RESEARCH RESULTS FROM ANALYSIS OF SATELLITE ORBITS

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ABSTRACT

The paper describes some of the discoveries about the Earth's upper atmosphere and the gravitational field resulting from the analysis of changes in satellite orbits. The density of the upper atmosphere at a height slightly above the perigee height can be determined from the rate of contraction of the orbit. The accumulation of such results led to the COSPAR International Reference Atmosphere 1972. The rotation rate of the upper atmosphere can be determined from the decrease in orbital inclination which it causes, thus giving a picture of the zonal winds. The orbit of a satellite is a most sensitive probe of the Earth's gravitational field, and orbit analysis is the best method for evaluating the longitude-averaged zonal harmonics. Also analysis of resonant orbits gives very accurate values of particular longitude-dependent harmonics, and these values serve as essential independent tests of the accuracy of the comprehensive models of the Earth's gravitational field.

Keywords: orbits, upper atmosphere, gravity field

1. INTRODUCTION

My aim is to describe some of the valuable results about the upper atmosphere and Earth's gravitational field obtained from analysis of satellite orbits. In the form of 'reference atmospheres' and geopotential models, these results have provided the basic information about the satellite's environment needed for many satellite missions. Some of the methods used will become outdated, but others will continue to thrive and will probably be exploited vigorously throughout the 1980s.

The upper atmosphere is discussed first, with sections 2, 3 and 4 covering air density, scale height and winds. Section 5 is devoted to the Earth's gravitational field.

2. UPPER-ATMOSPHERE DENSITY

2.1 Method

The method used is simple. A satellite in an elliptic orbit encounters the greatest air drag as it passes closest to the Earth, at perigee, and the deceleration produced by the air drag near perigee reduces the apogee height on the subsequent revolution. The orbit contracts and becomes more nearly circular: see Fig 1. The rate of contraction, which can be very accurately measured from the rate of decrease of orbital period, depends on the density of the air in the region near perigee and on the size, shape and mass of the satellite. If the mass and dimensions of the satellite are known, the density at a height a little above perigee can be quite accurately determined.

This technique was applied to Sputnik 1 within 2 weeks of its launch and gave the first scientific results from spacecraft, published in *Nature* (Ref 1) on 9 November 1957. It is interesting to look back at the accuracy of this first determination of density, namely 2.4×10^{-10} kg/m^3 at 241 km height. There were two errors, both quite small: the drag coefficient was taken as 2.0, whereas we now think it should be 2.2; and the perigee height derived was 6 km too high. With these corrections the result would be 2.2×10^{-10} kg/m^3 at 235 km height, which agrees with the density at this height, namely $(2.1 \pm 0.2) \times 10^{-10}$ kg/m^3, given by the COSPAR International Reference Atmosphere 1972

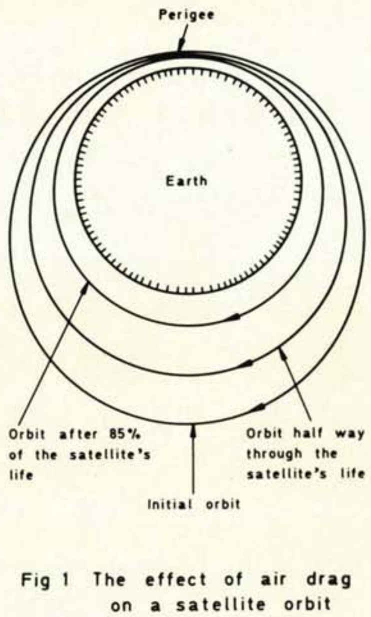


Fig 1 The effect of air drag on a satellite orbit

(Ref 2) for the appropriate exospheric temperature, about 1800 K. Previous estimates of the density at this height had been in error by a factor of about 10, so this good and rapid result from Sputnik 1 shows the method to be powerful, accurate and easy. It has continued in use, essentially unchanged though improved in detail. The error in the absolute values of air density is still about 5% because of uncertainty about the exact value of the drag coefficient for a particular satellite; but relative density, *ie* density variations, can usually be determined correct to 2% or better.

2.2 Results, 1958-1970

The evaluation of density from the orbital decay rate led to a long series of discoveries. It was found in 1958 that the density exhibited variations with a period of about 27 days in phase with solar activity; presumably, therefore, the density was controlled by the activity of the sun. Late in 1959, as the perigee points of several high satellites moved from daylight into darkness, it became obvious that the density was much lower by night than by day. In 1960 a semi-annual variation was detected, with maxima in April and October and minima in January and July. Analysis of orbits during the great geomagnetic storm of November 1960 showed increases of density by a factor of up to 8 on the day of the storm. As the solar cycle declined from its maximum in 1958 towards the minimum in 1964, there was a great decrease in density, as expected.

So, by 1970, we had a picture of an atmosphere under three strong solar controls: Fig 2. First, the density is greater at sunspot maximum than at sunspot minimum by a factor which increases from 2 at 200 km height to about 10 at 500 km, and then decreases to 3 by 1000 km. These are the figures for an average solar maximum like that of 1968-70. The present maximum is proving stronger and longer-lasting, so the factors are appreciably greater.

