ACCURATE ORBIT DETERMINATIONS FROM LASER RANGE OBSERVATIONS OF LAGEOS, STARLETTE AND GEOS-3

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ABSTRACT

For many geodetic and geophysical investigations accurate orbit determinations from satellite laser ranging play an essential role. To get a physical insight in how the spatial, temporal and geographical distribution of the laser data and the adoption of different perturbation models affect the accuracy of the orbital solutions, some detailed analyses have been performed for a limited number of selected data arcs of three geodetic satellites. These arcs were formed from laser measurements acquired during 504 satellite passes over three European and nine American laser stations. The laser data were used to solve simultaneously for the satellite's orbit, the coordinates of two European laser stations and some satellite parameters. In particular, attention has been paid to the sensitivity of the orbit determination to the modeling of the earth's gravitational field and atmospheric drag.

Keywords: Orbit determination, least-squares parameter estimation, geodetic satellites, laser range observations, gravity models, atmospheric models, accuracy analysis.

1. INTRODUCTION

Since summer 1979, the Working Group for Satellite Geodesy (WSG), the Geodetic Computing and Analysis Center (LGR) and the Section Orbital Mechanics (SOM) of Delft University of Technology take part in a NASA project, titled "Data use investigations for the Laser Geodynamics Satellite (LAGEOS) mission". The primary objective of the combined dutch efforts is to evaluate the reliability of geodetic network structures as computed from satellite laser range measurements, in particular with respect to the determination of relative groundstation positions. The availability of accurate relative positions is essential for the demonstration of relative movements and deformations of the earth's tectonic plates. The investigations at Delft University proceed along two inseparable main lines, briefly characterized by "statistical simulation" and "data utilization". WSG and LGR mainly work along the statistical simulation line; SOM is mainly involved in the data utilization approach. This paper deals only with the investigations by SOM.

Within the Dutch combined investigations, SOM also utilizes the laser ranging data to study: the detailed effects of atmospheric drag and solar radiation pressure on the orbits of geodetic satellites, the sensitivity of the computed orbit to different models of the earth's gravitational field, the application of Kalman filter techniques for semi-realtime improvement of the pointing accuracy of ground-based lasers, and the suitability of B-spline approximations for laser ranging data compression. In these studies, the NASA GEODYN/ORAN computer programs (Ref. 1) are used extensively. These programs were implemented in 1979 on the Delft University IBM 370/158 computer. This program system has the ability to determine orbits from a variety of tracking data types and is capable of estimating from single or multiple arcs various geophysical parameters, such as polar motion and earth rotation, tidal parameters, geopotential coefficients, as well as satellite parameters, such as drag coefficients. In addition, systematic timing and measurement errors may be estimated. It uses numerical techniques to integrate the equations of motion and the variational equations, and Bayesian least-squares techniques for the adjustment of the orbital and model parameters.

To characterize the quality of the laser ranging data used in these geophysical studies, the following example may serve. From the Kootwijk (Netherlands, station number 7833) laser ranging station, WSG routinely acquires day and night ranging data for the geodetic satellites STARLETTE and GEOS-3, and only at night for LAGEOS. The output energy of the ruby pulse-laser system is 1 to 2 J. The transmitted laser beam has a diameter of 19 cm and the divergence is adjustable from 1 to 20 arcminutes. Untill recently, the system generated 4 ns wide pulses at a maximum rate of 15 pulses per minute, producing measurements with an accuracy of about 25 cm root-mean-square (rms). In 1980 the accuracy level has been improved to about 15 cm, by reducing the pulse width to 2 ns. The "third-generation" NASA systems operate at an output energy of about 0.2 J per pulse, with a pulse length of the order of 0.2 ns and a repetition rate of several pulses per second. Single-shot accuracies of 7 cm have been demonstrated.

To get some experience in processing large amounts of laser data and in solving for geophysical parameters from long-arcs simultaneously with the orbits, preliminary studies have been performed by SOM, in which the coordinates of the Wettzell laser station (Fed. Rep. Germany, station 7834) were estimated. In the first of these studies (Ref. 2), 32 passes of LAGEOS, STARLETTE, GEOS-1 and GEOS-3 over only two stations, Kootwijk and Wettzell, were used. The Kootwijk coordinates were held fixed at values obtained from a recent NASA solution. The arc lengths used varied from 2 to 8 days. It was found that even from this very limited amount of data the Kootwijk-Wettzell baseline (chord distance) of about 600 km could be computed with an estimated accuracy of better than 0.7 m. In a follow-up study (Ref. 3) laser measurements from Kootwijk, Wettzell and San Fernando (Spain, station 7804) were processed. The data were acquired during 58 passes of LAGEOS, GEOS-1 and GEOS-3, distributed over 5 arcs with lengths of 2 to 8 days. Again the orbits and the Wettzell coordinates were solved for. That study yielded improved values for the coordinates of Wettzell and information on the sensitivity of the recovered coordinates to the different orbital characteristics of the satellites used in the solutions.

In this paper, results of a more-recent study are described. This study is based on laser ranging data of LAGEOS, STARLETTE and GEOS-3 acquired within the period July-October 1978 during 504 satellite passes over Kootwijk, Wettzell, San Fernando and nine laser station operated by NASA and the Smithsonian Astrophysical Observatory (SAO). General data of the satellites involved are given in Table 1.

Table 1. Satellite data

	STARLETTE	GEOS-3	LAGEOS
Satellite number	7501001	7502701	7603901
Launch date	Febr. 6	April 9	May 4
Shape	sphere	octahedron with radar altimeter dish on down end and truncated pyramid on top	sphere
Dimensions (cm)	24 Ø	122 wide 131 high	60 Ø
Mass (kg)	47.25	345.91	407.82
Cross-sect. area (m^2)	0.04524	1.4365	0.28263
Stabilization	-	gravity-gradient	-
Orbit			
a (km)	7335	7221	12271
e	0.0207	0.0014	0.0044
i (deg)	49.8	115.0	109.9
P (min)	104	102	225

The orbital parameters listed refer to the mean orbital elements of July-August 1978. The observations compiled by Kootwijk and Wettzell were taken from the Kootwijk databank; data from the NASA and SAO stations were obtained from the Goddard Space Flight Center, World Data Bank A. From these data a number of solutions was computed for the coordinates of Kootwijk and Wettzell, applying a long-arc analysis, with arc lengths of 4 to 13 days. The purpose of this investigation was not to produce extremely accurate results, but to obtain physical insight in how the choice of a particular set of observations, and the application of different perturbation models, influence the computed results. Data on the satellite's solar reflectivity, the atmospheric drag experienced by STARLETTE and GEOS-3 and an unmodeled along-track acceleration for LAGEOS were obtained. For each satellite the sensitivity of the orbital solution and the estimated parameters to the applied gravitation model was investigated. For GEOS-3 also the introduction of a time-varying drag coefficient to absorb atmospheric model deficiencies was studied.

