each other.

3.3 Albedo

The radiation reflected by the Earth and exerting a pressure on a spacecraft is rather complex. So only a simplified model will be considered. It is summarized in the following hypotheses:

- The spacecraft is spherical,
- The spacecraft is in the vicinity of the central body,
- Albedo is constant all over the Earth.

In this case, the acceleration generated by the reflected radiation pressure can be written:

$$\boxed{\vec{a} = \alpha\,|SRP| \iint (\vec{r}_{sun}.\vec{n}_c)(\vec{r}_{sat}.\vec{n}_c)\vec{r}_{sat}\,\frac{d\sigma}{\pi\, d^2}}$$

Where the integral is to be computed on the area on Earth visible from the satellite and lit by the Sun.

The notations used are the following:

$\vec{r}_{sun}$: unit vector from cell (on body surface) to Sun ~ unit vector from body center to Sun

$\vec{n}_c$: unit vector normal to cell surface, $\vec{r}_{sat}$: unit vector from cell to spacecraft

d : distance from cell to spacecraft

$|SRP|$: module of SRP acceleration assuming the spacecraft at the center of the central body

α : albedo (supposed constant all over Earth’s surface)

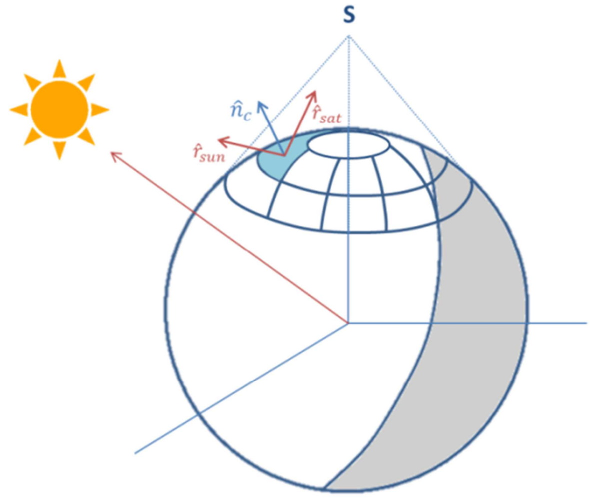

Figure 6: Geometry for albedo perturbation

If $\vec{r}_{sun}$ is assumed constant over the integration surface, the integral only depends on 2 parameters: the distance between the spacecraft and Earth center, and the angle between the Sun and the spacecraft from Earth center. It is then possible to pre-compute the expression for all possible values of these parameters and interpolate to obtain the results for any case. The computation of the acceleration due to albedo is then very efficient.

Below are plotted the 2 components of the acceleration divided by the module of SRP for an albedo value of 0.3. On the left: component on the radial direction (Earth center to spacecraft), on the right: component on the direction perpendicular to radial.

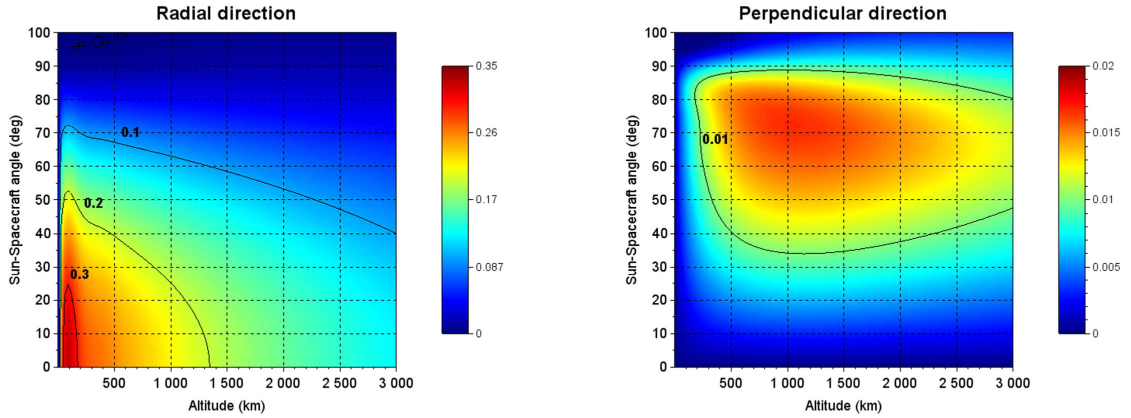

Figure 7: Instantaneous albedo acceleration divided by |SRP|

The maximum value for the perpendicular direction is 0.02, whereas the maximum value for the radial direction is 0.35 (acceleration is then about 1/3 of SRP).