The second aspect of solar control is geometric - the daily rhythm. The density is at a minimum in the early morning at 04h local time, and rises to a maximum in the afternoon at about 14h local time, then declines again. The maximum daytime density exceeds the minimum night-time density by a factor which increases from about 2 at 200 km height to 6 at 500 km and then decreases. These figures apply at sunspot minimum: at sunspot maximum the factor increases from 1.5 at 200 km height to 6 at 700 km and then decreases.

The third expression of solar control is the response of the atmosphere to short-term and transient solar activity, as evidenced by the tendency to 27-day variations in unison with those of the sun, and the day-to-day response to transient solar storms, when shock waves disrupt the solar wind and, impinging on the Earth, greatly increase the temperature and density of the upper atmosphere. For examples of these sudden storms, see Ref 3.

In addition to these complex solar influences, and independently of them, the atmosphere continued to pursue its strange semi-annual oscillation. This proved to be a combined annual and semi-annual variation, with a shallow minimum in January, a maximum in April, a deeper minimum in July, and another maximum in October, usually stronger than that in April. At a height of 500 km, the density at the maximum often exceeded that at the minimum by a factor of 3, but, to add to the confusion, the amplitude did not fall off much as height decreased, and even at a height of 200 km, a factor of 2 was not uncommon. Below 200 km, the semi-annual variation was often more important than the variations linked with the sun. To complete everyone's confusion, the amplitude of the semi-annual variation itself varied from year to year, and so did the dates of its maxima and minima.

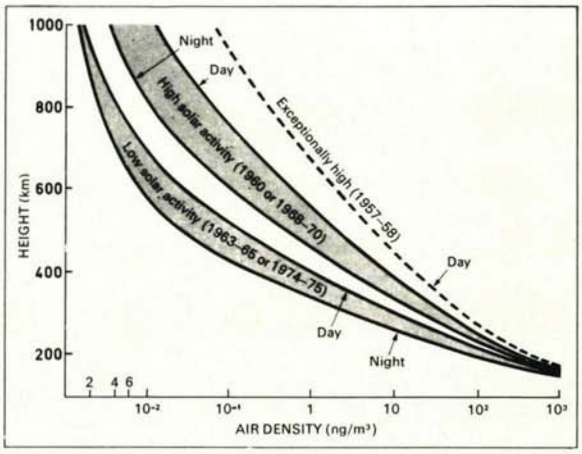


Fig 2 Variation of air density with height, by day and by night, for high and low solar activity

Another important feature of the atmosphere above 400 km, the winter helium bulge, was discovered in the late 1960s. Also instruments aboard satellites had revealed the existence of transient disturbances propagated as gravity waves and not recorded by the orbit decay method. For further references, and a fuller summary, see Ref 4.

2.3 The COSPAR International Reference Atmosphere 1972

During the 1960s all these complex influences were beautifully combined into one semi-theoretical, semi-empirical framework by L.G. Jacchia. His successive atmospheric models were based on the theoretical idea of static diffusion with a constant exospheric temperature at heights above 300 km. The actual atmosphere is highly dynamic, and the temperature can vary by 300 K between day and night, between solar minimum and maximum, or even in a matter of hours. But Jacchia succeeded in incorporating all the known effects, mostly by adjustments to the exospheric temperature, to produce a set of tables that are most serviceable, realistic and quite easy to use. These formed part of the COSPAR International Reference Atmosphere 1972, which was widely used during the 1970s in upper-atmosphere studies. (Later, in Ref 5, Jacchia further improved his atmospheric models.)

The COSPAR International Reference Atmosphere 1972, based on measurements made in the 1960s, proved successful in providing a general picture of the upper atmosphere environment for any satellite. The most serious defect of the model

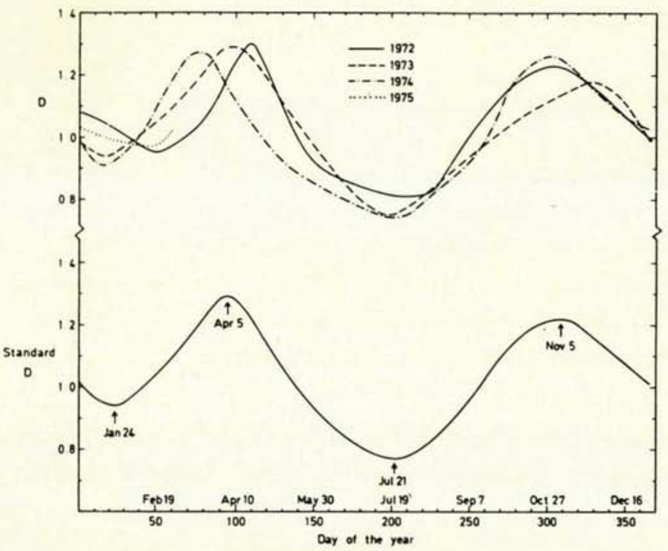


Fig. 3
Semi-annual variation of relative density D at heights of 200-250 km from 1972 to 1975, with standard variation, as given by Walker[6]

was the form of the semi-annual variation, which changed its behaviour quite considerably in the early 1970s, as shown by the detailed studies for 1972-5 made by D.M.C. Walker (Ref 6) and illustrated in Fig 3. The amplitude of the variation was greater than in the 1960s, the dates shifted a little, and the April maximum became higher than the October maximum. We still have no idea whether the semi-annual variation will revert to the pattern of the 1960s or go on to take yet another format. The semi-annual variation is important throughout the upper atmosphere from heights of 150 km up to 1000 km and it remains a difficult problem. It probably arises from seasonal variations in the lower atmosphere, which are notoriously difficult to predict.