2. ARC SELECTION

Orbit determinations are conventionally divided into short-arc and long-arc solutions. Typically, a short arc covers a period of a few minutes or part of a revolution. Long-arc solutions utilize tracking data over many satellite revolutions and from worldwide networks of tracking stations. Short-arc solutions require intense tracking coverage by several stations in the same geographical region for intervals that are fractions of an orbital period. The short-arc analysis is less influenced by small dynamic model errors than the analysis of arcs of much longer duration, but they have the disadvantage of being rather dependent on the data distribution within the arc.

Many geophysical applications, including absolute station positioning, require orbits with an accuracy of some decimeters or better over periods of a few revolutions to several days. This necessitates the accurate modeling of all perturbing forces acting on the satellite. However, our ability to determine the orbits of geodetic satellites has not yet reached the level of accuracy of the laser data. In fact, there is a time lag between the observational and model accuracies, since the highly accurate observations are also used to improve upon the perturbation models.

It will be clear that, in general, an accurate orbit determination requires the observations to be evenly distributed over the arc and taken at stations having a good geographical distribution. In addition, for the position determination of Kootwijk and Wettzell it would be attractive to have passes over these stations shortly before or after passes over the other stations of which the coordinates are accurately known. The orbital geometry also plays an important role. For the computation of baselines, for example, it is advantageous to have sub-satellite tracks nearly parallel to the interstation baseline. In reality, however, one has hardly any choice and one has to work with the available data.

As one of the intentions of the present study was to extend the results of our earlier investigations (Refs. 2, 3), data arcs had to be formed out of the available data acquired during summer and autumn of 1978. It turned out that there were only very few periods in which both sufficient Kootwijk and Wettzell data were available and a sufficient number of NASA and SAO laser stations had tracked the satellites, such that a reasonable orbital coverage was guaranteed. Even within these periods there were relatively large gaps between data passes over Kootwijk and Wettzell, necessitating the adoption of a long-arc analysis. Finally, only two arcs suitable for the determination of the coordinates of Kootwijk and Wettzell could be formed for each satellite. An additional arc with no data passes over Wettzell was selected for STARLETTE, and two additional arcs for GEOS-3. A summary of the data arcs is presented in Table 2. For each arc the following quantities are listed: the arc identification used in this paper, the satellite involved, the arc-length, the number of stations contributing to the observations of that arc, the total number of satellite passes, and the total number of ob-

Table 2. Arcs selected for data analysis.

Arc id.	Satellite	Length (day)	Stations	Passes	Observations
L1	LAGEOS	8.1	9	44	2549
L2	LAGEOS	11.6	6	56	4813
S1	STARLETTE	6.5	9	56	2755
S2	STARLETTE	4.1	7	41	1914
S3	STARLETTE	3.8	9	35	1781
G1	GEOS-3	7.1	8	59	2965
G2	GEOS-3	4.6	9	42	1903
G3	GEOS-3	11.6	8	75	3727
G4	GEOS-3	12.7	11	96	4172

servations used in the solutions. From all data available for a specific arc, this number of data points remained after some hand-editing had been performed, deleting passes or parts of passes that contained too many obvious wild data points, and after the number of data points for some stations had been reduced to prevent stations with a high laser pulse repetition rate to dominate the solution. The amount of tracking data averages at about 5, 9 and 8 passes per day for LAGEOS, STARLETTE and GEOS-3, respectively. All arcs, except for arc L2 comprise passes over Kootwijk, the average number of Kootwijk passes per arc being 5. Passes over Wettzell are present in the arcs L1, L2, S1 and G1 and average also 5. On many occasions there is a gap of one or two days between successive passes over Kootwijk or Wettzell. The arcs G3 and G4 were not used for the computation of station coordinates, but were selected specifically for the investigation of atmospheric effects. In Fig. 1 the sub-satellite tracks for the observed parts of the passes are plotted for arc 1 and 2 of each satellite. These plots give an indication of the limited geographical data distribution and clearly demonstrate the necessity of having additional laser stations on the African continent and in the far-east.

3. COMPUTATION MODEL

For Wettzell and the NASA stations, an a-priori observation accuracy level of 15 cm was assumed. The Wettzell data were first corrected for a time-tagging error in the data format (Ref. 2), but not for small pass-dependent range biases that contaminate the data due to laser system calibration problems (Ref. 4). For Kootwijk a value of 25 cm was taken and for all SAO stations, except Orroral Valley (Australia, station 9943), an accuracy of 70 cm was assumed. The data of Orroral Valley were corrected for a range-dependent error caused by instrumentation problems at that station (Ref. 5). This correction also necessitated the use of a range-dependent standard deviation, being over 1 m for LAGEOS.

For the numerical integration of the equations of motion and the variational equations, a fixed-step-size 11th-order Cowell predictor-corrector method was used. The stepsize was 120 s for LAGEOS, 100 s for GEOS-3 and 80 s for STARLETTE. The choice of the stepsize was a compromise between the desire to have a small stepsize in order to account properly for the high-frequency terms of the geopotential and the advantage of having a large stepsize to limit the computer time. The error resulting from too large a stepsize is predominantly along track. Thus, if drag or along-track acceleration is being adjusted in the orbit determination process, the integration error will be largely absorbed by these parameters. All tracking coordinates, except those of Kootwijk and Wettzell, were held fixed at their values in the NASA so-called Modified New Orleans (MNO) set of coordinates (Ref. 6).

