L-SAT STATION-KEEPING

J.J. Pocha A.F. Edwards

British Aerospace, Space and Communications Division

ABSTRACT

The station-keeping strategy for L-SAT is derived. The orbit perturbations that must be counteracted arise not only from natural sources such as triaxiality, solar radiation pressure, and luni-solar gravitation, but also from the spacecraft attitude control system operating in its normal and station-keeping modes. The strategy is designed to allow for worst-case conditions on a statistical basis, and includes the effects of tracking and manoeuvre implementation errors. It provides for station-keeping within the specified deadband with an optimum use of propellant.

Keywords: Station-Keeping, Spacecraft Operations

1. INTRODUCTION

A spacecraft in geo-stationary orbit changes its position with respect to an Earth-fixed frame of reference due to the action of a number of perturbations. The word "perturbation" is used to signify forces other than those due to the gravitational potential of a homogeneous spherical earth. The effect of the perturbations is to cause the spacecraft to drift away from its nominal station. If this drift was allowed to build up unchecked, the spacecraft could no longer be used as a communications relay station. "Station-keeping", therefore, has to be carried out to keep the satellite within strict latitude and longitude limits defining a "deadzone".

The magnitude of the deadzone depends upon the characteristics of the communications transponder on the satellite, and the ground station antenna. For example, satellites employing antennae providing global coverage may be maintained within a fairly large deadzone, i.e. $\pm 0.5^\circ$ of latitude and longitude, while those employing narrow beam, high gain, antennae must be kept within much stricter limits, i.e. $\pm 0.1^\circ$. The latter is typical of the OTS and ECS family of communications spacecraft.

The requirement for L-SAT is that the satellite must maintain station within a 0.1° half-cone angle as seen from the centre of the Earth, for the duration of its lifetime.

To good accuracy, the perturbations acting on a geosynchronous satellite can be divided into those causing longitude drift and those causing latitude drift. The longitude, or East-West, drift is due primarily to the gravitational potential component caused by the ellipticity of the Earth's equatorial cross-section, and to the solar radiation pressure acting on the large surface areas of the satellite. The latitude, or North-South, drift is caused by the gravitational forces exerted by the Sun and Moon.

In addition to the naturally occurring perturbations to the geosynchronous orbit, L-SAT suffers further perturbations through the operation of its attitude control system. L-SAT is maintained in a 3-axis stabilised attitude by a set of reaction wheels. The wheels maintain a net zero angular momentum about the spacecraft X and Z axes and a net constant and non-zero momentum about the spacecraft Y axis - the latter to keep the satellite Earth-pointing. The reaction wheels alter their spin rate to counteract the various disturbance torques acting on the satellite and from time to time reach the limits of their spin rate. At this point, the wheel spin rate is set to the opposite limit while the attitude is maintained by the actuation of small thrusters on board the satellite. Owing to their configuration on L-SAT, see Fig.1, the thrusters do not produce a pure torque. This results in a net ΔV being injected into the orbit, causing small changes to the orbit elements. Disturbance torques leading to thruster actuation occur during the satellite "normal mode" of operations, and also when orbit-adjust manoeuvres are being executed. During a North-South station-leeping manoeuvre, the satellite's attitude will be disturbed due to asymmetric thrusting, plume impingement effects, etc. As these manoeuvres tend to be rather large, special attitude control loops are designed into the satellite to cope with the disturbance torques they produce.

These loops actuate thruster firings to maintain satellite attitude within the allowed limits. These thruster firings, in turn, produce a net ΔV into the orbit.

These perturbations also have the effect of causing the satellite to depart from its nominal position, and must be counteracted by the station-keeping manoeuvres designed to satisfy the satellite's positional accuracy requirements.

2. EAST-WEST PERTURBATIONS

The perturbations in the East-West direction arise from

- triaxiality
- solar pressure
- operation of the attitude control system (ACS) in the normal mode
- operation of the ACS in the station-keeping mode.

The first two effects are well known and will only be covered briefly for completeness. The other two effects will be considered in more detail as they are L-SAT specific.

2.1 Triaxiality

The effect of triaxiality can be considered to be a small transverse acceleration on the spacecraft, the magnitude of which depends upon the location of the spacecraft with respect to the major and minor axes of the elliptic equatorial cross-section. Owing to the "resonance" between the orbit period and the earth's rotation rate, this transverse acceleration is almost constant when station-keeping within tight longitude limits is carried out. This transverse acceleration can be expressed in terms of the annual ΔV impressed upon the spacecraft, and the result is shown in Fig. 2.

L-SAT, placed at a nominal longitude of 19^{o}W experiences a transverse acceleration of -3.835×10^{-4} deg/day^2, and this remains essentially constant throughout the satellite's lifetime. The strategy for countering this acceleration is illustrated in Fig.3. The satellite is placed at the western edge of the deadband with an initial drift rate of such a magnitude that the transverse acceleration brings the satellite drift to a halt at the eastern edge of the deadband, halfway through the East-West station-keeping cycle. During the remaining half of the cycle the transverse acceleration causes the satellite to drift westwards until it reaches the western edge of the end of the cycle. If left unattended the satellite would transgress the deadband at this point. The station-keeping manoeuvre consists of nulling the westward drift rate and re-establishing the initial eastward drift rate. Since the orbit drift rate is the corrected parameter, the manoeuvre consists of changing the orbit's semi-major axis only. All other parameters are left unchanged.

2.2 Solar Radiation Pressure

The effect of solar radiation pressure is to induce an eccentricity into the orbit leaving the other parameters unchanged. The magnitude of the effect is governed essentially by the satellite's effective area-to-mass ratio. For L-SAT this has an average value of 0.076 m^2/kg in synchronous orbit. The induced eccentricity has the effect of causing a daily oscillation (libration) in the satellite's longitude about some mean value.

We define an eccentricity vector as having a magnitude equal to the orbit's eccentricity and a direction equal to the geocentre-to-perigee direction. Solar radiation pressure causes the eccentricity vector to change in a direction approximately perpendicular to the sun direction. The tip of the vector, therefore, moves along a nearly circular path during the year. The radius of the eccentricity circle depends only upon the effective area-to-mass ratio of the satellite, and not upon initial orbit conditions. For L-SAT the radius of the eccentricity circle is 8.474×10^{-4}.

The strategy for the control of the longitude libration due to solar pressure effects, consists of controlling the orbit's eccentricity and position of perigee with respect to the sun line. Initial conditions are set up such that the orbit has some eccentricity with the position of perigee at some angle behind the sun line. The eccentricity will then decrease until it reaches a minimum half-way through the cycle, at which point perigee lies along the sun line. At the end of the cycle the orbit eccentricity will have regained its initial value and the perigee will be as much in front of the sun line as it was behind it at the start of the cycle. At this point a manoeuvre is performed to regain the initial conditions. This consists of moving the perigee position, while leaving the eccentricity unchanged. The orbit configuration at the beginning, middle and end of a cycle is shown in Fig. 4.

