MODELLING OF ORBITAL PERTURBATIONS DUE TO RADIATION PRESSURE FOR HIGH EARTH SATELLITES

L.ANSELMO P.FARINELLA A.MILANI A.M.NOBILI

GRUPPO DI MECCANICA SPAZIALE
UNIVERSITY OF PISA-ITALY

ABSTRACT

The force model of radiation pressure is discussed for spacecraft of complex shape and structure. By general perturbations techniques, orbital effects in semimajor axis and in longitude are computed and divided in long and short-periodic ones.Long-periodic effects are excluded for constant-attitude s/c. Even for satellites with an Earth pointing antenna, for low inclination and eccentricity and for good antenna pointing, long-periodic perturbations in semimajor axis are small. However there are longitude perturbations of secular character. As an example the perturbations on the orbit of ESA SIRIO 2 satellite are computed. We conclude that an accurate orbit determination is possible for high (e.g. geosynchronous) satellites if orbit and attitude are carefully chosen.

Keywords:Radiation Pressure, Gauss'Perturbation Equations, Geosynchronous Orbit, High Gain Antenna.

1. INTRODUCTION

The main limitation to an accurate determination of satellite orbits (e.g. for geophysical purposes) is the poor modelling of non-gravitational perturbations (unless drag-free probes are used). For high orbiting satellites (e.g. geosynchronous) the dominant non-gravitational perturbation is the solar radiation pressure. Therefore a good model of the orbital perturbations due to radiation pressure is needed; but this is difficult for two reasons:

- the net acceleration results from the complex interaction between the sunlight and all the s/c surfaces (and also the power system);
- a small relative error in the force modelling can result in a big relative error in the orbital perturbations, because of the large secular effects produced by some components of the force.

1.2. Purpose of this paper

This paper introduces new analytical tools to evaluate the perturbative effects of radiation pressure for s/c of complex shape. For a large class of satellites, including most TLC satellites, this approach shows that many effects are negligible so that significant directions can be given to model radiation pressure perturbations within a given accuracy.

In this paper the perturbative analysis refers only to semimajor axis and longitude. The behaviour of eccentricity and inclination is better known (see e.g. [1]); anyway we will discuss the effects on inclination and eccentricity in a successive paper.

2. MODELLING OF RADIATION PRESSURE ACCELERATION

2.1 Radiation Pressure on an elementary surface

For each elementary surface dS the incident sunlight is absorbed, reflected and diffused with probabilities-respectively- α,ρ,δ. If we assume that the absorbed light is not reemitted, and that the diffusion lobe is perfectly spherical (Lambert law), the resulting force on dS is:

$$d\vec{F} = -\frac{\Phi}{c}\,|\cos\psi|\,[(1-\rho)\hat{s}+2(\frac{\delta}{3}+\rho\cos\psi)\hat{n}]dS \qquad (1)$$

where Φ is the solar flux, c the velocity of the light, ψ the angle between the normal $\hat{n}$ to dS and the sun direction $\hat{s}$. The absolute value takes into account the case of a surface that can be lightned on both sides.

This formula is also approximate because the real materials do not behave like a linear combination of a black-body, a perfect mirror and a Lambert diffuser , but the coefficients α,ρ,δ depend on the angle ψ .

2.2 Effects of the thermal state and the power system

The absorbed radiation is reirradiated in a highly anisotropic way due to the anisotropy of shape, surface temperature and emissivity . In particular, the surface temperature is very difficult to model, since it depends not only on the absorbed sunlight, but also on the internal heat sources and on the

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thermal properties of the whole s/c. This effect gives a relative contribution to the radiation pressure acceleration of the order of $\alpha\frac{\Delta T}{T}$, where ΔT is a typical surface temperature difference,and in most cases it is not negligible.

Another indirect effect of solar radiation, that is not negligible, is the spacecraft recoil due to radiowaves transmission towards the Earth. Since most satellites do use for this purpose a large fraction of their power supply, the relative contribution to radiation pressure acceleration is of the order of the efficiency of the solar cells system.