To model the earth's gravitational field three Goddard Earth Models (GEM) were used, along with a value of the earth's gravitational parameter $GM = 398600.64\ km^3s^{-2}$ and a mean equatorial radius of $a_e = 6378.138$ km. The reference model used in all comparisons was GEM-9 (Ref. 7), which is based solely on optical, laser and electronic observations taken on 30 satellites. The model is complete to degree and order 20 in the spherical harmonics, with selected coefficients to degree 30. For LAGEOS the effects of truncating this model to terms of degree and order 20 or 13 were investigated. For comparison, also the GEM-10B model (Ref. 8) was used. This model is a combination solution, containing a global set of 5° by 5° surface gravity anomalies along with the data in GEM-9 and over 700 passes of GEOS-3 altimetry, and is complete to degree and order 36 in the harmonic coefficients. GEOS-3 is known to be in resonance with the 13th and 15th order terms of the geopotential in a mild way, and has strong resonance perturbations from the 14th and 28th order terms. As all terms of degree 31 and above were derived solely from the altimeter and surface gravity data, the GEM-10B model slightly degrades the computation of the satellite's along-track component. To improve the along-track position computation of GEOS-3, Ref. 8 therefore recommends different values for some relevant geopotential coefficients. The gravity model thus obtained is indicated in this paper by GEM-10BM and was only applied to the GEOS-3 arcs.

For all satellites, the orbit perturbations due to solar and lunar attraction, direct solar radiation pressure and solid earth tides were accounted for. The JPL planetary ephemeris DE-96 was adopted along with the BIH polar motion and UT1 data. The luni-solar earth tides were modeled through a second-degree spherical harmonic, characterized by the Love number $k_2 = 0.29$ and a phase lag $\phi_2 = 2.5^\circ$. In addition, the geometric tracking station displacements due to tidal effects were taken into account, and modeled by the Love and Shida numbers $h_2 = 0.6$ and $l_2 = 0.075$, respectively. Atmospheric drag perturbations were computed for STARLETTE and GEOS-3, using the Jacchia 1971 reference atmosphere (Ref. 9). The adopted values of the satellite mass and constant cross-sectional area are listed in Table 1. Though the assumption of a constant cross-sectional area may seem unreal-

Fig. 1. Sub-satellite tracks of LAGEOS (top), STARLETTE (center) and GEOS-3 (bottom).

istic for a satellite like GEOS-3 with its irregular shape, it turned out to give satisfactory results, which is partly due to the fact that GEOS-3 is gravity-gradient stabilized. Preprocessing corrections, such as tropospheric refraction and transit time delay were applied to all laser observations as requested by the data-record. In addition, all range data were corrected for the range offset between the laser retroreflectors and the satellite's center of mass.

The "solve-for" parameters in the computation process were the satellite's state-vector at epoch, its solar reflectivity and drag coefficient (only for STARLETTE and GEOS-3), for LAGEOS an unmodeled along-track acceleration, and the Kootwijk and Wettzell coordinates within the MNO system. The reason that for LAGEOS an along-track acceleration was estimated is that the satellite's orbit is known to exhibit a small but discernable decrease of the semi-major axis by about 1 mm per day (Ref. 10) and that the atmospheric model used does not extend to the altitude of LAGEOS. The epoch of the state-vector was selected closely ahead of the first measurement of each arc.

Although the satellite's reflectivity and drag coefficient, or along-track acceleration, may be considered as simple scaling factors to absorb model errors, they do contain physical information. Therefore, in this study it was assumed that the values recovered for these parameters both give information on the actual surface forces acting on the satellite and provide an indication of the absolute accuracy of the total parameter estimation solution. The adjusted along-track acceleration of LAGEOS and drag coefficient of STARLETTE were considered to be time-independent during each arc, just as the reflectivity of all satellites. As the orbit of GEOS-3 is much more affected by atmospheric drag, due to its lower mean altitude and considerably larger area-to-mass ratio, the drag coefficient of this satellite was allowed to vary with time during periods of enhanced geomagnetic activity. In that case, a technique of adjusting multiple drag coefficients was applied in which each coefficient covers a time interval of one or more integral days. Consecutive coefficients may then take different values. The physical justification for this approach is the inability of the current atmospheric models to account for rapid variations in the atmospheric conditions, especially those related to geomagnetic storms. To compensate for this, multiple drag coefficients are used to absorb part of the atmospheric model deficiencies.

For the GEOS-3 arcs, which are the only ones that comprise observations from San Fernando, also a range bias for that station was solved for, because it was known that the data were in error due to incorrect tracking point corrections. A number of single-arc solutions for the Wettzell coordinates was obtained. In these cases arcs were used that did not contain Kootwijk observations, or for which the Kootwijk measurements were deliberately left out. Simultaneous Kootwijk and Wettzell coordinate solutions were obtained from two-arc solutions for each satellite.

Many different computer runs were made with GEODYN, each run being different in terms of perturbation models applied, observations processed or the number of adjusted parameters. In the following discussion only some representative examples of the results will be presented. This discussion will focus mainly at the orbital mechanics aspects, and not at the geodetic interpretation of the station coordinate solutions.

Table 3. Summary of range residual statistics for some selected stations. Listed are mean/rms values in meters.

Arc id.	Model	Kootwijk	Wettzell	Greenbelt	San Diego	Arequipa
L1	GEM-9(20)	0.02/0.48	-0.06/0.20	-0.15/0.20	0.28/0.37	0.01/1.07
L2		-	-0.00/0.20	-0.22/0.52	0.19/0.42	-0.19/1.23
S1	GEM-10B	0.78/2.27	-0.09/1.18	0.25/1.00	-0.57/1.51	-1.29/3.21
S2		-0.26/1.27	-	-	-0.37/1.37	-0.22/4.02
G1	GEM-9	0.39/1.99	0.03/0.94	-1.31/1.59	-	0.11/2.92
G2		-0.40/1.60	-	0.64/0.91	-0.18/1.73	-0.27/2.58
G1	GEM-10B	0.18/1.60	0.06/1.72	-0.72/1.83	-	-0.43/2.53
G2		-0.17/1.80	-	0.45/0.51	0.01/1.96	-0.17/2.71
G1	GEM-10BM	0.16/1.45	0.01/1.03	-0.54/1.65	-	-0.31/2.92
G2		-0.12/1.33	-	0.74/0.97	-0.13/1.87	-0.16/2.25

4. ORBITAL FIT

An important yardstick to judge the overall accuracy of the orbit determination and parameter estimation is the behavior of the range residuals. These are defined as the actual measurements minus the range values computed from the orbit determined within the parameter estimation process. These residuals are a measure of how well the computed orbit fits the actual measurements. If the majority of the residuals plotted per pass as a function of time do not lie within a band about zero, having a width in the order of the measurement's accuracy, it is a clear indication that the modeling was not optimal. On the other hand, if the residuals are nicely scattered about zero, this does not necessarily mean that the solution is correct. It only proves that some solution has been obtained that fits the observations; but the recovered values of the individual parameters need not to be physically correct. Statistical reliability analyses techniques, as in development by LGR, or a comparison with independently derived values for the parameters involved, are required to enable a definite judgement of the real accuracy of the results.

