PARTITIONING METHODS FOR IDENTIFICATION AND CONTROL OF FLEXIBLE SPACECRAFT

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ABSTRACT

This paper presents the results obtained in the estimation of the state vector and in the parameter identification of a linear model of European Satellite GEOS. The proposed method is the sequential decision technique based on a binary quantization procedure. This method is tested with simulated data as well as with in-flight data provided by telemetry.

Keywords : Estimation - Identification - Partitioning methods.

1. INTRODUCTION

This paper summarizes results obtained at the University of Louvain for the modelisation, the state vector estimation and the parameters identification of the European satellite GEOS. As proposed by ESA the problem can be described as follows. GEOS is a geostationnary satellite composed by a central rigid body carrying short antennas and two long flexible booms with tip masses (experimental devices). ESA is interested in the study of the attitude motion of GEOS, i.e. in the evolution of the orientation of the satellite with respect to some reference frame. This orientation is important for telecommunication antennas, solar pannels, instrumental devices... In order to perform this study it is necessary to elaborate a mathematical model for the attitude motion, and methods of estimation of the state and of identification of the parameters of this model. The knowledge or the monitoring of these parameters, which have a physical significance, is important in itself, in order to provide a correct interpretation of the scientific measurements or to check the stability of the attitude motion during configuration modifications (boom deployment...). The estimation and the identification have to be achived by processing telemetry data provided by two accelerometers and three magnetometers located on the central body.

Theoretically this satellite has to be described, as a continuum, by a partial differential equation. Practically a reduced order model has to be elaborated for estimation and identification purpose. This finite dimensional model is obtained by mode truncation. It has been checked that the first six modes are dominant and that the effect of the neglected modes on the accelerometers and magnetometers outputs is not significant (no observation spill-over). On the other hand this reduced model can be linearized and can therefore be described by a set of 12 first-order linear differential equations. The construction and properties of this model are discussed in section 2.

With this linear model the estimation problem is easily solved by use of the Kalman filter, provided the dynamical parameters of the model are perfectly known. As uncertainty on these parameters occurs due, for example, to fuel consumption, incomplete boom deployment, thermal effects..., it is necessary to implement an adaptive estimation procedure. As the knowledge of these dynamical parameters is important in itself we have chosen a method providing an explicit identification of them in order to follow the evolution of the dynamical structure of the satellite. A lot of identification methods are available, but, as the model is linear, it seems to be interesting to choose a method preserving this linear structure. This paper presents the application to GEOS of the sequential decision method and more precisely of the binary quantization procedure proposed by Lainiotis. The method and its implementation is described in section 3.

This method was tested successfully, firstly with measurements provided by simulations based on the reduced order linear model, and, in a second step, with real data

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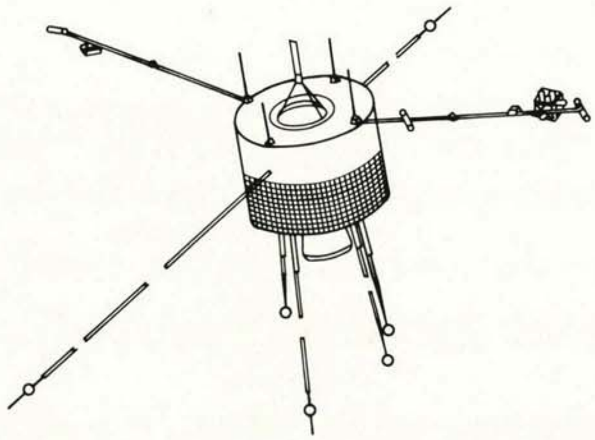

Total mass : 270 kg

Fig. 1 - GEOS I.

provided by telemetry. The results of this systematic studies are given in section 4.

2. MODEL DESCRIPTION

As said in introduction GEOS can be described as a main body with several appendages (antennas, radial booms and two 20 meters radial cables with instrumental devices). A representation of GEOS is given in fig.1. A dynamical analysis of this system has been performed by mean of the "multibody formalism", with a modelisation of the satellite as a set of interconnected deformable bodies ([1]). It has been shown ([2]) that the dynamical behaviour of the main body is almost unaffected by the high frequency modes induced by the vibrations of the short booms and the bending of the cables. For identification and estimation purpose it is therefore sufficient to consider a reduced order model representing GEOS as a main rigid body with two attached regid cables. The relative motion allowed for the cables in this model is the pendulum motion with respect to the main body.