Big thermal changes, with complex transients, are produced in the s/c by Earth eclipses of the sunlight. Moreover the effect of eclipses is very complex also for spherical satellites [2] , and is difficult to model in an accurate way because of penumbra effects [3] . Therefore there is no hope of modelling rad. pr. effects in the orbits in which eclipses do occur. In the case of a high satellite there are long orbital arcs without eclipses (e.g. arcs of about 140 days for a geosynchronous satellite), and we restrict our analysis to these arcs.

2.3 Force model for a complex spacecraft

On the basis of the previous considerations we model the radiation pressure orbital perturbations for a s/c consisting of an axially symmetric body plus an Earth-pointing antenna (Fig.1).

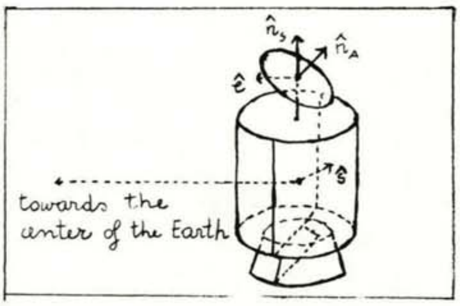

Figure 1. Spacecraft Schematization

If the spacecraft is spin-stabilized, the body irregularities are, in most cases, averaged out; the antenna is supposed to be despun . In these hypotheses the total acceleration is:

$$\frac{\vec{F}}{m_s} = [A+E(v)]\hat{s}+B\hat{n}_s+F(v)\hat{n}_A+D\hat{e} \qquad (2)$$

where m_s is the s/c mass, $\hat{n}_s$ the spin (or symmetry) axis direction, $\hat{n}_A$ the normal to the antenna, $\hat{e}$ the transmission direction, v the true anomaly of the s/c. A and B are functions of the sun position only; if the s/c has a shape not too complex, they are slowly varying with time and have order of magnitude $\frac{\Phi}{c}\ \frac{\text{Total Area}}{m_s}$. D is a slowly varying transmission flux,whose order of magnitude is w/cm_s (w is the transmitted power) and E, F depend both from the sun and the satellite position in a complex way. Starting from the formula (15), we will also assume that the antenna is flat, so that E and F can be calculated in an explicit way. The anisotropic thermal emission can be included in the B term if the s/c is spinning or its thermal properties are symmetric enough.

In the following analytical treatment the essential hypothesis is the constant attitude, i.e. $\hat{n}_s$ constant (or very slowly varying) in the inertial frame.

The main simplification consists in neglecting the mutual shadowing between body and antenna (and also the multiple reflections etc...). On the contrary the symmetry of the s/c body is not a very critical hypothesis, as will be remarked in §3. The effect of Eart's albedo is not considered either; this produces a small error for high satellites (e.g. for geosynchronous s/c the ratio between Earth's albedo and direct radiation from the Sun is about 0.01).

3. GENERAL PERTURBATIONS APPROACH

3.1 Gauss general perturbations equations

We use the Gauss equations to compute the perturbations in the satellite orbital elements a, e, i, Ω, ω, λ ($\lambda = \Omega+\omega+M$ is the mean longitude, M the mean anomaly). As an example, the Gauss equation for the semimajor axis a is:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\ [T+e(S \sin v+T \cos v)] \qquad (3)$$

S, T and W are the components of the perturbing acceleration in the orthonormal moving frame defined by the unit vectors $\hat{e}_S$, $\hat{e}_T$, $\hat{e}_W$ ($\hat{e}_S$ from the center of the Earth towards the s/c, $\hat{e}_T$ normal to $\hat{e}_S$ in the orbital plane, $\hat{e}_W$ normal to the orbital plane. n is the mean motion.