Using arcs of more than a few orbital revolutions one may expect that the history of the residuals per pass will show some signature, which reflects the effects of model errors. Of course, by increasing the number of adjustable parameters, one may artificially reduce this signature. Though this can be of significance for the analysis of the accuracy of the laser data themselves, it does not add anything to the real accuracy of the orbital solution.

As an example of the residual histories obtained in this study, Figs. 2 and 3 are included. These plots show residuals from the two-arc solutions L1+L2 and S1+S2, for passes of both satellites over Kootwijk (7833), Wettzell (7834) and Greenbelt (7063). In each plot, the satellite and station number, the date of the pass and the origin of the time scale for that pass are indicated. In Table 3 a summary is presented of the two-arc laser residual statistics for all passes over some selected stations. The laser positions at San Diego and Arequipa are specified by the station identification numbers 7062 and 9907, respectively. In these two-arc solutions, also the satellite's reflectivity and drag coefficient (STARLETTE and GEOS-3) or along-track acceleration (LAGEOS) were solved for, together with the coordinates of Kootwijk and Wettzell. When reading this Table it should be realized that the adjustment of the Kootwijk and Wettzell coordinates in these solutions has contributed to the relatively small

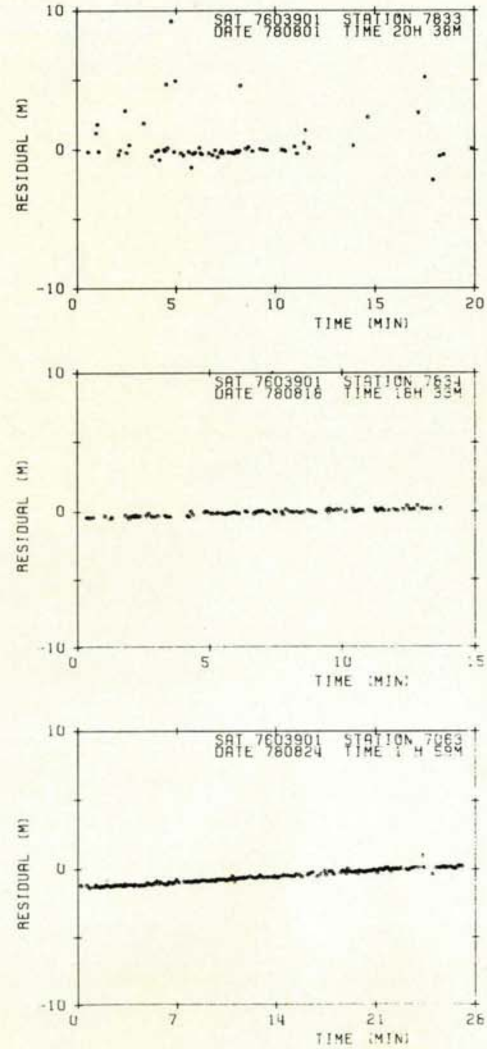

Fig. 2. Examples of range residuals for LAGEOS passes over Kootwijk (7833), Wettzell (7834) and Greenbelt (7063).

mean and rms values of the residuals for these stations. For LAGEOS an accurate orbital fit was obtained, having a mean residual value of less than 30 cm. The higher noise level of the SAO Arequipa observations is responsible for that station's residual rms values of up to 1.25 m. For STARLETTE and GEOS-3 less accurate orbital fits

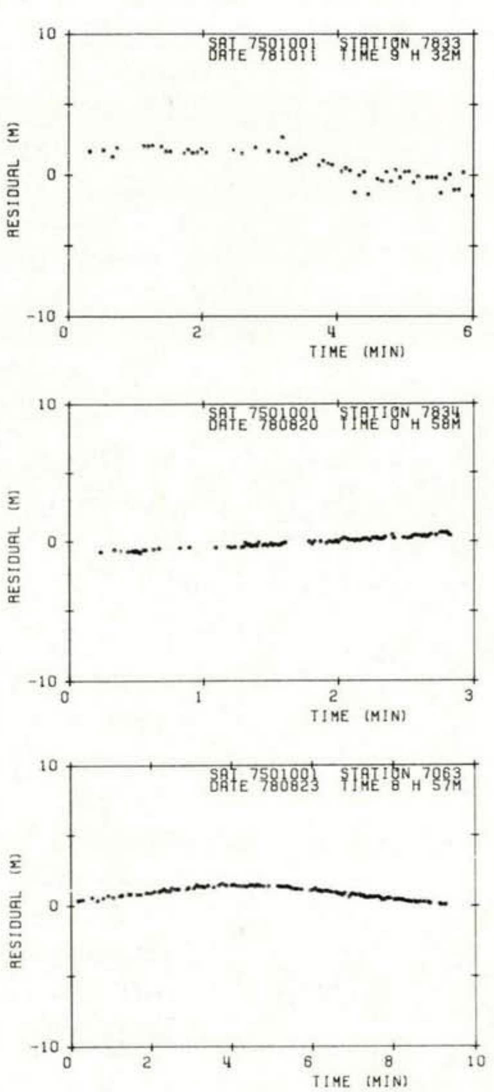

Fig. 3. Examples of range residuals for STARLETTE passes over Kootwijk, Wettzell and Greenbelt.

were obtained, the worst fit occurring for STARLETTE. This is for a large part a result of the fact that STARLETTE's orbit is sensitive to high-order terms of the geopotential, which are not modeled in GEM-10B. Table 3 also clearly shows that the GEM-10BM model leads to the best results for GEOS-3.

As another way of grasping the overall accuracy of the orbital fit, the two-arc solutions of LAGEOS, STARLETTE and GEOS-3 were, after convergence had been reached, extended with a final iteration in which the apparent range and timing biases were determined for each pass over a station. These biases were intended to represent the actual radial and along-track errors of the orbit in the region covered by that groundstation, assuming no tracking system errors. In these computations, the passes over Orroral Valley and San Fernando were excluded. As an example, Fig. 4 shows histograms of the range and timing biases for LAGEOS. The mean values and standard deviations of the apparent range and timing biases from all two-arc solutions are listed in Table 4. Interpreting

Table 4. Mean/standard deviation values of the apparent range and timing biases (excluding passes over Orroral and San Fernando).