The norm of the acceleration decreases moderately fast at low altitudes, and slightly faster as altitude increases. The values in the table below correspond to the case where the Sun and the spacecraft are aligned (case of maximum acceleration for a given altitude).

Altitude (km)	500	1000	2000	5000	10000	20000	36000
|albedo accel.| relative to SRP	0.26	0.22	0.17	0.09	0.04	0.014	0.005

As it can be seen, the ratio of the acceleration to SRP does not vary a lot between the altitudes 500km and 1000km.

4. Applications, derived effects

4.1 Effect of solid tides for a nearly circular Sun-synchronous orbit

As Sun-synchronous orbits have nearly a constant orientation with respect to the Sun, we may expect this fact should emphasize the impact of solid tides originating from the Sun on the orbit.

In order to evaluate the long-term effects of the perturbation, these effects are averaged over one revolution of the Sun.
Some usual hypotheses are considered: the Sun is supposed in a circular orbit around the Earth with a right ascension of the ascending node assumed to be 0.

The expression found for the average drift rate of inclination due to solid tides over one year is:

$$\overline{\overline{\frac{d\imath}{dt}}} = 3K \sin i \ \sin 2h \ \cos^4\left(\frac{\varepsilon}{2}\right)$$

In this equation ε is the inclination of the Sun's apparent orbit around the Earth (~ 23.5 deg), K has the same meaning as in section 3.1 and h is related to $MLTAN$ by the formula:
$12 * h = (MLTAN - 12) * \pi$.

The same can be done for the third body perturbation, using averaged equations from [3].
The result found for a circular orbit is:

$$\overline{\frac{d\imath}{dt}}_{3rd\ body} = \frac{3}{2}\frac{\mu_p}{n\, r_p^3} X_R Z_R$$

This expression is similar to the one found for solid tides, so we can immediately derive the ratio between the 2 perturbation doubly averaged effects:

$$\boxed{\frac{\overline{\overline{\frac{d\imath}{dt}}}\, Solid\ tide\ effect}{\overline{\overline{\frac{d\imath}{dt}}}\, 3rd\ body\ effect} = k_2\left(\frac{R_E}{a}\right)^5}$$

Figure 8 illustrates this result.

On the right, the doubly averaged effects are computed numerically (from the singly averaged ones) for the solid tides and third body perturbations. The orbit considered is circular, at an altitude of 700km. The ratio of the solid tides effect to the third body effect is effectively constant.
On the left, the ratio computed using the formula above is plotted. The ratio is close to 20% for an altitude around 600 km. It means for instance that the station keeping DV evaluated only

taking the third body effect into account, should be increased by the same amount (20%) to include the effect of solid tides, at least the major effects given by the degree 2 expansion. There is no need to compute the effect of solid tides in this particular case: it is enough to add a margin.