The actual configuration of this model in the inertial space is then described as follows (see fig.2). Define

- $\{\hat{I}\}$ as some inertial frame who's third axis is aligned with the nominal spin vector ω_0 and the angular momentum,
- $\{\hat{A}\}$ as the nominal rotating frame who's third axis is also aligned with $\underset{\sim}{\omega}_0$,
- $\{\hat{X}\}$ as the reference frame, fixed in the central body, but not necessarily coïnciding with its geometrical frame $(\{\hat{X}_1\})$,
- $\{\hat{X}^1\}$ as the geometrical frame of the main body,
- $\{\hat{X}^2\}\{\hat{X}^3\}$ as two frames aligned with the two (rigid) cables.

At the <u>nominal equilibrium</u> (consisting in a constant spin motion of the main body without any relative pendulum motion of the cables), the $\{\hat{A}\}$, $\{\hat{X}\}$, $\{\hat{X}^2\}$ and $\{\hat{X}^3\}$ frames coïncide with $\{\hat{X}^1\}$. This equilibrium is possible only if ω_0 coïncides with the axis of greatest moment of inertia of the whole system. If some bias exists, due to unbalanced mass in the main body or to assymmetric lengths of cables, $\{\hat{A}\}$ and $\{\hat{X}\}$ still coïncide but are tilted with respect to $\{\hat{X}^1\}$, and dito for the cables frames. Any <u>biased equilibrium</u> configuration is characterized by four constant angles (θ^1_{10}, θ^1_{20} for the main body and θ^2_{30}, θ^3_{30} for the two cables) depending only on the physical parameters of the satellite. When the satellite is deviated from its equilibrium configuration (nominal or biased) by some physical perturbation, the $\{\hat{X}\}$ frame (fixed in the central body) does not more coïncide with the $\{\hat{A}\}$ frame : three attitude angles are necessary to describe the relative time-varying misalignement between $\{\hat{A}\}$ and $\{\hat{X}\}$ (θ^1_1, θ^1_2 and θ^1_3). In addition two relative angles are necessary to describe the

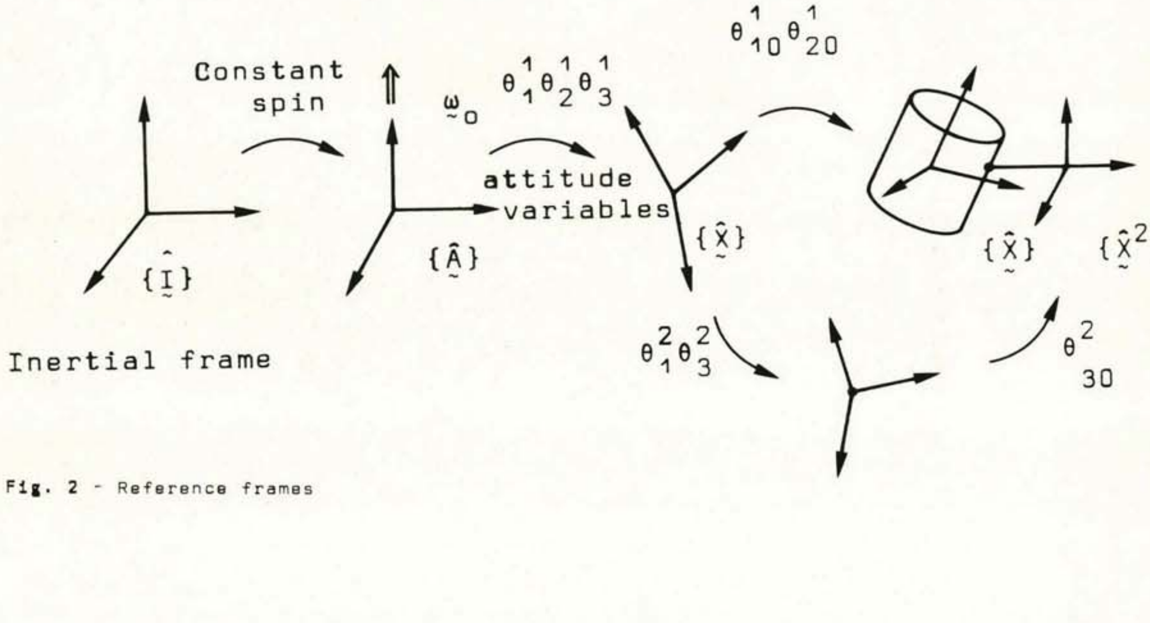

Fig. 2 - Reference frames

pendulum deviation of each cable with respect to $\{\hat{X}\}$: θ_1^2, θ_3^2 and θ_2^3, θ_3^3. Finally θ_3^1 may be eliminated by mean of the first integral of motion. All these transitions between frames are summarized in fig.2.