Arc id.	Model	Range bias (m)	Timing bias (ms)
L1 + L2	GEM-9(20)	-0.08/0.41	0.06/0.44
S1 + S2	GEM-10B	-0.37/1.84	-0.16/0.52
G1 + G2	GEM-9	0.10/1.49	-0.06/0.54
G1 + G2	GEM-10B	0.04/0.94	-0.13/0.59
G1 + G2	GEM-10BM	0.05/0.93	-0.04/0.54

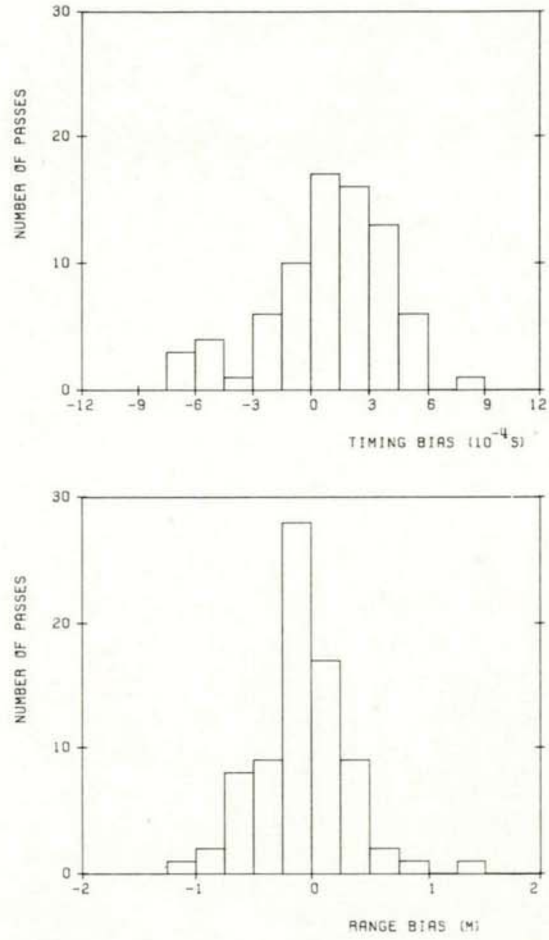

Fig. 4. Histograms of the apparent range and timing biases for LAGEOS as computed from a two-arc orbit solution.

this Table, it should be remembered that the laser data were recorded in regions of the world where the gravity model errors probably are minimal. This implies that the orbital accuracy may be worse over other parts of the world. Nevertheless, if the biases are taken as a measure of the global accuracy of the orbital fit, it may be concluded that the orbit of LAGEOS has been computed with an accuracy of about 0.5 m rms in radial direction and of about 2.5 m rms along track. For STARLETTE and GEOS-3, an along-track accuracy of about 4 m rms has been obtained. It is pointed out that these accuracies reflect the total model errors, including station position errors and perturbation model errors.

For a further indication of the strength of the orbital solution also the values recovered for additional model parameters may serve. By comparing these values with values that follow from physical theory or that were obtained by other investigators, one may obtain at least some impression of the quality of the solution. As an example, for LAGEOS an along-track acceleration of about -2.8×10^{-12} m s^{-2} was recovered, which corresponds to a semi-major axis decrease of about 1 mm per day. This value agrees well with the results of the extensive investigations described in Ref. 10. For STARLETTE, drag coefficients ranging from 3.2 to 3.6 were obtained, which is in complete agreement with the theoretical values for spherical satellites at this altitude. The recovered solar reflectivity values range from 1.10 to 1.15 for LAGEOS and 1.3 to 1.5 for STARLETTE and GEOS-3. These numbers are in accordance with what can be expected from the known surface characteristics of each satellite. The range bias recovered for the San Fernando observations is about 20.5 m, which is in agreement with NASA studies.

5. GRAVITY MODEL EFFECTS

One of the areas that has shown greatest improvement during the last decade is the modeling of the earth's gravitational field. The availability of high-precision measurements from doppler and laser tracking systems, and from satellite altimetry has enabled the definition of the geopotential to extend out to degree and order 36 and even higher. However, for geodetic satellites, in particular the satellites in relatively low orbits, the remaining deficiencies of these sophysticated gravity models still are an important source of orbital errors. The high-altitude orbit of the main geodetic satellite LAGEOS was purposely selected to minimize the effects of the total perturbation model errors. In fact, its orbit was a compromise between the requirement to minimize the influence of model uncertainties and specific geometrical orbit considerations demanding that the satellite could be tracked from many groundstations.

To get an impression of the accuracy of present gravity models, a number of numerical experiments was conducted. In this paper only the results for some single-arc solutions will be reported.
For each satellite an arc was selected that did not contain Kootwijk observations, or the observations from this station were deliberately left out in order to eliminate the chance of errors due to Kootwijk coordinate uncertainties. In the computations, the orbit, the satellite's reflectivity and drag coefficient, or, for LAGEOS, the along-track acceleration, were solved for. If Wettzell data were available for that arc, the Wettzell coordinates were also adjusted. For each arc, first a solution was made using the GEM-9 model. The satellite emphemerides at time intervals equal to the integration stepsize were extracted from the solution for the complete arc length. Then, the computations were repeated, using the same observations, but applying a different gravity model. Subsequently, the state-vector differences at every integration step were transformed to position differences in radial, cross-track and along-track components, relative to the orbit computed with GEM-9.

It was found that truncating the GEM-9 model to terms of order and degree 20 hardly affects the orbit determination of LAGEOS. A further truncation to order and degree 13 leads, as is shown in Fig. 5, to position differences of less than 15 cm in arc L2. The distinct periodic variation in the along-track component with a period of about 4 days is primarily due to the neglect of higher degree terms of the 13th and 14th order in the geopotential. According to Fig. 6, replacing GEM-9 by GEM-10B yields much larger effects of up to about

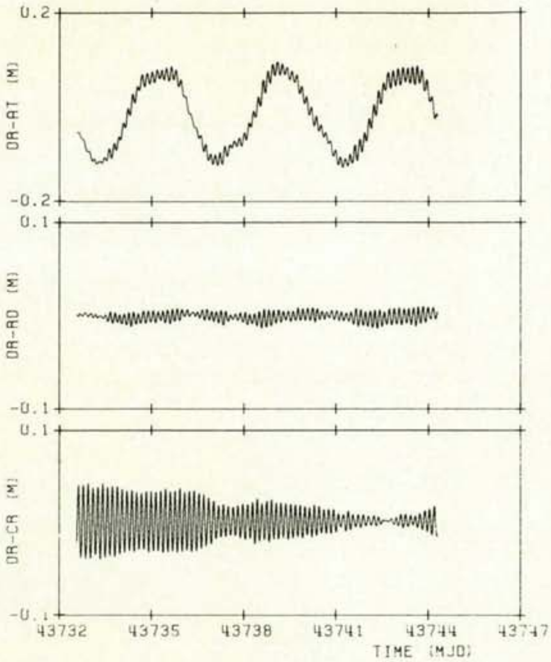

Fig. 5. The effects of truncating the GEM-9 gravity model to terms of order and degree 13 on a LAGEOS orbit solution. Position differences are given in cross-track (bottom), radial (center) and along-track (top) components.