2.4 Density from orbit analysis: limitations and future role

During the 1970s research scientists were no longer so interested in the values of the overall density of the upper atmosphere; they wished to make measurements of individual components of the upper air, the oxygen, nitrogen, helium and hydrogen at heights of 200-2000 km. To meet their wishes, a number of aeronomy satellites were launched, notably the NASA Aeronomy Explorer (AE) series, and detailed measurements of the individual species of gases were made. This has led to great progress, and to much more elaborate atmospheric models such as the Mass Spectrometer and Incoherent Scatter model (MSIS). Although several large volumes of MSIS are available (Ref 7), there are really too many variables for an adequate tabulation, and such models exist basically as computer programs. Existence only as a computer program may be no problem for the specialist continually using the model, but it does shelter the model from fruitful criticism by outsiders, and there is a danger that some particular model may attain a spurious sanctity, cherished by hierophantic groups who only walk along narrow paths and do not sample all the variables.

In particular, none of the models solves the problem of the semi-annual variation, and this is an area where orbit analysis should continue to make a strong contribution for many years to come. The orbital method, carefully used with a fairly long-lived satellite, gives the variations in density accurate to 2% over many years. This is just what is needed to monitor the variations in air density over an 11-year solar cycle, and to grasp and rationalize the vagaries of the semi-annual variation from year to year. Instruments aboard satellites cannot compete, because of problems of calibration, power supply and cost during many years of operations, and the need to produce quick results to justify expensive satellites, rather than the more valuable long-term studies. The work is quite laborious and needs to be done by those 'harmless drudges', the orbit analysts; it is foreign to the way of life of the high-flying and high-spending physicists whose great energies go into devising, presenting, promoting and coordinating proposals for prestigious projects - and into dancing like puppets on a string at the whim of NASA or ESA committees.

3. UPPER-ATMOSPHERE SCALE HEIGHT

The density scale height H, in an atmosphere where density ρ decreases exponentially with height y, is the height in which density falls off by a factor of 2.718. In other words, the density is given by $\rho = K\exp(-y/H)$. In practice the variation of ρ is not exactly exponential and it is better to define H differentially, as

$$\frac{1}{H} = -\frac{1}{\rho}\frac{d\rho}{dy} .$$

"Why bother with this rather recondite quantity?" is the first question that may spring to mind. The answer is that H is directly proportional to the air temperature divided by the mean molecular weight, and therefore provides a method of testing the accuracy of atmospheric models such as *CIRA 1972*, which use exospheric temperature as a guiding parameter. The scale height can be found by orbit analysis in two ways. First, a number of values of density at a given date can be calculated from a number of different satellites, and the variation of density with height can be established. This has the disadvantage of needing a number of orbit analyses to find the perigee height of each satellite. The second method is to measure the decrease in the perigee height for a single satellite, because this decrease is proportional to H. If the perigee height decreases by Δr_p while the orbital eccentricity decreases from e_0 to e_1, the value of H can be found from an equation in which the main term is $\Delta r_p = \frac{1}{2}H\,\ln(e_0/e_1)$. The main limitation on the method is the accuracy of Δr_p: even if all other perturbations are accurately removed, the change in Δr_p is likely to be small unless the satellite is near the end of its life, when the high drag militates against accurate values of perigee height.

However, accurate high-drag orbits can be determined, and in a recent analysis of Heos 2 second stage, H. Hiller obtained eleven values of scale height accurate to 2% in the last 14 days of the satellite's life (Ref 8). Fig 4 shows the variation of perigee height after removal of perturbations: clearly the curve is very well defined by

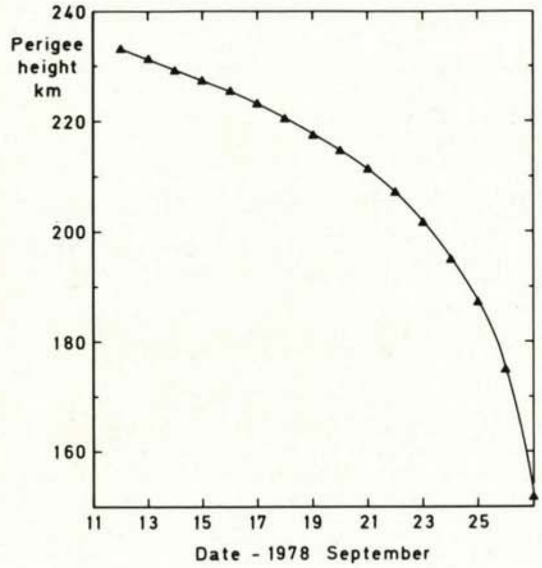


Fig 4 Perigee height of 1972-05B over a spherical Earth, cleared of gravitational perturbations