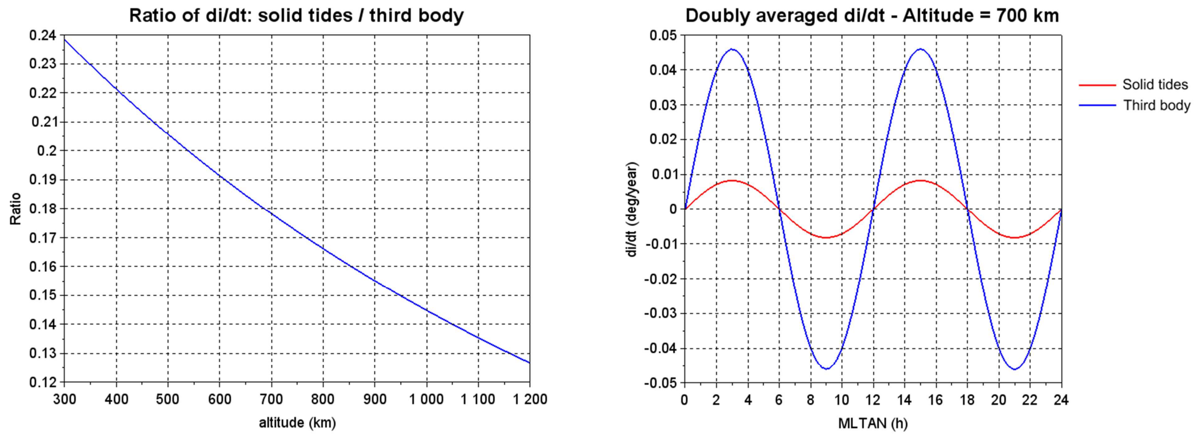

Figure 8: Solid tides versus third-body effects

4.2 Effect of apparent acceleration on the orbit's angular momentum vector

The objective is to derive a simple model for the evolution of inclination and RAAN when the orbital motion is affected by apparent acceleration.

Some simplifying hypotheses are assumed:

- The angular velocity of the celestial frame with respect to the inertial frame is constant.
- The components of the angular velocity vector are very small, so that the product of any two of them is considered negligible.

The formula given in 3.2 then becomes:

$$\frac{di}{dt} = \varOmega_Q \sin\omega - \varOmega_P \cos\omega$$

Replacing the components in the orbit frame (PQW) by components in the celestial frame yields:

$$\frac{di}{dt} = -\varOmega_X \cos\varOmega - \varOmega_Y \sin\varOmega$$

($\varOmega_X, \varOmega_Y$: components of the angular velocity vector, $\varOmega$: RAAN)

This equation can be easily integrated assuming $\dot{\varOmega}$ is constant, which gives:

$$\boxed{\Delta i = \Delta i_0 + \frac{-\varOmega_X}{\dot{\varOmega}} \sin\varOmega + \frac{\varOmega_Y}{\dot{\varOmega}} \cos\varOmega}$$

The same can be done for $\varOmega$, the inclination assumed nearly constant, and we get:

$$\boxed{\Delta\varOmega = \Delta\varOmega_0 - \varOmega_Z \Delta t - \left(\frac{\varOmega_X}{\dot{\varOmega}} \cos\varOmega + \frac{\varOmega_Y}{\dot{\varOmega}} \sin\varOmega\right) / \tan(i)}$$

Apart from the small secular drift on Ω, the angular momentum rotates "around" its unperturbed direction with an amplitude inversely proportional to $\dot{\Omega}$ and a period equal to $2\pi/\dot{\Omega}$. For a given angular velocity vector, the faster the orbit plane rotates (due to J2) around the pole axis, the smaller the amplitude of the oscillations.

We suppose the frame used for mission analysis is CIRF (see [5] pour definition), and the inertial frame is ICRF. The mean components of $\Omega_{CIRF/ICRF}$ in CIRF are close to [0, 0.56, 0] deg/century. Then we find for typical orbits the following results:

	Amplitude for incl. (deg)	Period (years)
LEO, Sun-synchronous (alt = 700 km)	9.1e-4	1
GEO (inclination = 1 deg)	6.7e-2	75
MEO (altitude = 20000km, inc = 55 deg)	2.2e-2	25
LEO (inc = 90 deg, alt = 700 km)	Secular, rate ~ 0.01 deg/year	

The effects are confirmed by a more accurate simulation for the GEO case (see **Figure 9**).
On the right, the tip of the angular momentum vector is shown in the reference frame such that the z-axis has the same direction as the reference angular momentum vector and the x-z plane contains the reference ascending node direction.
The plot on the left represents the evolution of inclination: the period of ~75 years is clearly visible as well as the amplitude close to the expected value. We see that the mean value is not 0 but is offset by a quantity equal to the amplitude of the oscillations, which causes a differential drift on the ascending node (due to J2).