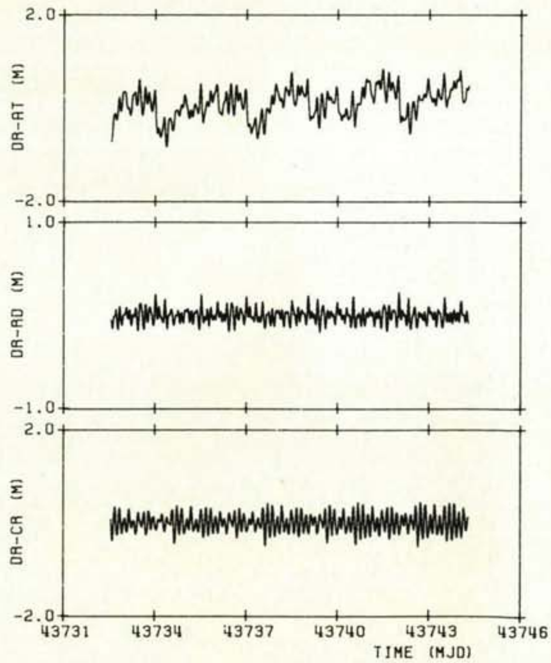

Fig. 6. The effects of replacing GEM-9 by the GEM-10B gravity model on a LAGEOS orbit solution.

Table 5. Summary of the gravity model effects on the orbit solutions. Mean/rms values of position differences are given in meters relative to GEM-9 orbit solutions.

Arc id.	Model	Orbital differences radial	cross-track	along-track
L2	GEM-9(20)	-0.00/0.00	0.00/0.00	0.01/0.01
L2	GEM-9(13)	-0.00/0.01	0.00/0.02	-0.02/0.07
L2	GEM-10B	-0.00/0.07	-0.00/0.20	0.11/0.33
S1	GEM-10B	-0.00/1.20	-0.02/1.15	0.88/4.24
G2	GEM-10B	0.01/1.60	0.01/1.48	-1.30/5.36
G2	GEM-10BM	0.01/1.46	0.01/1.77	-1.67/4.77

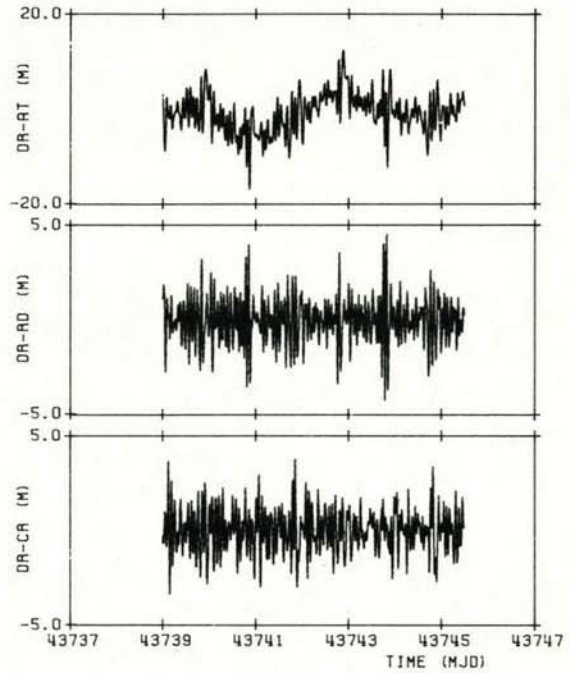

Fig. 7. The effects of replacing GEM-9 by the GEM-10B gravity model on a STARLETTE orbit solution.

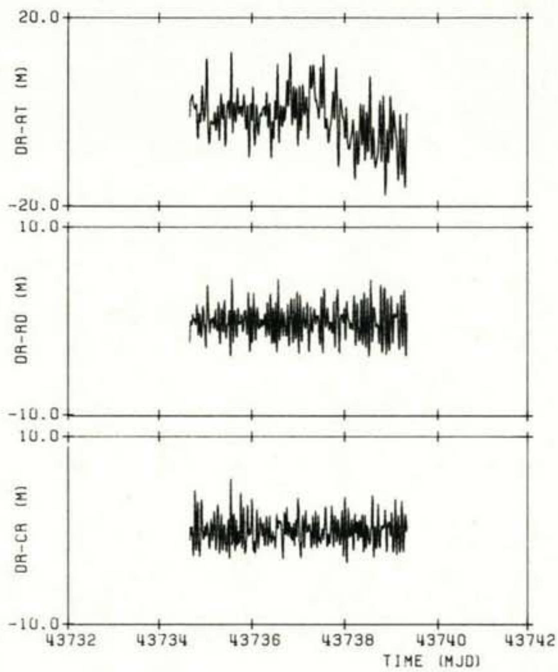

Fig. 8. The effects of replacing GEM-9 by the GEM-10B gravity model on a GEOS-3 orbit solution.

1 m in the along-track position.
This immediately implies that there is an appreciable difference between GEM-9 and GEM-10B in terms of the summed contributions of the geopotential coefficients below degree 14. In Figs. 7 and 8 corresponding information is given for STARLETTE and GEOS-3. For both arc S1 and arc G2 the orbital comparison between the GEM-10B and GEM-9 solutions is depicted. The differences are considerable larger than for LAGEOS, reaching 4.5 m for the radial component and 17 m for the along-track component. Clearly recognizable is an oscillating behavior of the along-track position difference for STARLETTE with a period of about 3.5 days. This could be the result of geopotential terms of degree 21 and above that are not modeled in GEM-9. In Table 5 the mean and rms values of the position differences are summarized.