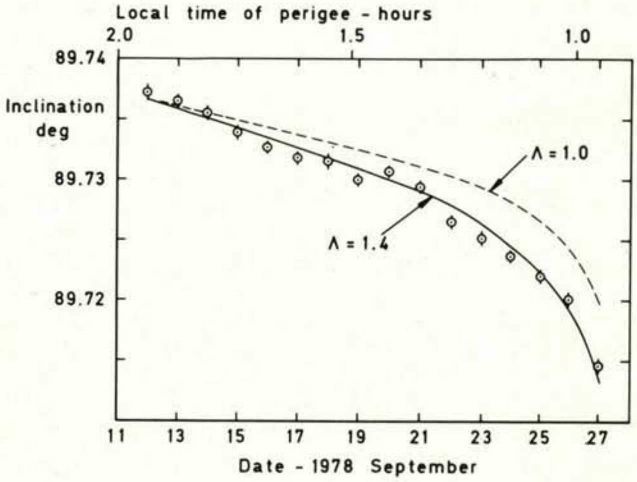


Fig 5 Values of inclination for 1972-05B, with Λ curves

the daily values. The values of H proved to be within 4% of those predicted by *CIRA 1972* and thus largely substantiate that model for September 1978. Continuing comparisons of this kind are, I think, essential to check the continuing validity of current atmospheric models.

4. UPPER-ATMOSPHERE WINDS

The great changes in air density between day and night imply corresponding changes in air pressure, and these pressure differences provide a driving force for winds in the upper atmosphere. The realistic calculation of such winds is still a difficult and disputed procedure, and it is valuable to have observational measurements, which orbit analysis can fortunately provide.

If the atmosphere did not rotate, the inclination of a satellite's orbit to the equator would remain unchanged as its life proceeded, if irrelevant gravitational perturbations are removed. But the west-to-east rotation of the atmosphere slightly reduces the inclination, and the rate of decrease is proportional to the atmospheric rotation rate, Λ . So the observed values of inclination can be fitted with theoretical curves to determine Λ . As with density, the main effects occur on a small section of the orbit near perigee, so the appropriate height, latitude and local time can be specified. If Λ = 1.0 rev/day, there is no west-to-east (zonal) wind; if Λ > 1.0 rev/day, the atmosphere is rotating faster than the Earth ('super-rotation') and there is a west-to-east wind. The first application of the method, to Sputnik 2, indicated a strong west-to-east wind at heights near 250 km (Ref 9).

The method can be powerful when accurate high-drag orbits are obtained, and a recent example, from Hiller's analysis of 1972-05B (Ref 8), is shown in Fig 5, where the values of inclination have been cleared of gravitational perturbations. An average rotation rate of Λ = 1.4 rev/day fits the points and Λ = 1.0 rev/day is quite untenable.

The orbital inclination is also affected by meridional (N-S) winds, but to a much smaller extent because the effects tend to cancel out. Meridional winds are of greatest importance for near-equatorial orbits and the best results on zonal winds are obtained from analysis of near-polar orbits, as in Fig 5.

Many values of Λ have been obtained over the years from various satellites, and Fig 6 presents an interpretation of the results, which have been divided into three categories, 'morning' (04-12 hours local time), 'evening' (18-24 hours local time), and 'average', which covers results from satellites in circular orbits and those for which perigee samples all local times without much bias. The categories are rather ill-defined, but the sub-division proves fruitful, as Fig 6 shows. The average rotation rate increases from a little above 1.0 rev/day at a height of 200 km to a maximum of 1.3 at 350 km, and then seems to decrease to near 0.8 by 600 km. The evening values of Λ are higher, indicating west-to-east evening winds of 80 m/s at a height of 200 km, increasing to nearly 150 m/s at 350 km, with a decrease at greater heights. The morning winds are from east to west but rather weaker, of order 50-100 m/s. Fig 6 is from Ref 10.

The general rotation rate of the upper atmosphere is a fundamental parameter not directly accessible by other methods of measurement, such as vapour trails from sounding rockets (which are local measurements) and radar backscatter (which is at a fixed latitude and involves a model atmosphere for the computations). So Fig 6 is one of the successes of orbit analysis. It is, incidentally, due for revision. More points need to be added and the tendency for the rotation rate to be lower in summer than in winter, as found by Walker (Ref 11) needs to be allowed for.

The future of this technique is difficult to predict. Attempts to determine wind velocity by

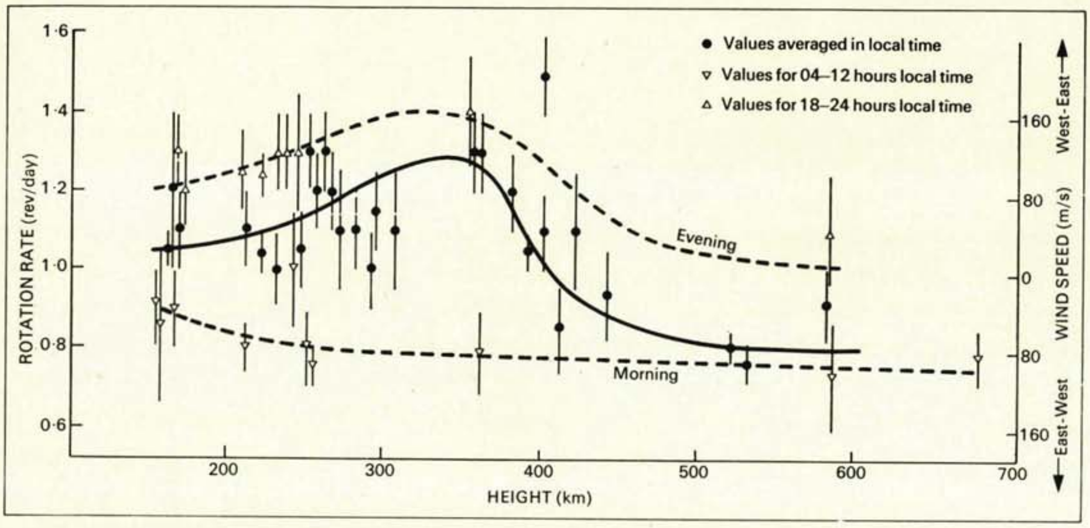


Fig 6 Upper-atmosphere rotation rate Λ, with corresponding wind speed at latitudes near 30°. The full line respresents average conditions, the broken lines morning and evening