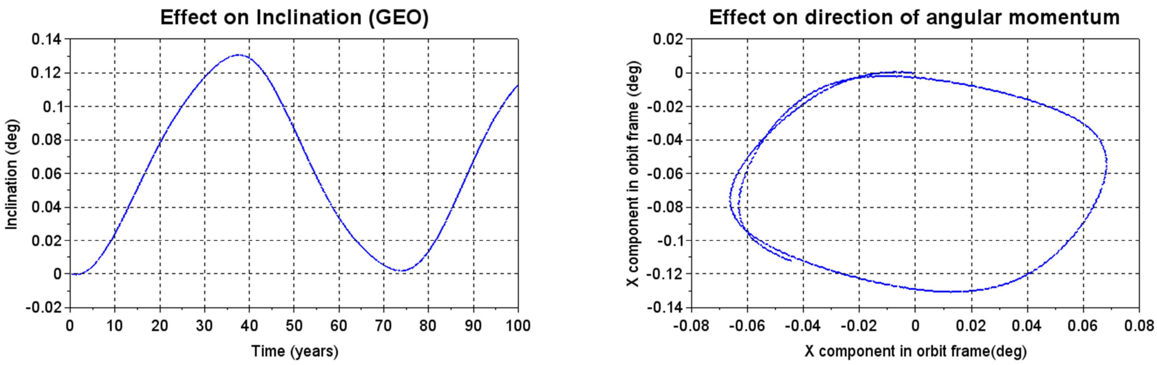

Figure 9: Apparent acceleration effect on GEO orbit

4.3 CIRF/ICRF frame transformation

The purpose here is to find the most simple frame transformation in order to compute the effect of apparent acceleration on the orbital elements with sufficient accuracy.

The reference frame used to define and study the orbit is CIRF ([5]). The inertial reference frame is ICRF (assumed identical to GCRF). The exact transformation from ICRF to CIRF requires the computation of lots of nutation terms which is time consuming. One possibility could be interpolation using pre-computed data, but the amount of data would have to be huge (several hundreds of years). Another way is to use an approximate frame transformation.

In the IERS conventions 2010, the transformation from ICRF to CIRF is function of 3 variables: X, Y and s.

One of the most simple transformation, called *Rep1*, uses the following definitions:

```
F = 1.6279050815 + 8433.4661569164 * TT
D  = 5.1984665887 + 7771.3771455937 * TT
om = 2.1824391966 - 33.7570459536 * TT

X = (2004191898 * TT - 6844318 * sin(om) - 523908 * sin(2*F-2*D+2*om))
Y = (-22407275 * TT^2 + 9205236 * cos(om) + 573033 * cos(2*F-2*D+2*om))
s = 0
```

(X and Y are in micro-arcseconds, TT is the number of centuries since J2000).

This transformation gives reasonably accurate results over a short time period, but diverges after about one century.

New terms (with decreasing amplitudes, see [6]) are added to X and Y in order to build other candidate frames:
Rep2: 1 term in TT^2 added to X,
Rep3: 2 terms added to X and 1 to Y.
Rep3 seems (at first sight) to satisfy the requirements as the number of terms is small and the angular error obtained on a position transformation is much smaller than with *Rep1* and almost stable over 200 years (see **Figure 10**, left).

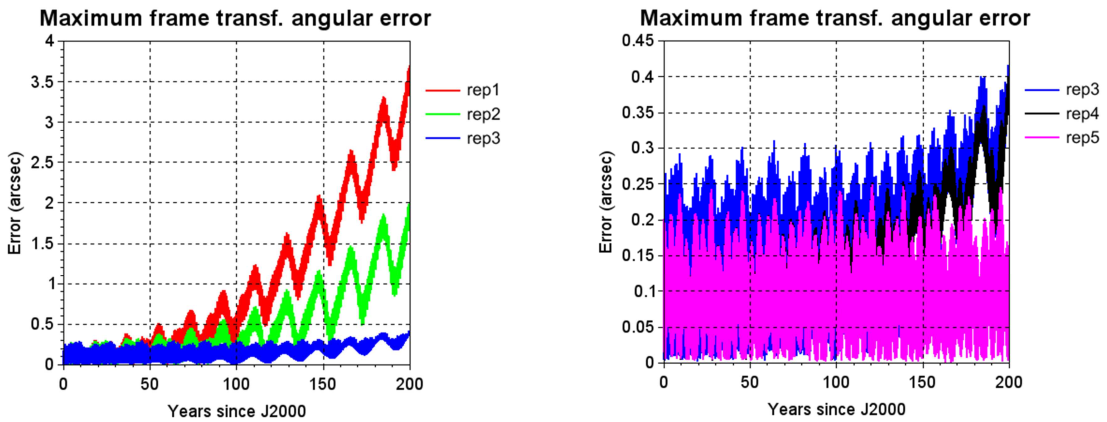

Figure 10: Frame transformation accuracies

The frame transformation is then tested on various orbits over 50 years. The effect of apparent acceleration is computed as explained in 2.2, including the indirect effects due to J2. **Figure 11** shows the averaged derivative of inclination for a nearly circular and Sun-synchronous LEO orbit using the full model (left) and the error when using the simplified model *Rep3* (right). The error is not negligible compared to the effect of the perturbation itself: about 30%.