2.3 East-West Perturbations due to Normal Mode Operations

During normal mode operations, the control system will have to counter various disturbance torques. These torques are a function of the time of year, position in the orbit, and a number of parameters associated with the satellite itself. The most significant torques are those due to solar radiation pressure, gravity gradient, and residual magnetic moment. The most important of these are torques due to solar radiation pressure, and these originate from:

- centre of gravity off-set from the centre of pressure caused by centre of gravity uncertainty and solar array drive mis-alignment
- dihedral effects caused by array bending and misalignments
- windmill effects caused by array and array drive misalignments
- effective centre of pressure offsets.

The North-South offset component is caused mainly by the array while the East-West offset component is caused by solar array drive misalignment.

The L-SAT control system is based on reaction wheel control about all three axes and hence only secular disturbance torques need be off-loaded by thruster actuation. Owing to the L-SAT thruster configuration this results in net velocity increments, leading to orbit perturbations. Desaturation about the roll axis results in radial and North-South ΔV's. Desaturation about the pitch axis results in radial and East-West ΔV's. Desaturation about the yaw axis results in North-South ΔV's. This is illustrated in Fig.1. Below, we shall consider the torques about each axis in turn.

2.3.1 Pitch Torques

The main secular component of the pitch torque is caused by CG/CP offset in the East-West direction. The magnitude of the torque depends upon the magnitude of the offset, the satellite's reflectance properties, and the declination of the sun. The direction of the torque and therefore, the direction of the off-loading will be constant throughout the satellite's lifetime, but the direction cannot be predicted before launch. Simulations performed using the 99% worst offsets indicate that the maximum number of thruster actuations will be 52 per day occurring at the equinoxes. The number will decrease by 15% to a minimum at the solstices. This behaviour is shown in Fig. 5.

2.3.2 Roll and Yaw Torques

The main secular components of these torques are also caused by CG/CP offsets along the Z and X axes respectively. The sign of the off-loading torque will change as a function of sun declination, being zero at the equinoxes and maximum and minimum at the solstices, as shown in Fig. 6. Simulations performed, using the 99% worst case offsets, indicate that the maximum number of thruster actuations will be 27 per day about each axis.

2.3.3 Orbit Perturbations

The effects on the orbit of the pitch, roll, and yaw thruster actuations are best measured in terms of their in-plane and out-of-plane ΔV components. For a single thruster pulse, the resulting ΔV's are shown in Table 1. We will first show that the orbit perturbations due to the out-of-plane (or North-South) ΔV's are negligible, and then consider the in-plane ΔV's.

The change in orbit pole position in degrees is related to the applied ΔV_N by the relation.

$$\Delta P = \frac{180}{\pi} \cdot \frac{\Delta V_N}{V_S}$$

Where ΔV_N is the applied North-South velocity increment and V_S the velocity in synchronous orbit. For yaw pulses, over a 15 day station-keeping cycle embracing the peak disturbance torque period when 27 pulses per day are required, the change in pole position is 2×10^{-4} degrees. This is negligible and can be ignored over a single cycle. Because the sign of the yaw torque changes half-way through the period of a year, any increase in inclination over half a year will be compensated by a decrease in the following half year resulting in no manoeuvre penalty over the year as a whole.

A similar argument applies to the out-of-plane ΔV induced by roll torque compensation. The influence of out-of-plane ΔV's on the orbit is, therefore, negligible and non-penalising.

We consider, now, the in-plane perturbations. The change in drift rate produced by a transverse velocity increment, ΔV_T, is

$$\Delta D = \frac{1080 \, \Delta V_T}{V_S} \text{ deg/day, positive eastwards.}$$

Radial ΔV's have a negligible effect upon drift rate, and the change in drift rate produced by transverse ΔV's is independent of the position in the orbit where the thrusts are applied. The pitch torques, therefore, are the only ones to affect the drift rate. In the period of peak torques, the effect of 52 pulses per day is to produce a daily change in drift rate of $\pm 1.708 \times 10^{-3}$ deg/day. To good approximation, this can be regarded as a longitude acceleration of $\pm 1.708 \times 10^{-3}$ deg/day^2. Owing to the nature of the pitch disturbance torque, this acceleration will remain constant in direction, and near constant (to 15%) in magnitude throughout the satellite's lifetime. We can therefore consider it as an enhanced triaxiality type perturbation to the orbit, and add its effect directly to the triaxiality effect.

In order to examine the effect of in-plane components upon the orbit's eccentricity, the concept of the eccentricity vector, defined in Section 2.2, is used. For a near circular orbit, the change in the eccentricity vector $(\Delta e_1, \Delta e_2)$ produced by an in-plane burn with radial component ΔV_R and transverse component ΔV_T, is given by :

$$\Delta e_1 = \frac{2\Delta V_T}{V_S} \cos\theta + \frac{\Delta V_R}{V_S} \sin\theta$$

$$\Delta e_2 = \frac{2\Delta V_T}{V_S} \sin\theta - \frac{\Delta V_R}{V_S} \cos\theta$$

Where θ is the right ascension of the burn application point relative to Aries, e_1 is the component of e along the direction of Aries, and e_2 is the component of e perpendicular to this direction.

For L-SAT, to avoid flexure excitation of the arrays, the thruster pulses must be at least 200 secs apart. To evaluate the change in the eccentricity vector due to a train of pulses, each one separated by $\Delta\theta$ from the next (for L-SAT, $\Delta\theta = 0.86^o$), we note that no orbit elements appear on the right hand side of the above equations to first order. After much algebra, we find that the change in the eccentricity vector for a train of N pulses starting at θ_o to be given by :

$$\Delta e_1 = A \cos \theta_o + B \sin \theta_o$$

$$= \sqrt{A^2 + B^2} \cos (\theta_o - \varepsilon)$$

$$\Delta e_2 = A \sin \theta_o - B \cos \theta_o$$

$$= \sqrt{A^2 + B^2} \sin (\theta_o - \varepsilon)$$

$$\varepsilon = \arctan (B/A)$$

$$A = \frac{2\,\Delta V_T}{V_S} K_1 + \frac{\Delta V_R}{V_S} K_2$$

$$B = \frac{\Delta V_R}{V_S} K_1 - 2\frac{\Delta V_T}{V_S} K_2$$

$$K_1 = \sum_{i=o}^{N-1} \cos i\Delta\theta = \frac{1}{2} + \frac{\sin (N-\frac{1}{2})\Delta\theta}{2 \sin \frac{\Delta\theta}{2}}$$

$$K_2 = \sum_{i=o}^{N-1} \sin i\Delta\theta = \frac{\sin\frac{(N\Delta\theta)}{2} \sin\frac{(N-1)\Delta\theta}{2}}{\sin \frac{\Delta\theta}{2}}$$

Note that, to good approximation,

$$\sqrt{A^2 + B^2} = N\left(\frac{2\,\Delta V_T}{V_S}^2 + \frac{\Delta V_R}{V_S}^2\right)$$

For N = 52, this approximation is accurate to 97%. The values of K_1 and K_2 for varying numbers of pulses are given in Table 2.