instruments aboard satellites have so far proved expensive and rather inaccurate, but an accurate wind gauge on an orbiting satellite could put orbit analysis in the shade. At present, however, it continues to compete with and complement the other methods, such as vapour trails, radar back-scatter and doppler shifts in airglow spectra, which all have their special virtues and defects.

5. THE EARTH'S GRAVITATIONAL FIELD

5.1 Introduction

Orbits respond not only to atmospheric forces, but also to the peculiarities in the Earth's gravitational attraction, and it is fortunate for the orbital analyst that the perturbations produced by these two sources are usually quite different. For example, air drag reduces the semi major axis, which is generally unaffected by gravitational forces, while the oblateness of the Earth greatly changes the orientation of the orbital plane, which is only very slightly affected by atmospheric forces.

Orbit analysis has two distinct applications in the study of the gravitational field - for evaluating (a) the longitude-averaged (zonal) harmonics and (b) the longitude-dependent (tesseral) harmonics. These will be discussed separately in the next two sections.

5.2 Zonal harmonics

The Earth is flattened at the poles, the polar diameter being about 43 km less than the equatorial diameter. The gravitational pull of the extra 'bulge' of material near the equator makes the plane of a satellite's orbit rotate about the Earth's axis in the direction opposite to the satellite's motion, as in Fig 7, while the orbital inclination remains constant. The rotation of the orbital plane can be quite rapid, 5 degrees per day being typical, and measurements of this rotation rate with Sputnik 2 in 1958 led to better estimates of the Earth's flattening, the flattening being defined as the equatorial diameter minus the polar diameter, divided by the equatorial diameter. The favoured pre-satellite value was 1/297.1, but Sputnik 2 suggested that the value was nearer 1/298.1 (Ref 12), and it is now known to be 1/298.257 (Ref 13).

The evaluation of the flattening was the first step in satellite geodesy. To proceed further, we assume first that the Earth is symmetrical about its axis, and that the shape is made up of a series of harmonics. The third harmonic is often called pear-shaped, the fourth is square-shaped, and so on (Fig 8). Each one is a shape that would be found by slicing the Earth through the poles, if that particular harmonic were the only one that existed. Mathematically, the longitude-averaged potential $\bar{U}$ at an exterior point distant r from the Earth's centre at co-latitude θ may be written (Ref 12)

$$\bar{U} = \frac{\mu}{r}\left\{1 + \sum_{n=2}^{\infty} J_n\left(\frac{R}{r}\right)^n P_n(\cos\theta)\right\} \qquad (1)$$

where μ is the gravitational constant for the Earth (398600 $km^3\ s^{-2}$),

R is the Earth's equatorial radius (6378.14 km),

$P_n(\cos\theta)$ is the Legendre polynomial of degree n and argument $\cos\theta$,

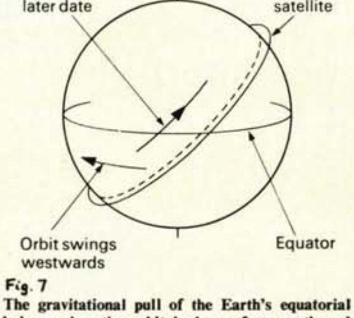


Fig. 7
The gravitational pull of the Earth's equatorial bulge makes the orbital plane of an eastbound satellite swing westward.

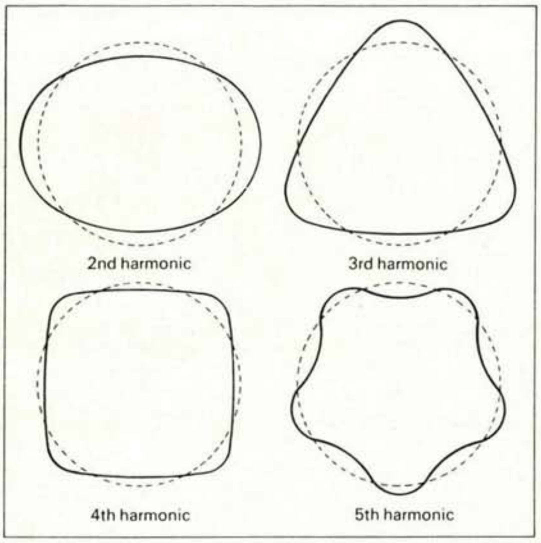


Fig 8 Exaggerated forms of the second to fifth zonal harmonics

and the J_n are constant coefficients which have to be determined. The first harmonic (n = 1) is missing if the origin is at the Earth's centre of mass. The value of J_2 is of order 10^{-3}, while $J_3, J_4, J_5, \ldots$ are of order 10^{-6}.