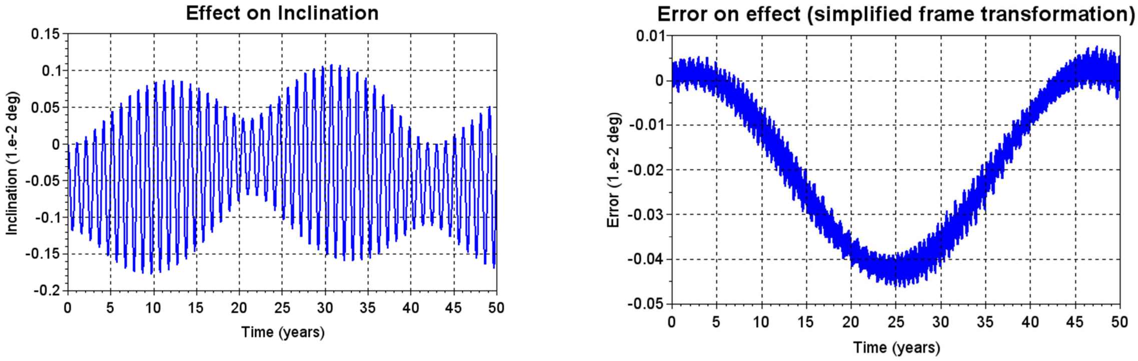

Figure 11: Effect on inclination of LEO orbit

The reason is that not only the amplitude of the terms used in the frame transformation matters, but also their frequency.
Among the terms not included are ones with periods close to 1 year, and 1 year is approximately the period of the ascending node. Some kind of resonance appears due to the missing terms, which explains the error.

A new frame (*Rep3b*) is then defined for which additional terms with periods of about 1 year are added: 2 to X and 2 to Y.
Looking at the position frame transformation error, this new frame doesn't seem to bring much improvement as the maximum angular errors are: for *Rep3*: 0.3 arcsec, and for *Rep3b:* 0.27 arcsec. Yet the effect of the additional terms is spectacular: the error on inclination has shrinked by a factor of 10.

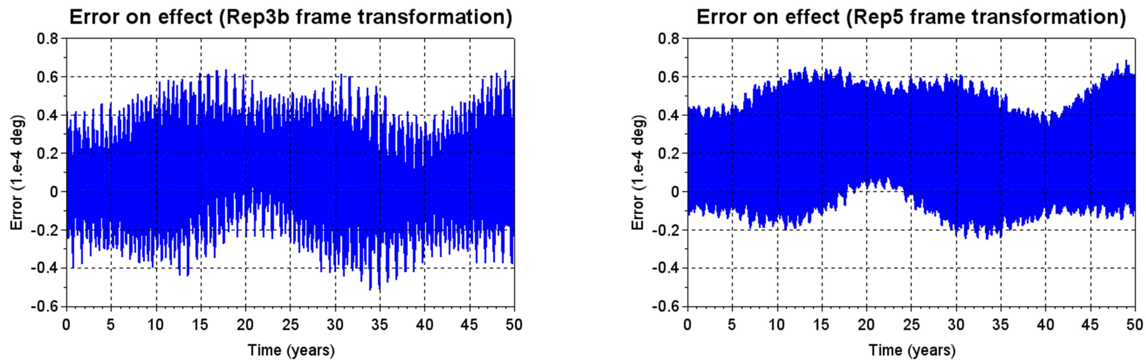

Figure 12: Effect on inclination – frame comparison

Other slightly more accurate reference frames have been defined to limit the risk of errors with other orbit types:
Rep4: additional terms added with periods of about 1/2, 1/3 and also 18 and 9 years (10 terms added to X and 9 to Y).
Rep5: same as *Rep4* for X and Y, s changed according to [5].