The above equations show that for the pulses to have no effect upon orbit eccentricity, they must be distributed in such a way that the effect of each pulse is cancelled by a pulse 180^o around the orbit. A minimum of 2 pulse trains, on opposite sides of the orbit, is, therefore, required to avoid perturbations to the orbit eccentricity. By the same token, if the pulses are fired in a single group on one side of the orbit, the change in the eccentricity vector $(\Delta e_1, \Delta e_2)$ will depend upon the position in the orbit at which the pulse train is initiated. We, therefore, have the means of favourably influencing the orbit eccentricity by suitably timing the torque offloading pulses, and this is possible owing to the freedom available for the selection of burn times.

The strategy selected is designed to negate a portion of the naturally occurring eccentricity perturbation on a daily basis. The variation in the eccentricity vector due to solar radiation pressure is given by :

$$\frac{de_1}{dt} = - K \sin \theta_S(t)$$

$$\frac{de_2}{dt} = K \cos \theta_S(t)$$

where K is a constant (a function of the satellite's effective area-to-mass ratio), and $\theta_S(t)$ is the right ascension of the sun at time t. The daily variation of the eccentricity vector is given by :

$$\Delta e_1 = - K \sin \theta_S$$

$$\Delta e_2 = K \cos \theta_S$$

The change in the eccentricity vector caused by a group of pulses initiated at a right ascension of θ_o is given by :

$$\Delta e_1 = \sqrt{A^2 + B^2} \cos (\theta_o - \varepsilon)$$

$$\Delta e_2 = \sqrt{A^2 + B^2} \sin (\theta_o - \varepsilon)$$

If $\theta_S - 90^o = \theta_o - \varepsilon$ then the eccentricity change due to the thruster firings will oppose that due to the solar radiation pressure.

As $(180^o + (\theta_o - \theta_S))$ represents the solar array angle - see Fig. 7 - the pulse train would be initiated whenever the solar array angle reached $90^o + \varepsilon$. This provides a simple method of sequencing the burns.

To a good approximation, the net effect of such a strategy is to reduce the eccentricity vector circle radius through the relation.

$$\text{radius} = \text{constant} \times K^1$$

$$K^1 = K - \sqrt{A^2 + B^2}$$

The number of pitch and roll thruster firings varies throughout the year, and the effect on ε and $\sqrt{A^2 + B^2}$ is shown in Table 3. It can be seen that the effect of roll firings is very much lower than that of the pitch firings. Maximum roll torque occurs when the pitch torque is a minimum, and the combined effect of the two is near constant. For pitch, the variation in the angle ε is small, indicating that it may be possible to use a single array angle throughout the mission to initiate the pulse train. If the minimum torque angle is chosen, the maximum misalignment of the pitch produced eccentricity change would be about 5°, resulting in an eccentricity change in the radiation induced eccentricity change direction of 99% of the value without misalignments.

The angle ε in the case of roll torques varies by a much larger amount throughout the year. However by selecting a constant value corresponding to the maximum torque case i.e: $\varepsilon = 78^\circ$, the best results are achieved when considered in combination with the pitch firings.

For L-SAT, the radius of the eccentricity circle due to the solar radiation pressure perturbation is 8.474×10^{-4}. When the torque induced compensation, using the strategy defined above, is included, this reduces to 6.522×10^{-4}. It can be seen, therefore, that a worthwhile alleviation is obtained by correctly phasing the thruster firing.

2.4 East-West Perturbations due to Station-Keeping Loops

The North-South station-keeping manoeuvre induces appreciable disturbance torques on the satellite, to the extent that special control loops are used to control the spacecraft's attitude during station-keeping. The main torques acting on the spacecraft are due to plume impingement, thruster misalignments, variations in thrust level, and centre-of-gravity offsets consisting of a small non-rotating component and a larger rotating component caused by the dihedral effect of the solar array - see Fig. 8.

The main roll and yaw disturbance torques are the misalignment and thruster mismatch torques and torques due to array dihedral. The last is dominant, and as a consequence, the direction of the disturbance torque will change (as the solar array drive rotates) with a period of one year - see Fig. 9.

The main torque along the pitch axis is due to plume impingement on the large solar arrays. The torque is essentially zero when the array is either normal or parallel to the plane through the axes of the two North-South station-keeping thrusters, and a maximum when the array is at 45° to this plane. See Fig. 9.

The effects of the torques are counteracted by thruster actuation yielding the in-plane and out-of-plane components per thruster pulse as shown in Table 4. Note that the yaw torque is absorbed by the off modulation of the thrusters.

For L-SAT, during the station-keeping burn the spacecraft will most probably be operated in a one-sided limit cycle, because the disturbance torques should be in the same sense throughout the burn. The frequency, f, of thruster firings required to counteract the constant disturbing torque T_D is given by :

$$f = T_D/T_A \cdot \Delta t$$

Where T_A = torque applied by the thrusters per second and Δt is the thruster pulse duration. For the 2.9kW array on L-SAT the maximum disturbance torques and the corresponding thruster firing frequencies are shown in Table 5. The time of a typical North-South station-keeping burn (including 18% off-modulation) is 118 secs; so that the required number of firings at the torque levels of Table 5 are 166 for pitch and 221 for roll. The resulting ΔV components for each are shown in Table 6.