S, T and W are easily computed from (2) if the unit vectors $\hat{s}$, $\hat{n}_S$, $\hat{n}_A$ and $\hat{e}$ are expressed in the $\hat{e}_S$, $\hat{e}_T$, $\hat{e}_W$ frame. For the r.p. force on the body we use:

$$\hat{s} = \begin{pmatrix} A \cos v+B \sin v \\ B \cos v-A \sin v \\ C \end{pmatrix} \qquad \hat{n}_s = \begin{pmatrix} \eta \cos(z-v) \\ \eta \sin(z-v) \\ \sqrt{1-\eta^2} \end{pmatrix} \qquad (4)$$

where η is the sine of the angle between $\hat{n}_S$ and $\hat{e}_W$; since usually the rotation axis of a spin stabilized s/c is in the N-S direction, η is of the same order of magnitude as sin i.

3.2 Long-period perturbations in semimajor axis

To understand the relevance of the perturbations, the first step is to evaluate the long-period or secular perturbations in semimajor axis.

For the s/c body, we will show that only short periodic perturbations are present. In this case (only A, B terms):

$$S = \Lambda(A \cos v + B \sin v) + \eta B \cos(z-v)$$
$$T = \Lambda(B \cos v - A \sin v) + \eta B \sin(z-v) \qquad (6)$$

hence:

$$\frac{n\sqrt{1-e^2}}{2}\frac{da}{dt} = \Lambda(B \cos v - A \sin v) + B\eta \sin(z-v) + e(\Lambda B + \eta B \sin z) \qquad (7)$$

To evaluate the long-periodic or secular term contained in the equation (7), the classical developments of sin v, cos v in D'Alembert series can be used:

$$\cos v = \cos M - e + e \cos 2M + \ldots$$
$$\sin v = \sin M + e \sin 2M + \ldots\ldots \qquad (8)$$

By substituting the developments (8) in the (7), and averaging over one orbital period of the satellite, we obtain:

$$\int_0^{2\pi} \frac{da}{dt}\, dM = 0 \qquad (9)$$

hence no secular or long period (~1 year) perturbations are present.

This result is in fact a particular case of a more general theorem stating that for any constant attitude s/c, not necessarily spin stabilized, at the first order in the perturbation, the semimajor axis does not undergo any secular or long-period variation due to direct solar radiation (supposing no eclipses), but only variations with periods of the order of the orbital period (announced in [4] ; the proof will appear elsewhere).

4. DEVELOPMENT IN SMALL ORBIT AND ATTITUDE PAPAMETERS

4.1 Gauss equations linearized in e, i

For a satellite with small inclination and eccentricity, the perturbative equations can be written neglecting all the terms containing e^2, i^2 or $i\,e$. For a we obtain:

$$\frac{da}{dt} = \frac{2}{n}\,[T + e(S \sin M + T \cos M)] \qquad (10)$$

For the mean longitude λ , we must remember that its time derivative is not the mean motion n, because of the perturbations in Ω , ω and in the mean anomaly at epoch. Therefore we put:

$$\lambda = \int n\, dt + \varepsilon = \rho + \varepsilon \qquad (11)$$

where the perturbing components affecting the semimajor axis give rise to changes in ρ , the others in ε . Always neglecting quadratic terms, the perturbative equation for ε is:

$$\frac{d\varepsilon}{dt} = \frac{1}{2na}\,[2Te \sin M - (4+5e \cos M)\, S] + i\,\frac{\sin(\omega+M)}{2na}\, W \qquad (12)$$

while for ρ we have simply:

$$\frac{d^2\rho}{dt^2} = -\frac{3}{2}\,\frac{n}{a}\,\frac{da}{dt} \qquad (13)$$

4.2 Antenna pointing parameters

To compute the perturbing acceleration components S, T, W coming from the antenna (E,F terms) and from the radiowaves emission (D terms) we need the expressions of $\hat{n}_A$, $\hat{e}$ in the Gauss reference frame $\hat{e}_S$, $\hat{e}_T$, $\hat{e}_W$.