With the description we obtain a six degrees of freedom dynamical system whose linearized equations of motion around the equilibrium are deduced from the multibody approach under the following matricial form

$$M\ddot{y} + G\dot{y} + Ky = 0 , \quad (1)$$

where

$$y = [\theta_1^1,\ \theta_2^1,\ \theta_1^2,\ \theta_3^2,\ \theta_1^3,\ \theta_3^3]^T , \quad (2)$$

and where the three (6x6) matrices M,G,K depend on the physical parameters of the satellite. A (12x12) transition matrix Φ can then be defined, correspondingly to the following state vector x

$$x = [y^T,\ \dot{y}^T] \quad (3)$$

in order to describe completely the attitude behaviour of the satellite :

$$x(t+\Delta t) = \Phi(\Delta t)x(t) . \quad (4)$$

Two accelerometers and three magnetometers, fixed in the central body, provide measurements which, in linear approximation, are given by

$$z_a(t) = z_{ao} + H_a x(t) \quad (5)$$

$$z_m(t) = z_{mo}(t) + H_m(t)x(t) ,$$

where z_{ao} and $z_{mo}(t)$ represent the output signals obtained on equilibrium motion and where the elements of $H_m(t)$ and $z_{mo}(t)$ are polynomials in sin $\omega_o t$ and cos $\omega_o t$. The characteristic elements of the accelerometers outputs (z_{ao} and H_a) are time invariant and depend only on the equilibrium configuration, while the characteristic elements of the magnetometers outputs ($z_{mo}(t)$ and $H_m(t)$) depend on the equilibrium configuration but also explicitly on time as these outputs are directly related to an absolute reference : the earth's magnetic field.

In order to provide a flexible tool for the estimation/identification algorithm, a numerical programme has been elaborated to generate, for any set of physical parameters characterizing GEOS,

- the equilibrium configuration : θ_{10}^1, θ_{20}^2, θ_{30}^3
- the state transition matrix ($\phi(\Delta t)$) corresponding to the motion around this equilibrium
- the output characteristic elements : z_{ao}, $z_{mo}(t)$, H_a, $H_m(t)$.

The programme allows the choice of 13 parameters : the six characteristic elements of the main body inertia matrix, its mass, the coordinates of its center of mass in the $\{\hat{X}_1\}$ frame, the spin speed and the lengths of the two cables.

3. SEQUENTIAL DECISION METHOD

The sequential decision technique and its implementation in GEOS' particular problem are now discussed in details. The general estimation/identification problem is defined as follows. Consider a stochastic system satisfying a discrete-time dynamical equation

$$x(k+1) = \phi(k+1,k,\theta)x(k) + G(k)v(k), \quad (6)$$

and observed through a measurement equation

$$z(k) = H(k,\theta)x(k)+w(k) . \quad (7)$$

In these relations x(k) and z(k) are respectively the n-state vector and the m-output vector at time k, while θ is the q vector of the unknown parameters to be identified. $\phi(k+1,k,\theta)$ and $H(k,\theta)$ are respectively the (nxn) state transition matrix and the (mxn) output matrix and depend on θ. $\{v(k)\}$ and $\{w(k)\}$ are white gaussian uncorrelated noise sequences, with zero mean and, respectively, $Q(k)(\geq 0)$ and $R(k)(>0)$ as covariance matrices. It is assumed, in addition, that the a priori probability density function (p.d.f.) of the initial state x(0), say p[x(0)], is known and that the a priori p.d.f. of θ, $p(\theta)$, defined on the subset Θ of the possible values of ($\theta \in R^q$), is given. The problem is to estimate x(k) and to identify the values of the unknown parameters. Let's note θ^* the actual (unknown) value of θ.

The sequential decision method can be introduced in two steps. In a first step consider a discretization of the parameter space Θ into M possible values of θ, say θ_i (i=1,...,M). For each i, construct a Kalman filter based on the corresponding θ_i, i.e. on $\Phi(k+1,k,\theta_i)$ and $H(k,\theta_i)$. These M filters generate the conditional p.d.f. of the state given the past measurements and a particular θ_i :

$$p[x(k)|Z_o^k,\theta_i] \quad , \ i=1,\dots,M \quad (8)$$

where

$$Z_o^k = \{z(0),\dots,z(k)\} .$$

It is possible, in addition to compute recursively the evolution of a posteriori probabilities of the θ_i, i.e.

$$Pr[\theta_i|Z_o^k] \quad . \quad (9)$$

The initial condition of this evolution is deduced from $p(\theta)$, while the evolution itself is described by recursive relations deduced from Lainiotis ([3]) and Magill ([4])

$$Pr[\theta_i|Z_o^k] = \frac{\Lambda[k,z(k),\theta_i]Pr[\theta_i|Z_o^{k-1}]}{\sum_{j=1}^{M} \Lambda[k,z(k),\theta_j]Pr[\theta_j|Z_o^{k-1}]}, \quad (10)$$

where

$$\Lambda[k,z(k),\theta_i] = |P_{\tilde{Z}}(k,\theta_i)|^{-1/2} \exp\{-\frac{1}{2}||\tilde{z}(k,\theta_i)||^2 P_{\tilde{Z}}^{-1}(k,\theta_i)\} \quad (11)$$

In this expression $\tilde{z}(k,\theta_i)$ represents the innovation process based on θ_i ; it is known to be a white gaussian process, with zero mean and with $P_{\tilde{Z}}(k,\theta_i)$ as covariance matrix given recursively by the Kalman filter. These Λ are in fact likelihood ratios and their effect consists in penalizing the probabilities according to the lack of consitency of the innovation process based on θ_i with its computed statistics. Details can be found in [5].