The maximum transverse component will cause a change in drift rate of $\pm 3.723 \times 10^{-2}$ deg/day, while the maximum change in the eccentricity vector will be :

$$\text{for pitch: } \Delta e_1 = (\pm 6.58 \cos\theta - 5.4 \sin\theta) \times 10^{-5}$$

$$\Delta e_2 = (\pm 6.58 \sin\theta + 5.4 \cos\theta) \times 10^{-5}$$

$$\text{for roll : } \Delta e_1 = -7.26 \times 10^{-5} \sin\theta$$

$$\Delta e_2 = 7.26 \times 10^{-5} \cos\theta$$

The North-South station-keeping strategy involves correcting only for the secular motion of the orbit pole and prevents the build-up of execution and tracking errors. The magnitude and direction of the required burn will vary over the lifetime of the satellite due to the variation of the position of the Moon's ascending node. The prime thrusters will apply a northerly impulse at a right ascension of $270^\circ + \delta$, whereas the back-up thrusters willapply a southerly impulse at a right ascension of $90^\circ + \delta$. Here δ is the direction of the secular motion of the orbit pole relative to the Vernal Equinox - its value varies between approximately $+ 10^\circ$ and $- 10^\circ$. The position of application of station-keeping manoeuvres determines the eccentricity change induced, according to the expressions below :

Pitch Torque Compensation

Firing Position	$\Delta V_T > o$	$\Delta V_T < o$
$\theta = 90 + \delta$	$\Delta e_1 = -8.5\text{x}10^{-5}\cos(\delta-50.6)$	$\Delta e_1 = -8.5\text{x}10^{-5}\cos(\delta+50.6)$
	$\Delta e_2 = -8.5\text{x}10^{-5}\sin(\delta-50.6)$	$\Delta e_2 = -8.5\text{x}10^{-5}\sin(\delta+50.6)$
$\theta = 270 + \delta$	$\Delta e_1 = 8.5\text{x}10^{-5}\cos(\delta-50.6)$	$\Delta e_1 = 8.5\text{x}10^{-5}\cos(\delta+50.6)$
	$\Delta e_2 = 8.5\text{x}10^{-5}\sin(\delta-50.6)$	$\Delta e_2 = 8.5\text{x}10^{-5}\sin(\delta+50.6)$

Roll Torque Compensation

Firing Position

$\theta = 90 + \delta$

$$\Delta e_1 = -7.26\text{x}10^{-5}\cos\delta$$

$$\Delta e_2 = -7.26\text{x}10^{-5}\sin\delta$$

$\theta = 270 + \delta$

$$\Delta e_1 = 7.26\text{x}10^{-5}\cos\delta$$

$$\Delta e_2 = 7.26\text{x}10^{-5}\sin\delta$$

The change in the eccentricity vector caused by the pitch and roll firings is illustrated in Fig.10. Because the eccentricity control strategy involves keeping the perigee directed nearly towards the sun,for half a year, the eccentricity change caused by the North-South station-keeping manoeuvre tends to increase the magnitude of the eccentricity, while for the other half of the year it tends to diminish it. The increase in eccentricity must be accommodated within the East-West station-keeping strategy, and because of its relatively large magnitude it has a significant impact upon it.

3. EAST-WEST STATION-KEEPING STRATEGY

The perturbations leading to longitude drift, mentioned above, must all be corrected by a maneouvre strategy designed to maintain the satellite within its stipulated deadband with a probability of 99%. The requirement for L-SAT is that the satellite must be kept within 0.1° half cone angle about its nominal longitude, as seen from the earth's centre. As the North-South and East-West perturbations can be de-coupled, we can satisfy this requirement by maintaining a longitude deadband of ± 0.087° and a latitude deadband of ± 0.05°. In addition, to ease the workload of the ground station, every effort has been made to reduce the frequency of manoeuvres. For L-SAT, a minimum East-West station-keeping cycle duration of 14 days has been considered.

The station-keeping strategy must not only account for the perturbations outlined in Section 2, but also for manoeuvre execution errors arising from inaccuracies in the orbit determination and thruster actuation procedures. Depending upon the on-station longitude of the satellite, station-keeping manoeuvres are made on always one side of the deadband, or on both sides. The latter occurs if the on-station longitude is very close to a triaxiality perturbation null. For L-SAT, stationed at 19° W, the strategy involves a one-sided limit cycle for East-West perturbation control.

The East-West deadband is subdivided as shown in Fig. 11.

Here A + E is the sum of:

- libration due to solar pressure effects.
- uncontrolled second order perturbations due to in-plane luni-solar gravitational effects.

The twice daily longitudinal perturbation caused by the Moon has an amplitude of ± .0025°. Because the eccentricity strategy to be described maintains the perigee pointed nearly towards the sun, the twice daily tidal longitude libration caused by the sun is out of phase with the uncontrolled solar pressure libration. As long as the perigee line is within 30° of the sun line only the deadband allocated to uncontrolled solar pressure eccentricity libration is required. The longer period perturbations (period 2 weeks to 1 year) fall into two classes. Those caused by the sun, despite having significant amplitudes, have low peak drift rates and even lower accelerations which, over a station-keeping cycle, may be ignored. The lunar-caused oscillation of half-monthly period is uncontrolled, and an allowance must be made equal to its amplitude of ± .005°.

B is the root sum squared value of longitude tracking error plus libration due to errors in the eccentricity correction manoeuvre. X represents the switching boundary i.e. manoeuvres are initiated when the satellite's measured position reaches this value. It is indented from U by an amount B in order to compensate for tracking errors in longitude and eccentricity.

The line V is a hypothetical boundary indented by an amount C from X to allow for position tracking errors only. C is .707 x longitude tracking error and is used in the determination of the maximum and minimum cycle times.

Taking V as the initial longitude, the triaxiality correction manoeuvre must be designed such that even with 2.58σ errors in drift rate estimation and 2.58σ errors in manoeuvre execution tending to compound each other, the satellite does not overshoot the longitude limit P which is indented from Q by an amount D which is the libration due to errors in the eccentricity correction manoeuvre. The maximum cycle time will occur when the satellite's initial position is at V, the errors conspire to give a larger initial drift rate than nominal, and the final position is at U' - where U' is indented from X by an amount equal to .707 x longitude tracking error. Note that both at the start and end of the cycle, the satellite's computed position is at X. The minimum cycle time will occur when the initial position is at U', the errors conspire to give a lower drift rate than nominal, and the final position is at V. Once more at the start and end of the cycle, the satellite's computed position is at X.

The above considerations allow the determination of the nominal East-West cycle time. Whatever the initial conditions, we aim for the nominal East-West cycle. Such a strategy allows us to remain within the allowed deadband to a 2.58σ probability level in an efficient and economical manner.

For L-SAT, the following tracking accuracies have been used :

longitude errors at the 2.58σ level = 0.0073 deg.

drift rate error at the 2.58σ level = 1×10^{-3} deg/day

These are typical of on-station accuracy levels obtainable when ranging and angular measurements are taken over a period of 48 hours.

3.1 Triaxiality Control

For L-SAT at 19°W longitude, the triaxiality longitude acceleration is -3.835×10^{-4} deg/day². We have seen in Section 2.3 that the effect on drift rate of the normal mode pitch compensation thruster firings may be considered to lead to an effectively enhanced triaxiality acceleration which is near constant and equal to $\pm 1.708 \times 10^{-3}$ deg/day² to a 99% probability level. If we now assume that this acceleration is in the same direction as the triaxiality-induced acceleration, we get a total longitude acceleration of -2.092×10^{-3} deg/day².