If we assume that the antenna is spinning around the axis $\hat{n}_S$ in such a way to beam the radiowaves towards the Earth, with an error ξ in the east-west direction, then also an error (of the order of η) in the north-south direction results. Neglecting higher-order terms in η, ξ we have:

$$\hat{e} = \begin{pmatrix} -1 \\ -\xi \\ -\eta \cos(z-v) \end{pmatrix} \qquad (14)$$

If the antenna is an high-gain one, both η and ξ must be small, say of the order of 1 degree ≃ 1/57.

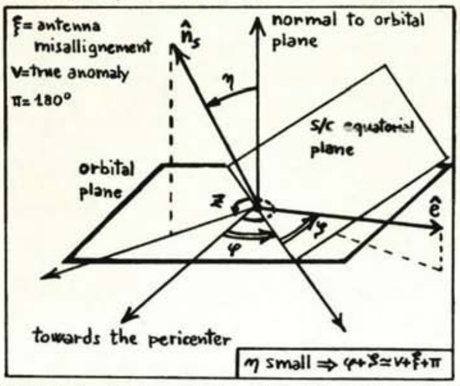

Figure 2. Gauss frame and antenna pointing angles

To compute the effects of direct r.p. on the antenna we assume that it is a portion of plane (of any shape), inclined by an angle σ with respect to $\hat{n}_S$. To reflect the radiowaves coming from the s/c body in the $\hat{e}$ direction, $\hat{n}_A$ must lie in the $\hat{e}$, $\hat{n}_S$ plane:

$$\hat{n}_A = \begin{pmatrix} \cos\sigma + \eta \cos(z-v) \sin\sigma \\ \xi \cos\sigma + \eta \sin(z-v) \sin\sigma \\ \sin\sigma - \eta \cos(z-v) \cos\sigma \end{pmatrix} \qquad (15)$$

If the antenna is not flat, but it is axially symmetric around the axis $\hat{n}_A$, formulas (2), (14), (15) still hold. Anyway the following analysis is based on the hypothesis that the antenna is flat.

4.3 Linearization in the small parameters

If the orbital small parameters e, i and the attitude small parameters η, ξ are such that quadratic terms can be neglected, also the mixed terms $e\eta$, $i\eta$, $e\xi$, $i\xi$ will be negligible. Therefore the full expression of the perturbative equations (10), (12), (13) as a function of the parameters A, B, D, E, F of (2) is simplified.

For the semimajor axis we have:

$$\frac{n}{2}\frac{da}{dt} = [A+E(M)](\mathcal{B}\cos M-\mathcal{A}\sin M)+ \tag{16}$$
$$+\eta[B+F(M)\sin\sigma]\sin(z-M)+\xi[F(M)\cos\sigma-D]+$$
$$+e[F(M)\cos\sigma-D]\sin M+eA(\mathcal{B}\cos 2M-\mathcal{A}\sin 2M)+$$
$$+eF_1(M)\cos\sigma+eE_1(M)(\mathcal{B}\cos M-\mathcal{A}\sin M)$$

where E_1, F_1 are defined by the developments $F(v)=F(M)+eF_1(M)+\ldots, E(v)=E(M)+eE_1(M)+\ldots$

As we know, the A, B terms are only of periodic character. Therefore the most important long-period effect is produced by the only E term not containing the small parameters. Since we assumed a flat antenna, its contribution is

$$E(v) = R'\cos\psi + R''|\cos\psi| \tag{17}$$

where the two terms take into account the possibility of different optical properties and /or temperatures between the two faces and $R'\pm R''$ is of order of magnitude $R=\frac{\Phi}{c}\frac{\text{Area antenna}}{m_s}$ and is slowly changing (i.e.with the sun). $\cos\psi$ is given, for $\eta=\xi=0$, by

$$\cos\psi=\langle\hat{n}_A,\hat{s}\rangle=(\mathcal{A}\cos v+\mathcal{B}\sin v)\cos\sigma+\mathcal{C}\sin\sigma \tag{18}$$