Measuring the rotation rate of the orbital plane for a particular satellite gives a value for a linear sum of the even J coefficients, say $A_2J_2 + A_4J_4 + A_6J_6 + \ldots$. The coefficients A differ for each orbital inclination; so, by using many satellites at different inclinations, we can evaluate the individual coefficients $J_2, J_4, J_6, \ldots, J_{2n}$, if those of degree $> 2n$ are neglected. The choice of n is a delicate matter, and at present harmonics up to degree 30 or 40 are usually evaluated.

The odd zonal harmonic coefficients $J_3, J_5, J_7, \ldots$, which are asymmetrical about the equator, are evaluated by measuring a quite different perturbation - the oscillation in the perigee distance from the Earth's centre as the perigee moves from the northern to the southern hemisphere and back. The amplitude of this oscillation varies with inclination, but is typically about 5 km, perigee usually being nearer to the Earth's centre when in the northern hemisphere. At inclinations near 63°, however, the amplitude of the oscillation can exceed 50 km. The measured amplitude gives a value of a linear sum of the odd J coefficients, say $B_3J_3 + B_5J_5 + B_7J_7 + \ldots$, and again the coefficients B vary with inclination, so that by using satellites widely distributed in inclination it is possible to solve for the individual J coefficients.

By using these methods, values of both even and odd J coefficients of steadily improving accuracy have been derived, and the resulting section of the Earth's sea-level surface (geoid) averaged over all longitudes, is shown in Fig 9. This is based on the values of even harmonics from Ref 13 and odd harmonics from Ref 14. The profile is relative to a spheroid of flattening 1/298.25, drawn as a broken line, and the diagram shows that sea level at the north pole is 45 m further from the equator than is sea level at the south pole.

Orbital analysis is a very powerful method for determining longitude-averaged coefficients in the geopotential, (a) because the continual spinning of the Earth automatically averages its effects in a perfectly uniform manner, and (b) because the effects are very large. The effect of the flattening is to move the orbital plane in longitude by up to 500 km per day, and since that longitude can be measured accurate to 0.1 km without too much trouble, the flattening should be evaluated with an accuracy of one part in 5000 from one day's observations, or one part in a million from observations over 200 days. Also the odd zonal harmonics, which create a variation of ±20 m in the sea-level surface, give rise to an orbital perturbation which is typically about 5 km - nearly 300 times the irregularity in the Earth which causes it. So the shape of the geoid should be definable with an accuracy about 300 times better than the average orbital accuracy of the satellites. Fig 9 is believed to be accurate to about 50 cm, and this is reasonable since it is derived from orbits with accuracies of about 100 m.

Because orbital analysis is so powerful a method and measures precisely the right quantity, it seems likely to continue as the prime technique for determining zonal harmonics.

5.3 Tesseral harmonics

In the full expression for the geopotential U, a double infinite series of tesseral harmonics has to be added to the longitude-averaged potential $\bar{U}$ defined in equation (1). If λ denotes longitude, positive to the east, the standard form is

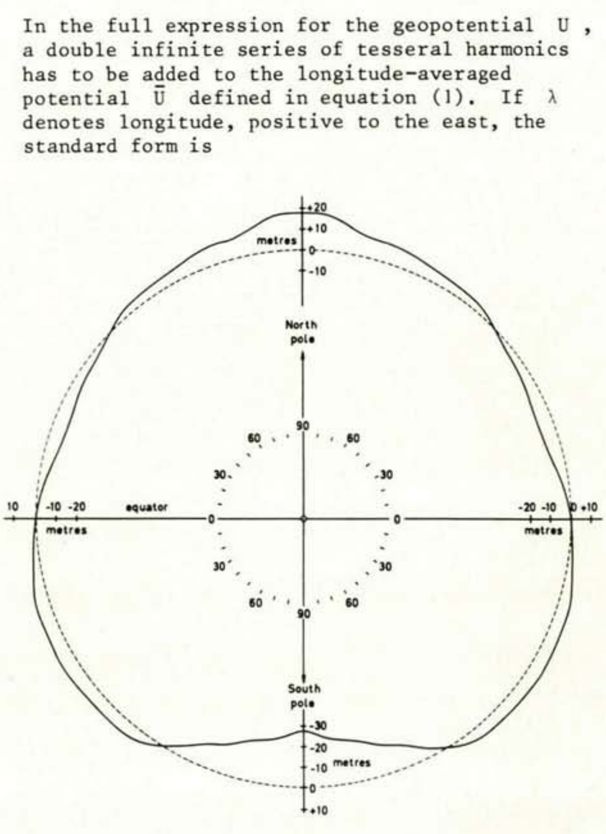


Fig 9 Height of the meridional geoid section (solid line) relative to a spheroid of flattening 1/298.25

$$U = \bar{U} + \frac{\mu}{r}\sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m}(\cos\theta)\left\{\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right\}N_{\ell m}$$