The minimum cycle initial drift rate is determined from the longitude tracking error, the minimum cycle time (= 14 days), and the longitude acceleration, by the expression :

$$\frac{2\Delta\lambda}{\sqrt{2}} = \dot{\lambda}_{min}\, t_c + \tfrac{1}{2}\,\ddot{\lambda}\, t_c^2$$

where $\Delta\lambda$ = longitude tracking error

$\dot{\lambda}_{min}$ = initial drift rate of minimum cycle

t_c = minimum cycle time

$\ddot{\lambda}$ = longitude acceleration

The nominal cycle and maximum cycle initial drift rates are obtained from the minimum cycle drift rate by ensuring that the maximum and minimum occur at 2.58σ levels with respect to the nominal, in opposite senses. The random errors in drift rate are caused by tracking and thrust level variations. The L-SAT thruster system is based on bi-propellant technology and the thrust level should be accurate to about 2.5%. Since in the nominal case, the magnitude of the burn induced drift rate is twice the nominal drift rate, the total error will be 5% of the nominal drift rate. We therefore have :

$$\dot{\lambda}_{nom} = \dot{\lambda}_{min} + \{(\Delta D)^2 + (0.05\,\dot{\lambda}_{nom})^2\}^{\frac{1}{2}}$$

where ΔD = drift rate tracking error (= 1.0×10^{-3} deg/day)

$\dot{\lambda}_{nom}$ = initial drift rate of nominal cycle.

The nominal cycle time is given by :

$$t_c = 2\,\dot{\lambda}_{nom}/\ddot{\lambda}$$

similarly,

$$\dot{\lambda}_{max} = \dot{\lambda}_{nom} + \{(\Delta D)^2 + (0.05\,\lambda_{nom})^2\}^{\frac{1}{2}}$$

$$\text{and } t_c = 2\,\dot{\lambda}^2_{max}/\ddot{\lambda}$$

The maximum cycle governs the triaxiality deadband allocation and this is given by :

$$\Delta\lambda_{max} = \frac{\dot{\lambda}_{max}^{\;2}}{2\,\ddot{\lambda}} + \frac{\Delta\lambda}{\sqrt{2}}$$

For L-SAT, the results obtained are shown below:

	Cycle Time (day)	Initial Drift Rate (deg/day)	Longitude Allocation
Minimum Cycle	14	0.01537	
Nominal Cycle	15.9	0.01667	
Maximum Cycle	17.7	0.01797	0.0823°

On the basis of the above results the deadband available for eccentricity libration can be determined given that the longitude error, due to eccentricity burn error, is 0.0054° at the 2.58σ level. It can be shown that this is equal to ± 0.0304°.

The total East-West deadband allocation is shown below:

Uncontrolled perturbations	=	0.015°
Triaxiality allowance	=	0.0823°
Error allowance at West edge (B in Fig.11)	=	0.0096°
Error allowance at East edge (D in Fig.11)	=	0.0054°
Eccentricity libration	=	0.0608°

Next we consider how the eccentricity is to be controlled so that the eccentricity-induced libration does not exceed the allocated amount.

3.2 Eccentricity Control

L-SAT has an average effective area-to-mass ratio of 0.076 m^2/kg, and the radius of the eccentricity circle is therefore 8.474×10^{-4}. We have seen that this can be reduced to about 6.522×10^{-4} with suitable timing of the normal mode attitude control burns. Owing to the large radius of the eccentricity circle compared to the allowable eccentricity libration of ± 0.0304°, a two burn eccentricity control manoeuvre is necessary. This manoeuvre involves a rotation of the line of apsides without changing the eccentricity. The strategy for eccentricity control is as follows. The line of apsides is set behind the sun line at the beginning of the cycle by a certain amount. The value of the eccentricity is the maximum that the allowable libration will allow - here 2.66×10^{-4}. As time proceeds, the eccentricity vector will evolve such that the eccentricity decreases and the line of apsides gradually overtakes the sun line. Half-way through the cycle, the eccentricity is a minimum, and the line of apsides coincides with the sun line. During the second half of the cycle the eccentricity will increase and the line of apsides will lead the sun line, until at the end of the cycle, the eccentricity is again at its maximum allowable value and the line of apsides is as far ahead of the sun line as it was behind at the start of the cycle. A two-burn manoeuvre now restores the initial conditions, and the cycle begins again.

Since the triaxiality and solar pressure corrections are combined into one manoeuvre, the eccentricity control strategy must be planned to accommodate the maximum triaxiality cycle time. Because the occurrence of cycle times other than nominal is assumed to be random, the nominal cycle determines the total ΔV requirements. The following manoeuvre details are derived for L-SAT :

- Allowable libration = ± 0.0304
- Allowable eccentricity vector radius = 2.66×10^{-4}
- Initial eccentricity vector radius = 2.61×10^{-4}
- Initial angle between line of apsides and sun line = 12.09°
- Magnitudes of the two burns when combined with triaxiality = 0.131 and -0.036 m/s

The above analysis has included only the effects of solar radiation pressure and the normal mode thruster firings. We shall now consider the effect of North-South station-keeping attitude control thruster firings. It can be shown that the effects of the North-South station-keeping manoeuvres themselves are negligible. During North-South station-keeping attitude control thrusting in roll and pitch produce eccentricity vector changes as shown in Fig. 10. To accommodate this change the eccentricity vector at the start of the cycle is adjusted to ensure that the total eccentricity at the end of the cycle stays within the allowable limit.

When the eccentricity vector is in the positive e_1 half-plane North-South manoeuvres executed near a right ascension of 270° result in an increase in the eccentricity vector, while when it is in the negative e_1 half-plane such manoeuvres decrease the eccentricity magnitude. Since the eccentricity vector position is governed by the sun position, it will at some time or other occupy both halves of the e_1 e_2 plane.

The North-South station-keeping attitude control burns will, therefore, both increase and decrease the eccentricity vector through the year. To estimate the impact on the station-keeping fuel budget we determine the average effect on the eccentricity vector, taking into account that the magnitude of the eccentricity change produced by the station-keeping burns varies according to the solar array angle.