6. ATMOSPHERIC DRAG EFFECTS

Due to its relatively low mean altitude and high area-to-mass ratio, the orbit of GEOS-3 is appreciably influenced by atmospheric drag. If the satellite's aerodynamic characteristics and the atmospheric conditions were known perfectly, this would not hamper the precision of the orbit computations. However, it is known that the current atmospheric models are far from perfect and that the computed atmospheric density exhibits both systematic and short-period errors. The first type of errors does not affect the orbital solutions when a drag coefficient is also solved for. However, the effects of sudden solar or geomagnetic disturbances on the atmospheric density may cause serious problems. These phenomena lead to short-period density fluctuations that are not modeled accurately in the atmospheric models. This results in short-term errors in the computed atmospheric drag, which cannot be absorbed by solving for a single drag coefficient.

A possible approach to handle this type of atmospheric model errors is to use multiple drag coefficients. In principle, when the drag coefficient is assumed to remain constant for a period of only a few hours and if a series of such drag coefficients is solved for, covering the whole arc length, they may accomodate atmospheric model errors. Then, it can be expected that the time history of the drag coefficients will exhibit a correlation with the solar and geomagnetic activity, expressed by the usual parameters: solar radio flux density at 10.7 cm wavelength, $F_{10.7}$, and the planetary geomagnetic index, A_p. From the orbit dynamics point of view, this approach is somewhat questionable as it may deteriorate the strength of the orbital solution, in particular when only a few data passes per day are available. Therefore, the selection of the periods during which the drag coefficient is allowed to vary with time has to be made very carefully.

The arcs G3 and G4 were deliberately selected for testing the validity of the multiple drag coefficient technique, because near the middle of each arc a short period of enhanced geomagnetic activity was present. The selection criterion applied for the drag coefficient time intervals imposed that if the geomagnetic index did not change too much during some consecutive days, a single drag coefficient was to be used for that sequence of days. Only during the days that the planetary geomagnetic index exhibited significant variations, the drag coefficient was allowed to change from day to day. In Fig. 9 the recovered values of the drag coefficients in arc G4 along with the actual daily values of $F_{10.7}$ and A_p are plotted, reveiling a strong correlation between the drag coefficient and the geomagnetic index. The sharp decrease in the drag coefficient time history after the peak of the geomagnetic activity may be attributed to discontinuity effects. In order to enable a smooth transition of the orbit from the day of the high geomagnetic activity to the following quiet period, the value of the drag coefficient first drops below its nominal value to compensate for the unrealistic high value during the previous day, before returning to normal. In this orbit computation the GEM-9 gravity model was used and, apart from the multiple drag coefficients, the state-vector at epoch, a San Fernando range bias and the satellite's solar reflectivity were solved for.

To demonstrate the improvement in the accuracy of the orbit computation by using the multiple drag coefficients, the computations were repeated for a single drag coefficient covering the whole arc. The two resulting orbits, which were derived from exactly the same laser range data, were then differenced at every integration step over the arc length. The differences in radial, cross-track and

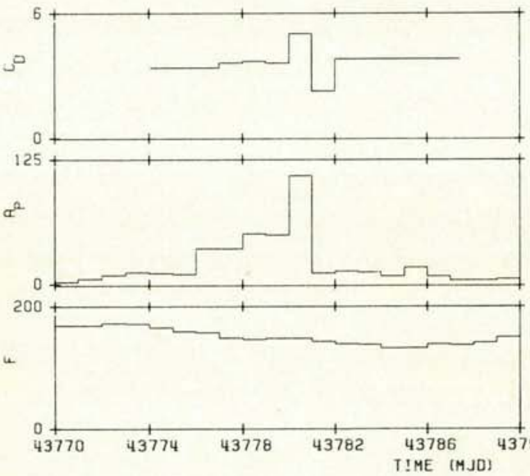

Fig. 9. The actual values of the 10.7 cm solar radio flux density and the geomagnetic index along with the recovered GEOS-3 drag coefficients for arc G4.

along-track components, relative to the orbit computed with the multiple drag coefficients, are plotted in Fig. 10. This Figure clearly shows that if the usual approach of using a single drag coefficient had been adopted, the errors in modeling the effects of geomagnetic disturbances on the atmospheric density would have led to along-track position errors of more than 30 m for this arc.

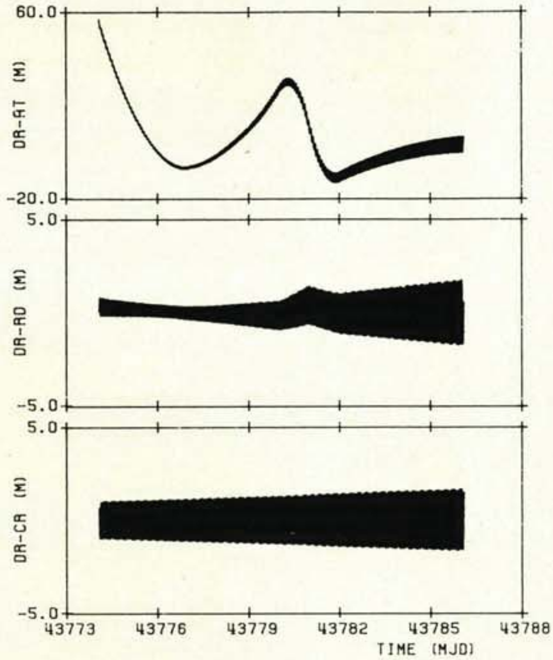

Fig. 10. The effects of replacing the multiple drag coefficient approach by a single drag coefficient on the orbit determination of GEOS-3 for arc G4.

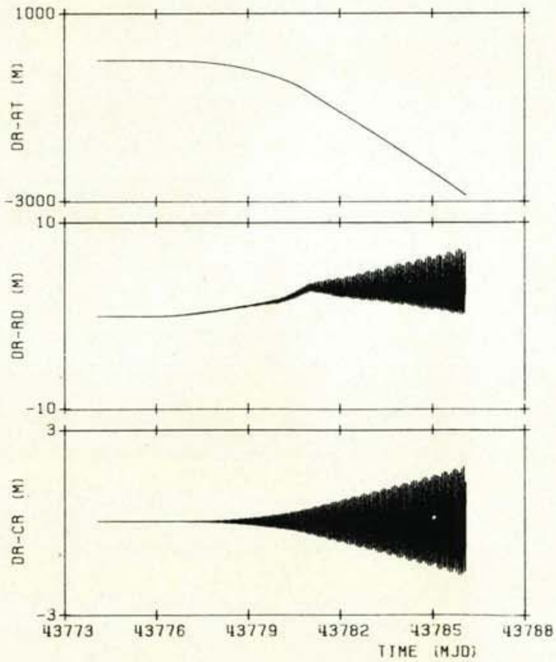

Fig. 11. Orbit prediction errors for GEOS-3 if the geomagnetic storm occurring in arc G4 had not been modeled.