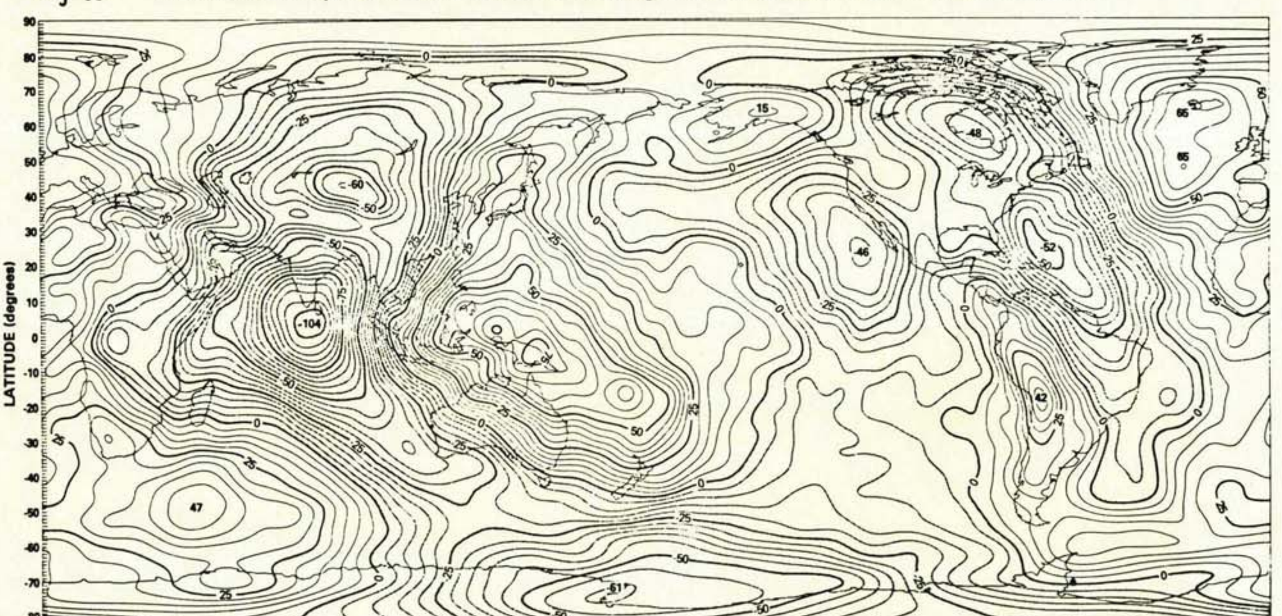
where $P_{\ell}^{m}(\cos\theta)$ is the associated Legendre function of order m and degree ℓ, the $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are constants to be determined, and $N_{\ell m} = \left\{2(\ell - m)!\ (2\ell + 1)/(\ell + m)!\right\}^{\frac{1}{2}}$ is a normalizing factor (see Ref 15). The suffix ℓ may be regarded as specifying latitudinal variations, the suffix m meridional variations.

Each harmonic coefficient $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ creates an orbital perturbation, which has to be taken into account in an accurate orbit determination. But the perturbations are generally small and periodic, and do not offer the same opportunity for accurate measurement as the much larger effects of the zonal harmonics. Consequently the recent comprehensive models of the Earth's gravitational field have been derived from a mixture of measurements: geometrical geodesy, orbit analysis, satellite altimetry, terrestrial gravity and Doppler and laser tracking have all played a part. The best recent model appears to be the Goddard Earth Model 10B (Ref 13), the geoid map being shown in Fig 10. Although the detail makes the map difficult to read, the familiar features can be picked out: the dips south of India 104 m deep and south of New Zealand 61 m deep; the humps in New Guinea 75 m high and south of Iceland 65 m high, etc.

In deriving these comprehensive gravity models, more than a million observational equations are solved for more than a thousand coefficients: GEM 10B goes to degree and order 36 and has 1300 coefficients. The correlations between these coefficients provide an obvious source of error and, although the complete gravitational field may be quite accurately represented, the individual coefficients could still be greatly in error. This is where orbital analysis has an important role, because accurate values of harmonics of certain orders can be determined by analysing orbital resonances of that order. These provide a stringent and independent test of the accuracy of the comprehensive models.

A primary orbital resonance occurs when a satellite repeats its track over the Earth after 1 day. The most frequent resonance is that of 15th order when the orbital period is such that the track repeats after 15 revolutions of the satellite while the Earth completes one rotation. In a general β:α resonance the track repeats after β revolutions of the satellite and α rotations of the Earth. For example, in the 29:2 resonance, the satellite makes 29 revolutions in 2 days and then repeats its ground track. When an orbit experiences β:α resonance, the harmonics of order β have the same effect day after day, and perturbations in the orbital inclination and eccentricity can build up until they are large enough to be accurately measured. In practice the orbits are 'dragged' through resonance, as the orbital period slowly decreases under the action of air drag. But the effects of resonance can last for several years: with one satellite recently analysed (1971-54A) the perturbations went on for 5 years and the inclination decreased continuously for 2 years by a total of 0.1° or 12 km. By fitting theoretical curves to these variations, values of lumped 15th-order coefficients - linear sums of individual coefficients - were obtained accurate to about 2% (Ref 16).