The pitch torque has a 6-month period, and attains its maximum value when the array plane is at 45° to the plane containing the axes of the two North-South thrusters. The roll torque has a one-year period. Consider the three-month period between pitch torque zeros. Presuming that six station-keeping cycles are distributed evenly throughout this period, the average torque is 0.613 of the peak value. Likewise for the three-month period between a zero and a maximum in roll torque, the average torque is 0.725 of the peak value. Assuming that these variations are in phase, the average value of the eccentricity change is 9.465×10^{-5}. For the following three months the pitch torque will be in the opposite direction, and the roll torque will decrease to zero. The eccentricity change will have the same magnitude but will be negative in angle. If we now assume that the average angle between the eccentricity vector and the eccentricity change vector is 45° we have the result that the average radial eccentricity increase has a value of 6.69×10^{-5}. This must be allowed for in the station-keeping strategy as described above. The following manoeuvre details are derived :

- Eccentricity at start of cycle = 1.987×10^{-4}
- Initial angle between line of apsides and sun line = 18.8°
- Magnitude of the two burns when combined with triaxiality = 0.172 and −0.077 m/s

In the second half of the year, when the eccentricity vector is in the negative e_1 plane, the eccentricity change is negative and the initial eccentricity vector can be set to the value derived without consideration of North-South station-keeping perturbations. The yearly average of the ΔV for an East-West station-keeping manoeuvre is then 0.18 m/s giving a 7-year ΔV requirement of 34 m/s.

4. NORTH-SOUTH STATION-KEEPING

North-South station-keeping is designed to counteract the effects of luni-solar gravitation, and keep the spacecraft within its latitude deadband. For L-SAT the latitude deadband is $\pm 0.05^\circ$. The strategy consists of correcting for the secular motion of the orbit pole in such a way that the ΔV required for the manoeuvres is minimised.For compatibility with the East-West station-keeping manoeuvres, the nominal cycle time is 16 days and the manoeuvre is performed to interleave with the East-West manoeuvres as explained in Section 5. A portion of the deadband is allocated to tracking and manoeuvre execution errors and the satellite is station-kept within the remaining deadband. For L-SAT an allowance of 0.006° is made on both sides of the deadband for such errors and the satellite is maintained within a deadband of $\pm 0.044^\circ$.

The strategy consists of setting up the initial conditions in such a way that the orbit inclination at the start and end of the cycle is the same.

The orbit plane orientation is transformed into the X, and Y co-ordinates of the orbit pole, giving :

$$X = i \sin \Omega \quad ; \quad Y = - i \cos \Omega$$

where i = orbit inclination

Ω = right ascension of the ascending node

X = is directed along the vernal equinox.

Now we define the following :

T = start time of cycle

$X_S(T), Y_S(T)$ = instantaneous pole position at start of cycle

$X_e(T), Y_e(T)$ = instantaneous pole position at end of cycle

T_C = cycle length

$\dot{x}, \dot{y}$ = rate of change of pole position due to secular effect

$\Delta X(t), \Delta Y(t)$ = cyclic offset from secular motion at time t

As the initial and final inclinations are the same, we have:

$$X_S(T)^2 + Y_S(T)^2 = X_e(T)^2 + Y_e(T)^2$$

At the start of the cycle we set the Y co-ordinate of the secular motion to zero thus :

$$Y_S(T) = \Delta Y(T)$$

The Y co-ordinate of the instantaneous pole position at the end of the cycle is, therefore,

$$Y_e(T) = \dot{y}\, T_C + \Delta Y(T + T_C)$$

The X co-ordinate of the instantaneous pole position at the end of the cycle will be,

$$X_e(T) = X_S(T) - \Delta X(T) + \dot{x}\, T_C + \Delta X(T + T_C)$$

We can now solve explicitly for the X co-ordinate at the start of the cycle, and obtain:

$$X_S(T) = -\tfrac{1}{2}(\Delta X(T+T_C) - \Delta X(T) + \dot{x}T_C) + \frac{\Delta Y(T)^2 - \Delta Y(T+T_C)^2 - \dot{y}T_C(2\Delta Y(T+T_C) + \dot{y}T_C)}{2\ (\Delta X(T + T_C) - \Delta X(T) + \dot{x}\, T_C)}$$

The strategy thus provides, for any cycle, a start point $(X_S(T), Y_S(T))$ such that the end point is $(X_e(T), Y_e(T))$. The instantaneous pole position would evolve from $(X_S(T), Y_S(T))$ to $(X_e(T), Y_e(T))$ over the next T_C days. A North-South station-keeping manoeuvre could then be designed to move the pole from that point to $(X_S(T + T_C), Y_S(T + T_C))$. The magnitude of the manoeuvre is given by :

$$\Delta V = V((X_S(T+T_C) - X_e(T))^2 + (Y_S(T+T_C) - Y_e(T))^2)^{\frac{1}{2}}$$

Where V is the velocity of the satellite in geosynchronous orbit.

The right ascension at which the burn is applied is :

$$\theta_N = \arctan \frac{X_S(T+T_C) - X_e(T)}{Y_S(T+T_C) - Y_e(T)}$$ for northerly thrusts,

$\theta_S = \theta_N + \pi$ for Southerly thrusts.

A computerised implementation of this strategy has produced the orbit pole history shown in Fig. 12. Assuming that L-SAT acquires its operational station in mid.June 1984, the ΔV requirement for this strategy is 357.4 m/s for 7 years. For a nominal 16 day cycle, the maximum inclination over the 7-year lifetime will be 0.036^o. Fig. 12 also shows that though for some times during the year the maximum inclination is greater than the initial and final inclinations of a cycle, the latitude deadband is not violated. Fig. 13 shows the variation in burn size through the satellite's lifetime. The manoeuvre ΔV varies from a maximum of 2.77 m/s to a minimum of 0.91 m/s.

5. INTERACTION BETWEEN NORTH-SOUTH AND EAST-WEST STATION-KEEPING

We have already seen the effect on East-West station-keeping of attitude control thrusting during North-South station-keeping. In this section we investigate the impact of the North-South station-keeping burns themselves. If the North-South burns were perfectly nominal there would be no coupling into the East-West direction. Thruster misalignments and thrust vector orientation errors, however, make a degree of coupling inevitable. For L-SAT the thrust direction errors are assumed randomly distributed around zero, with a 99% probable error of less than 0.35^o, being composed of a 0.25^o thruster error and a 0.1^o attitude control error in the yaw direction during North-South station-keeping. If we assume that a maximum North-South manoeuvre has taken place and include a 2.5% thrust magnitude error, the in-plane velocity increment becomes 0.011 m/s. Here account is taken of the fact that two thrusters are used for the North-South manoeuvre.