In orbit predictions, one of the problems is that solar and geomagnetic disturbances are hardly or not predictable. To investigate the effects of a sudden unpredicted geomagnetic storm on the orbit of GEOS-3, the following computational experiment was conceived. Starting from the solution of arc G4 in which the multiple drag coefficients were used, an orbit prediction was generated, covering the time span of the complete arc. The force model and the recovered parameters were left unchanged, except for the drag coefficient and the geomagnetic index, which were both fixed at their epoch values. By differencing the two orbits the effect of the geomagnetic disturbance occurring in the middle of this arc could be identified. The results are plotted in Fig. 11, showing the immense effects of geomagnetic storms and illustrating the considerable difficulties in generating very accurate orbit predictions for GEOS-3.

7. INTERSTATION BASELINES

From the laser range observations of each satellite a number of individual solutions was obtained for the coordinates of Kootwijk and Wettzell. In this paper, only some results of two-arc solutions for the relative station positions will be presented. In these two-arc solutions the coordinates of Kootwijk and Wettzell were adjusted simultaneously, while the coordinates of all other stations were held fixed at their MNO values (Ref. 6). In addition to the state-vectors at epoch and the station coordinates, for each arc also the values of the satellite's reflectivity and drag coefficient, or, for LAGEOS, an along-track acceleration were recovered.

In Table 6 the different solutions are listed in terms of the interstation baselines between Kootwijk, Wettzell, Greenbelt and Arequipa. Because the orbital fit of the LAGEOS arcs has proven to be superior to that of the other satellites, the baseline values derived from the LAGEOS observations are supposed to be the most accurate. Focussing on the results for the Kootwijk-Greenbelt and Kootwijk-Arequipa baselines, one may note that the GEOS-3 solution with GEM-10BM most closely approximates the LAGEOS results. The results for STARLETTE are worst, reflecting the relatively bad orbital solution for this satellite. The Kootwijk-Wettzell baseline solutions exhibit an unexpected tendency since it is evident that the GEOS-3 GEM-10BM solution is worse than the STARLETTE solution. This tendency is also present in the other baselines where Wettzell is involved. This might be a direct consequence of the small biases which are known to be present in the Wettzell data but for which no corrections could be made.

The solutions for the baselines have been compared with a European doppler-derived solution for the Kootwijk-Wettzell baseline (Ref. 11) and with a number of laser solutions obtained by NASA Goddard Space Flight Center and the University of Texas. Though such a comparison is hampered to some extend by different definitions of the scale, origin and orientation of the reference system used, it was found that our LAGEOS Kootwijk-Wettzell and Kootwijk-Greenbelt baselines agree to within 25 cm with the doppler solution and the best American laser solutions. Our previous solutions of the Kootwijk-Wettzell baseline (Ref. 2, 3) differ less than 20 cm from the present LAGEOS solution. The real accuracy of the other LAGEOS baseline solutions presented in Table 6 is hard to estimate; it is believed, however, that a sub-meter accuracy level has been reached.

8. CONCLUSIONS

Laser range measurements of LAGEOS, STARLETTE and GEOS-3 during 504 satellite passes over the European stations Kootwijk, Wettzell and San Fernando and nine laser stations in North- and South-America and in Australia were processed. Nine data arcs were formed with lengths of 4 to 13 days. These data arcs were used for the orbit determination of the satellites and for the recovery of satellite parameters, like the drag coefficient and the solar reflectivity. In addition, a number of solutions for the coordinates of Kootwijk and Wettzell was obtained. The earth's gravitational field was modeled by the NASA GEM-9 or GEM-10B gravity models. For each satellite the sensitivity of the orbital solution and parameter estimation to the applied gravity model has been investigated. For GEOS-3, also the effects of introducing multiple drag coefficients to absorb atmospheric model deficiencies have been studied.

The orbital fitting of the range measurements indicates that the position of LAGEOS has been computed with an accuracy of about 0.5 m rms in radial direction and about 2.5 m rms along-track. For STARLETTE and GEOS-3, radial rms values of about 1.8 m and 1.0 m, respectively, were reached. The along-track position accuracy of STARLETTE and GEOS-3 was about 4 m rms.

An unmodeled along-track acceleration of -2.8×10^{-12} m s^{-2} was recovered for LAGEOS, corresponding to a semi-major axis decrease of about 1 mm per day. For the San Fernando observations a range bias of about 20.5 m has been identified.

It was shown that truncating the GEM-9 gravity field to terms up to order and degree 13, yields

Table 6. Comparison of solutions for the interstation baselines (m) between Kootwijk, Wettzell, Greenbelt and Arequipa.

Arc id.	Model	Kootwijk Wettzell	Kootwijk Greenbelt	Kootwijk Arequipa	Wettzell Greenbelt	Wettzell Arequipa
L1 + L2	GEM-9(20)	602423.32	6003351.41	9417064.37	6522113.91	9667528.88
S1 + S2	GEM-10B	23.19	50.30	61.55	13.34	28.24
G1 + G2	GEM-9	23.79	52.56	64.23	15.47	29.41
G1 + G2	GEM-10B	22.31	51.22	64.26	12.74	28.24
G1 + G2	GEM-10BM	21.39	51.24	64.37	12.66	29.56

for LAGEOS position differences of less than 15 cm. Replacing GEM-9 by GEM-10B leads to position differences of up to 1 m along-track. For STARLETTE and GEOS-3, the comparison between the GEM-9 and GEM-10B models results, for the arcs considered, in position differences of up to 4.5 m in the radial direction and up to 17 m along-track.

It has been demonstrated that by solving for multiple drag coefficients the atmospheric drag modeling for GEOS-3 could be substantially improved during periods of enhanced geomagnetic activity. For one arc, in which a geomagnetic storm had occurred, it was found that if a single drag coefficient is used, an along-track position error of more than 30 m builds up.

Solutions for the interstation baselines between the Kootwijk, Wettzell, Greenbelt and Arequipa stations were presented. It is believed that all baselines are accurate to the sub-meter level; the Kootwijk-Wettzell and Kootwijk-Greenbelt solutions agree to within 25 cm with the results of other investigators.

9. ACKNOWLEDGEMENT

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