Fig. **10** **Geoid Surface Computed from the GEM 10B Model (Height in Meters Above the Mean Ellipsoid, f = 1/298.257)**



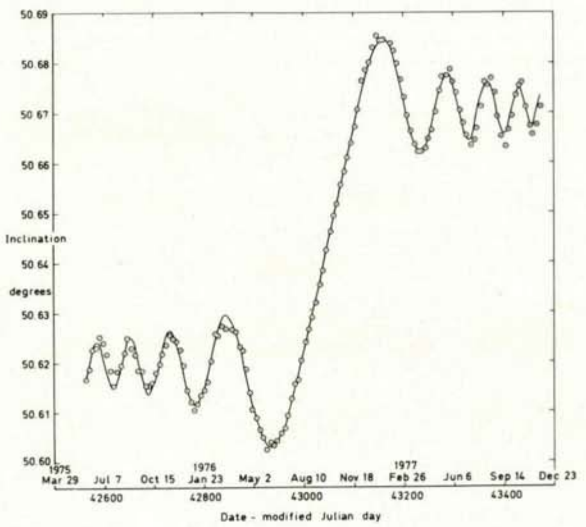


Fig 11 1974-34A: variation of inclination near 15th-order resonance

Another example, shown in Fig 11, is a recent analysis of Intercosmos 11 (1974-34A) by D.M.C. Walker (Ref 17). Here the resonance lasted for about 2 years and there was an increase in inclination of 0.08° (10 km) between June and December 1976, the date of exact resonance being 1 October 1976. Again the fitting gave values of lumped 15th-order coefficients accurate to 2%.

We have recently used these results and similar analyses of 21 other 15th-order resonant orbits to determine individual coefficients of order 15 and degree 15, 16, 17, ..., 35 (Ref 18). The orbits covered a wide range of inclination, and excellent accuracy was achieved: the coefficients of degree 15, 16, 17, ..., 23 have standard deviations of 1.4×10^{-9}, equivalent to 1 cm in geoid height.

These new values of 15th-order harmonics provide a severe test of existing gravitational field models, and most of the models fail the test. However, GEM 10B fits our values quite well. Fig 12 shows the comparison between GEM 10B and the most accurate of our values, for two of the six lumped coefficients. So, at least in its 15th-order coefficients and probably in others, GEM 10B provides a realistic model of the gravitational field.

Resonances of order 14, 13, 12, ..., can be analysed just as accurately as those of order 15 - more accurately if the drag is lower. But if these orbits are near-circular, as is desirable for accurate analysis, they are so high and suffer so little drag that most of them are unlikely to reach resonance for 20, 50 or even 1000 years. And we cannot wait so long. There have been a number of useful 14th-order resonant satellites, and many synchronous satellites from which coefficients of order 2, 4 and 6 may be obtained. But resonant orbits of order 7, 8, 9, ..., 13 are few, and progress is slow.

The 2-day resonances, particularly the 29:2 and 31:2, when the satellite makes 29 or 31 revolutions in 2 days before repeating its track, are also of interest, but the effects are much smaller than for 15th-order resonance, so the results are

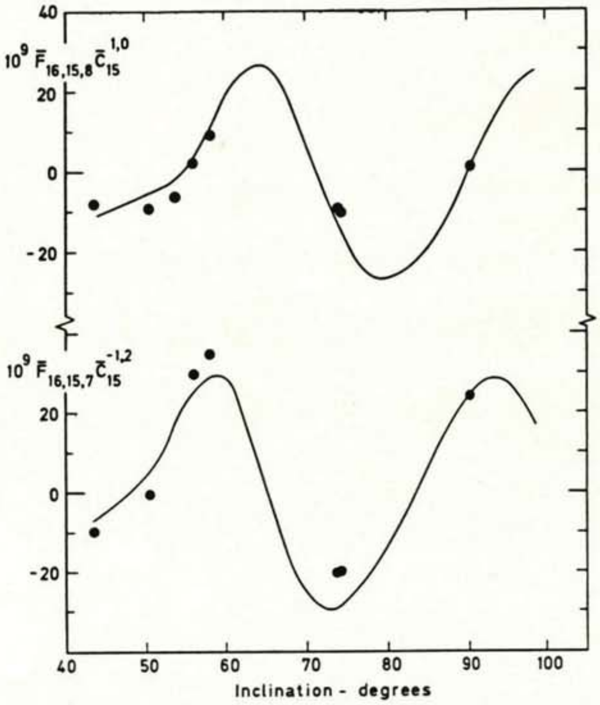


Fig 12 Values of lumped 15th-order coefficients from orbital resonance (circles) and variation given by GEM 10B

less accurate. However, good values have recently been obtained for 31:2 resonance (Ref 19) and fairly good values for 29:2 resonance (Ref 20). It is also possible to obtain values of 30th-order harmonics from the 15th-order resonances.

As the comprehensive models of the gravitational field are extended to higher order and degree, the independent check provided by the orbital resonance method seems likely to become even more important, because it is very difficult to find other independent methods of checking the accuracy of the coefficients. Already GEM 10C exists, extending to degree and order 180, with 32000 coefficients, and there is a need to estimate the likely errors from using this model in many different applications.

6. CONCLUSIONS

The analysis of changes in satellite orbits caused by aerodynamic and gravitational forces has in the past proved to be a reliable and cheap method for measuring the properties of the upper atmosphere and the Earth's gravitational field. The technique has limitations: for example, it is not possible to measure very rapid variations in air density, nor to distinguish between species of gases. But orbit analysis seems likely to remain a valuable technique in all these areas of application. It is the best method for tracing long-term variations in air density, particularly the semi-annual variations and changes over a solar cycle. It is useful in giving observational values of scale height for comparison with atmospheric models. It provides information on the rate of rotation of the upper atmosphere which is complementary to other methods. In the study of the Earth's gravitational field, orbit analysis offers the best method for evaluating zonal harmonics, and the analysis of orbital resonances gives accurate values of lumped harmonics of a particular order, which are most valuable in providing an independent test of the comprehensive models of the gravity field.

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