If the coupling is completely radial the change in eccentricity is 3.5×10^{-6}, which can be neglected when root-sum-squared with other eccentricity error components. If the coupling is entirely transverse, the change in eccentricity can still be neglected, but the change in the drift rate is 0.0038 deg/day, and must be taken into account. The equation below describes the satellite's longitude history:

$$\lambda = 0.01633t - 1.046 \times 10^{-3}\, t^2$$

$$\pm\, 0.0038(t - t_{NS}) \text{ for } t > t_{NS}$$

where λ = spacecraft longitude in degrees

t = time in days since start of cycle

t_{NS} = time in days of North-South manoeuvre

The longitude requirement is that the deadband must not be violated before the effect of the North-South manoeuvre is assessed, and corrective action computed - i.e. before $(t_{NS} + 2)$ days.

First consider the positive sign in the above equation. There is the possibility that the deadband limit opposite the switching boundary may be transgressed hence the North-South manoeuvre must not be performed too soon after the satellite has begun to return to the switching boundary. In the case of the negative sign in the above equation, there is the possibility that the switching boundary will be violated, hence the North-South manoeuvre must not be performed too late after the satellite has begun to return to the switching boundary. This places constraints on the timing of the North-South manoeuvre.

For L-SAT, the North-South manoeuvre must take place between 10.4 and 13.8 days after the start of the cycle.

Finally, we consider the effect of the North-South station-keeping attitude control burns on the satellite's drift rate. As shown in section 2.4, the maximum transverse component will cause a change in the drift rate of $\pm$ 0.0372 deg/day. If this is directed towards the switching boundary it is possible that deadband violation will occur before 2 days have elapsed after the North-South station-keeping manoeuvre. Hence corrective action must be taken. Consider a nominal cycle, then assuming that a North-South manoeuvre is performed 10.4 days after the beginning of the cycle, a drift rate of 0.0191 deg/day must be removed. Now, considering that there are two three-month periods during the year when this unfavourable situation occurs, the ΔV penalty of having to arrest this large drift rate comes to 2 m/s over a 7-year period.

6. CONCLUSION

The station-keeping strategy for L-SAT has been derived with a view to satisfying a number of constraints. These are :

- the satellite shall be maintained within 0.1^o of its nominal position as seen from the centre of the Earth, to a probability of 99%
- the minimum cycle time is to be 14 days,
- the ΔV for the manoeuvres is to be minimised.

The resulting strategy takes account of natural and manoeuvre-induced perturbations as well as random tracking and manoeuvre execution errors. The ΔV budget for a 7-year life is as follows:

for East-West Station-keeping, ΔV = 36 m/s

for North-South Station-keeping, ΔV = 357 m/s

Total ΔV = 393 m/s

High manoeuvre efficiency is obtained by suitable selection of strategies and be deriving as much benefit as possible from attitude control thruster burns.

Acknowledgements

The work reported here has been carried out under the ESA L-SAT Phase B contract, and is reproduced by permission.

TABLE 1

ΔV Induced by Single Thruster Pulse

(Normal Mode)

Axis \ ΔV	ΔV Radial (m/s)	ΔV Transverse (m/s)	ΔV North-South (m/s)
Roll	-7.988×10^{-5}		$\pm 9.352 \times 10^{-5}$
Pitch	-7.988×10^{-5}	$\pm 9.352 \times 10^{-5}$	
Yaw			$\pm 1.23 \times 10^{-4}$

TABLE 2

No. of Pulses	K_1	K_2
5	4.996	0.145
20	19.739	2.745
27	26.302	5.217
40	37.863	11.036
52	46.988	18.991

TABLE 3

Axis	Maximum Torque			Minimum Torque		
	Pulses	ε	$\sqrt{A^2 + B^2}$	Pulses	ε	$\sqrt{A^2 + B^2}$
Pitch	52	-45.1° ($\Delta V_T > 0$)	3.35×10^{-6}	44	-41.6° ($\Delta V_T > 0$)	2.82×10^{-6}
	52	1.1° ($\Delta V_T < 0$)	3.35×10^{-6}	44	4.57° $\Delta V_T < 0$)	2.82×10^{-6}
Roll	27	78.8	6.967×10^{-7}	0	0	0

TABLE 4

ΔV's Induced by Single Thruster Pulse

(Station-Keeping Mode)

Axis	ΔV Radial (m/s)	ΔV Transverse (m/s)	ΔV North-South (m/s)
Roll	-5.923×10^{-4}		$\pm 6.935 \times 10^{-4}$
Pitch	-5.923×10^{-4}	$\pm 6.935 \times 10^{-4}$	

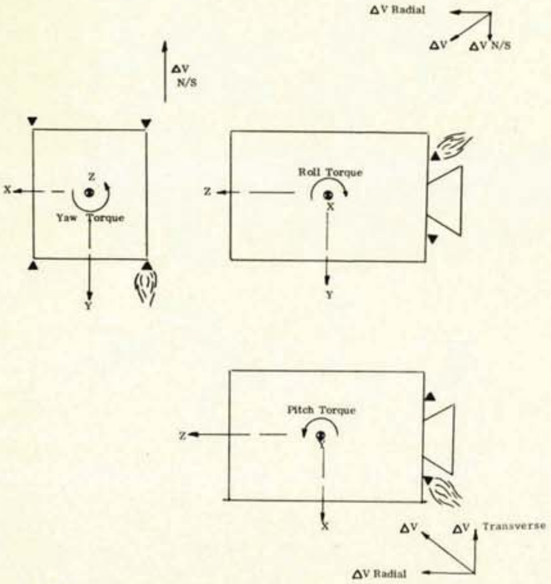
TABLE 5

Axis	Peak Disturbance Torque (Nm)	Thruster Frequency (sec^{-1})
Roll	2.0	1.872
Pitch	1.5	1.404

TABLE 6

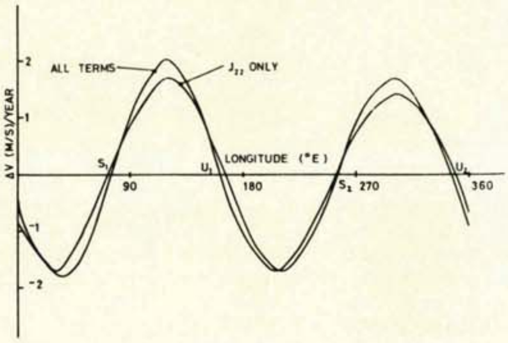
Total ΔV's Induced during a Station-Keeping Manoeuvre

Axis	ΔV Radial (m/s)	ΔV Transverse (m/s)	ΔV North-South (m/s)
Roll	-0.117		$\pm$ 0.141
Pitch	-0.087	$\pm$ 0.106	