The accuracy obtained with *Rep5* is quite good over a long period of time. But the results for the inclination of the LEO orbit are not much impacted.
It has also been checked that the error is acceptable on all orbital elements for classical orbits (LEO, MEO, GTO, GEO).

The results presented above have successfully been implemented in STELA ([8]).

4.4 Albedo effect on orbit

When computing the acceleration due to albedo using the formula given in section 3.3, the norm of the (instantaneous) acceleration appears to be about 1/4 of that due to SRP for an altitude of about 500 km, which seems quite large. The objective is then to have estimates of the averaged effects.
We'll consider only the case of a nearly circular Sun-synchronous orbit, case supposed to be an unfavorable one. The SRP coefficient used (Cr * A/M) is 1.5e-2 m^2/kg.

The time derivatives of the orbital elements (adapted to circular orbits) are averaged numerically. The following table gives the maximum direct effects for any time of year and any MLTAN:

	sma	ex	ey	inc	Ω	pso
Time derivative	0.8e-3 m/day	2.5e-8 day^{-1}	4e-8 day^{-1}	2.5e-7 deg/day	2.5 e-7 deg/day	7.e-6 deg/day (⇔ 0.9 m/day)

The amplitudes found are effectively quite small. At an altitude of 1000 km, the results would be about the same. The conclusion is that there does not seem to be any good reason to have to take this perturbation into account in early design phases.

5. Conclusion

This paper has shown some illustrations of the processing of perturbations applied to small forces. Most results are based on averaging that is well adapted to the mission design process. Of course, some caution is necessary to be sure that averaged effects can be added or integrated, particularly if (long-term) coupling exists between forces. But provided the conditions are met, the evaluation of perturbation effects is much simpler, more efficient and less time consuming.

Useful equations for the averaged effects of solid tides and apparent acceleration on the Keplerian orbital elements have been given and demonstrated in practical applications.

One interesting result is the effect of solid tides (using the Love model expansion up to degree 2) on circular Sun-synchronous orbits. The long-term drift rate on inclination due to solid tides is related to the one originating from the third body acceleration through a simple formula. The ratio is about 20% for LEOs and should be taken into account in the station keeping cost.

Another result is related to apparent acceleration: the analytical developments have led to simple formulas governing the evolution of the orbit's inclination and RAAN. The effect of the perturbation can then be better understood. The perturbation effect can also be roughly predicted (amplitude, period) using the simple formulas.

The study of the effects of apparent acceleration on the orbit's long-term evolution has also shown how the orbital elements can be sensitive to frequency aspects. The frame transformation used to compute the effect of the force has to be accurate enough and should include the right combination of frequencies otherwise the long term evolution is impacted.

The third perturbation studied in this paper is albedo. The main conclusion is that there does not seem to be any good reason to consider this perturbation in early design phases.

All the developments presented in this paper have been implemented in Scilab for various mission analyses. Some aspects are present in CelestLab, CNES space mechanics toolbox for Scilab, and in STELA, CNES software used for orbit long-term propagation.

6. References

[1] J.-P. Carrou et al, Spaceflight dynamics, vol. 1, CNES, 1995.

[2] A. Lamy et al, *Resonance Effects on Lifetime of Low Earth Orbit Satellites,* ISSFD 2012

[3] A. Lamy et al, *Analysis of Geostationary Transfer Orbit Long Term Evolution and Lifetime*, ISSFD, 2011.

[4] H. Fraysse et al, *Long Term Orbit Propagation Techniques Developed in the Frame of the French Space Act*, ISSFD, 2011

[5] Gerard Petit and Brian Luzum (eds.), IERS Conventions (2010), IERS Conventions Centre, http://www.iers.org/IERS/EN/Publications/TechnicalNotes/tn36.html

[6] N. Capitaine, P. T. Wallace, *Concise CIO based precession-nutation formulations*, A&A 478, 277–284

[7] David P. McKinley, *Long Term Mean Local Time of the Ascending Node Prediction*, a.i. solutions

[8] STELA software download site: *https://logiciels.cnes.fr/content/stela?language=en*

[9] CelestLab: Scilab space mechanics toolbox for mission analysis, *https://atoms.scilab.org/toolboxes/celestlab*