To compute the long-period effect produced by the main E term, we average over one revolution:

$$I=\int^{2\pi}E(M)(\mathcal{B}\cos M-\mathcal{A}\sin M)dM=R'I_1+R''I_2 \tag{19}$$

The integrals I_1, I_2 are recognized to be zero by putting

$$\mathcal{A} = S\cos\theta;\ \mathcal{B} = S\sin\theta \tag{20}$$

so that they become of the form:

$$I_1=\int_0^{2\pi}[\alpha\cos(\theta-M)+\beta]\sin(\theta-M)d(\theta-M)$$
$$I_2=\int_0^{2\pi}|\alpha\cos(\theta-M)+\beta|\sin(\theta-M)d(\theta-M) \tag{21}$$

and $I_1=I_2=0$ because they are integrals of odd functions.

Therefore even taking into account the despun antenna, the secular or long periodic effects in semimajor axis contain the small parameters η,ξ or e. The conclusions coming from (16) are summarized in table 1.

TABLE 1

Perturbations in semimajor axis a

ORDER		1	e	i	η	ξ
Long-periodic or secular	Body	0	0	0	0	$2Dy\xi$
	Antenna	0	2Rye	0	$2Ry\eta$	$2Ry\xi$
Orbital period	Body	2A	2De	0	$2B\eta$	0
	Antenna	2R	2Re	0	$2R\eta$	$2R\xi$
1/2 orbital period	Body	0	Ae	0	0	0
	Antenna	R	Re	0	$R\eta$	$R\xi$

All the entries of table 1 are orders of magnitude of Δa, to be divided by n^2. y is the number of s/c orbits in 1 year, and shows the relative weight of the long-period (annual) effects. The entries containing D are of secular character; their amplitude grows to the indicated value in about 60 days. The higher armonics are present, but their orders of magnitude are smaller or equal to those indicated (e.g. the semiannual term and the 3M, 4M etc. terms).

The effects in longitude of mean motion perturbations can be computed from table 1 and the formula:

$$a\Delta\rho = -\frac{3}{2}\Delta a \quad \text{for terms with orbital period}$$
$$a\Delta\rho = -\frac{3}{2}y\Delta a \quad \text{for annual terms.} \tag{22}$$

For the ε component of longitude the linearized perturbative equation is:

$$2na\frac{d\varepsilon}{dt}=4D-4F(M)\cos\sigma-4E(M)(\mathcal{A}\cos M+\mathcal{B}\sin M)-$$
$$-4A(\mathcal{A}\cos M+\mathcal{B}\sin M)+$$
$$+\frac{1}{2}\mathcal{A}Ae-\frac{11}{2}Ae(\mathcal{A}\sin 2M+\mathcal{B}\sin 2M)-$$
$$-4B\cos(z-M)\eta+5De\cos M+ \tag{23}$$
$$+i\sin(\omega+M)\{AC+B\}+$$
$$+\text{first order E or F terms.}$$

In this case, secular or long-periodic terms do occur, both because of transmission and because of direct radiation pressure on the antenna (D, E, F terms in the first line of (23)); the E, F terms give effects of order of magnitude R, that can be computed in an explicit way by evaluating integrals similar to those of (19), but containing even (and not odd) functions. The first order antenna terms are all estimated as order - of - magnitude by 2R, as in the case of a. The results are summarized in table 2.

TABLE 2

Perturbations in the element ε

ORDER		l	e	i	η	ξ
Long-periodic	Body	2Dy	Aye/4	0	$2By\eta$	0
	Antenna	2Ry	2Rye	2Ryi	$2Ry\eta$	$2Ry\xi$
Orbital periodic	Body	2A	5De/2	i(AC+B)/2	$2B\eta$	0
	Antenna	2R	2Re	2Ri	$2R\eta$	$2R\xi$
1/2 orbital period	Body	0	11Ae/8	0	0	0
	Antenna	R	Re	Ri	$R\eta$	$R\xi$

5. EXAMPLE: THE SIRIO-2 SATELLITE

To give an idea of the orders of magnitude of the effects listed in tables 1 and 2, and to show an example of accuracy analysis, we will analyse the r.p. perturbations on the geosynchronous satellite SIRIO-2, to be launched by ESA late this year.