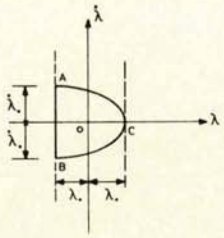


LSAT THRUSTERS — INDUCED TORQUES AND VELOCITY INCREMENTS

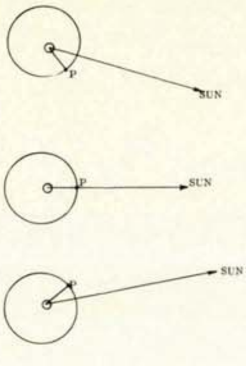
FIG. 1



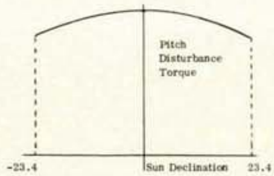
TRIAXIALITY PERTURBATION FIG. 2



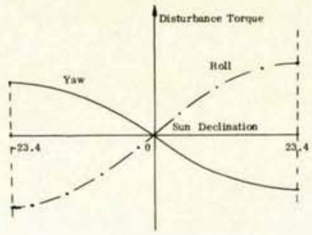
MANOEUVRE STRATEGY FIG. 3



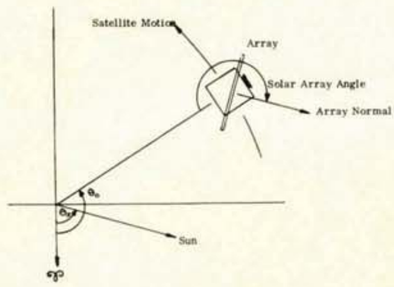
ECCENTRICITY STRATEGY FIG.4



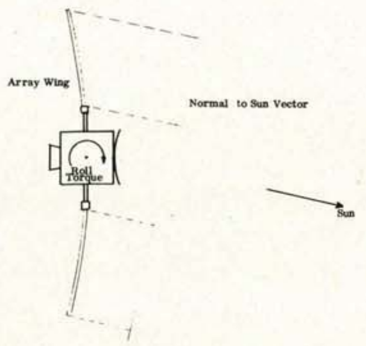
PITCH DISTURBANCE TORQUE VARIATION FIG.5



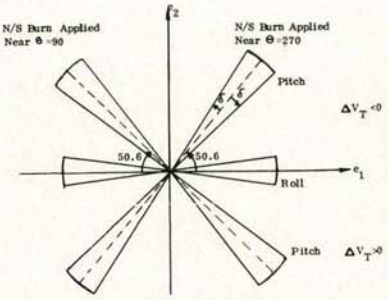
ROLL AND YAW TORQUE VARIATION FIG.6



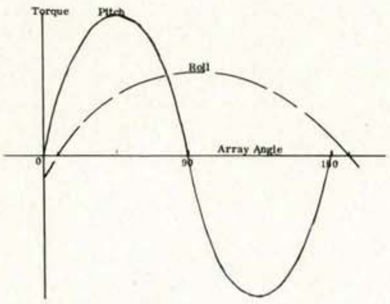
SATELLITE ARRAY GEOMETRY FIG.7



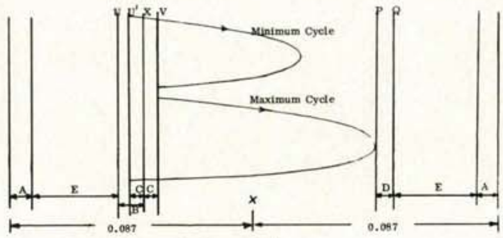
ARRAY DIHEDRAL EFFECT FIG.8



ECCENTRICITY INCREMENT DUE TO STATION-KEEPING FIG.10



PITCH AND ROLL TORQUE VARIATION FIG.9



A=Librations due to second order perturbations
B=RSS of longitude and eccentricity tracking errors
C=0.707*longitude tracking error
D=Libration due to eccentricity tracking error
E=Libration due to eccentricity control

EAST-WEST DEADBAND ALLOCATION FOR LSAT FIG.11

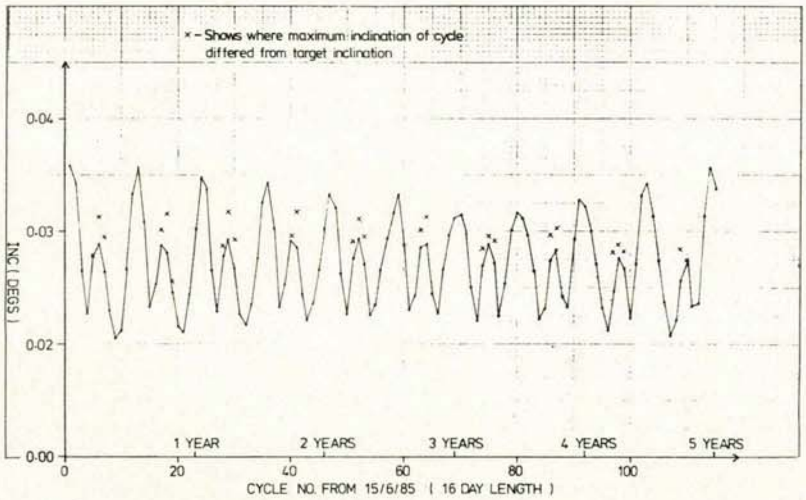


FIG 12 VARIATION OF TARGET INCLINATION AND MAXIMUM INCLINATION DURING CYCLE

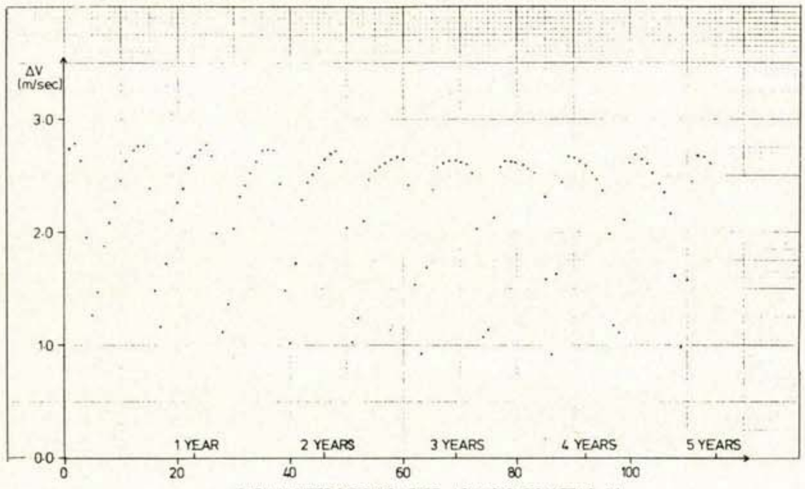


FIG 13 ΔV REQUIREMENTS PER CYCLE FOR N/S STATION KEEPING STATEGY OF 16 DAY LENGTH