Sirio-2 consists of a s/c body quite axially symmetric and spin-stabilized, with the spin axis slowly precessing around the north-south direction (with a precession amplitude smaller than 0.5 degrees), plus an high-gain flat antenna, despun and inclined 45 degrees with respect to the spin axis. Its orbital eccentricity and inclination are specified at small values, $e \leq 0.001$ and $i \leq 1$ degree. Hence η will always be $\leq 1/40$ and ξ is supposed to be small, say $\leq 1/60$.
The orders of magnitude of r.p. acceleration are, in m/sec^2:

$$A \simeq 2\times10^{-8} \quad B \simeq 10^{-8} \quad R \simeq 5\times10^{-9} \quad D \simeq 10^{-9} \quad .$$

Table 3 summarize the effects on semimajor axis; η means negligible (< 0.1 m); the first and second term in any case refer to the A, B, D and to the E, F terms respectively.

TABLE 3

Perturbative effects on SIRIO-2 semimajor axis (meters)

	l	e	i	η	ξ
Long-periodic or secular	0+0	0+0.7	0+0	0+15.5	2.4+12
Diurnal	7.5 1.7	~+~	0+0	~+~	0+~
1/2 day period	0+0.8	~+~	0+0	0+~	0+~

The main effects on longitude will therefore came from long-periodic $a\Delta\rho$: by (22) the η, ξ terms can be of the order of 5 kilometers. But also some of the $a\Delta\varepsilon$ long-periodic terms are significant. Table 4 summarized the $a\Delta\varepsilon$ perturbations (here ~ means $<$ 1 m).

TABLE 4

Perturbative effects on SIRIO-2: $a\Delta\varepsilon$ (meters)

ORDER	l	e	i	η	ε
Long-periodic or secular	140+700	~+~	0+12	35+17	0+12
Diurnal	7.5+1.7	~+~	~+~	~+~	0+~
1/2 day period	~+~	~+~	0+~	0+~	0+~

An accuracy analysis can be performed on the hypotheses that the r.p. force is modelled, both for the body and the antenna, with a 10% uncertainity, that η is well known and ξ is poorly known.

In these hypotheses, for a and ρ the long-periodic ξ terms is the most difficult to treat and a better antenna pointing is needed to achieve an accuracy of 1-2 meters in a, 0.5-1 Km in longitude. (Since the antenna despin motor is not likely to mantain ξ constant, it is enough to have a smaller average value of ξ). If this better pointing is achieved, the most important long-period terms in longitude became the η term in ρ and the main term in ε; then a good r.p. model for the antenna becomes very useful. On the contrary, modelling accurately the r.p. on the s/c body is not very important.

Because of the geometry of the range measurements to a geosynchronous s/c, the longitude uncertainity produces a distance uncertainity 1 order of magnitude smaller. We conclude that the uncertainity in the r.p. effects will produce a range uncertainity of the order of tenths of meters, much bigger than the laser ranging errors, still small enough for the planned geophysical use of the SIRIO-2/LASSO mission.

6. CONCLUSIONS

It is not true that the next generation of satellite geodesy and geophysics experiments will use only drag-free or cannon-ball s/c .

Also s/c with complex shape and structure, including many TLC satellites, can be used for high accuracy tracking experiments, provided that:
- high orbits with low e, i are chosen;

- the problems of r.p. modelling are taken into account in the design of the antennas and of their pointing system;
- the optical coefficients α,ρ,δ of the external surface of the s/c are known before launch with a reasonable accuracy;
- the manoeuvre arcs are long enough to allow the decoupling of the long-and short-periodic perturbations in the data analysis.

On the contrary, a complex shape of the s/c body is not a big problem if it is spin-stabilized.

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