Because of the discretization of H it is quite probable that the actual value, θ^*, does not belong to the family of θ_i. But, even in this case, Magill has shown ([4]) that the algorithm converges to the θ_i which is the closest, in the Euclidian norm sense, to the actual θ^*. That means that the corresponding a posteriori probability tends to one, for k tending to infinity, while the probabilities corresponding to other values of θ tend to zero. This convergence property constitutes the foundation of the simplified procedure proposed in the following second step.

If many parameters have to be identified and if a good accuracy is required, it is necessary to consider simultaneously a large number of θ_i, increasing significantly the computational work. A method to avoid this limitation has been proposed by Lainiotis and Sengbush ([6]). Instead of considering simultaneously the M values θ_i, only two values, θ^1 and θ^2, are compared. The two Kalman filters based on these values produce recursively the corresponding a posteriori probabilities, using eq. (3) and (10). From the convergence property presented above, it can be expected that the probability of the best value of θ will tend to one, so an elementary choice between θ^1 and θ^2 will be made possible. The convergence of the algorithm is assured by the following rule of construction of the alternatives$[\theta^1, \theta^2]$.

Define an initial value of θ, say $\theta(1)$, a q vector of increments, say $e(1)$, and a q-vector of directions, say $d(1)$, with either $d_i(1) = 1.0$, either $d_i(1) = -1.0 (i=1,...q)$.

a) For the first elementary choice define $\theta^1(1)$ and $\theta^2(1)$ as follows :

$$\theta^1(1) = \theta(1)$$

and

$$\theta_i^2(1) = \theta_i^1(1) \qquad (i=2,...,q)$$

while

$$\theta_1^2(1) = \theta_1^1(1) + e_1(1)d_1(1)$$

b) When, accordingly to some decision rule, a decision has been taken, the best value of θ is chosen as $\theta^1(2)$ for the second choice, and $e(2)$ and $d(2)$ are modified as follows :
If the best value is $\theta^1(1)$,

then
$$e_1(2) = \delta e_1(1)$$
$$e_i(2) = e_i(1) \quad (i=2,...,q)$$
$$d_1(2) = -d_1(1)$$
$$d_i(2) = d_i(1) \quad (i=2,...,q)$$

where δ is a reduction coefficient with $0<\delta<1$.
On the other hand, if the best value is $\theta^2(1)$, then $e(2)$ and $d(2)$ are imposed to be equal to $e(1)$ and $d(1)$.
In both cases, the new $\theta^2(2)$ is defined as coinciding with $\theta^1(2)$, except for the second component

$$\theta_2^2(2) = \theta_2^1(2) + e_2(2)d_2(2) .$$

c) A new elementary choice has now to be made between $\theta^1(2)$ and $\theta^2(2)$. When, after q successive elementary choices, the q components of θ have been inspected, we go back to the first component with the modified values of e and d and the process is repeated. As the reduction coefficient δ is comprised between 0 and 1, this procedure assures the convergence of the algorithm to θ^*. The procedure is stopped when all increments fall below a prespecified level.

The advantage of this technique is that it preserves the linear structure of the system and does not complicate it artificially. The method is quite flexible and the corresponding computer programmes are self-contained : they do not need preliminary programmes such as sensitivity analysis with respect to the physical parameters. This algorithm gives the possibility to identify a large number of parameters with a small computer. Eventually a long sequence of output data has to be processed before obtaining the convergence. If long records are not available it is, nevertheless, possible to process off-line the same sequence several times, each time improving the quality of the identification. On the other hand, as it decomposes the original problem into elementary (linear) subproblems, this algorithm belongs, in some sense, to the class of the partitioning algorithms.

With the linear model described in section I it is straightforward to implement the sequential decision method for the identification of GEOS. In order to provide a flexible tool for the investigation of the problem several options are left open and have to be specified as inputs for the identification programme. They concern

a) The characteristic elements of the estimation problem. The a-priori p.d.f. of the initial state has to be specified through its mean and covariance matrix, as well as the covariance matrices of the noises [$Q(k)$ and $R(k)$].

b) The initial conditions of the identification process. It is possible to identify any subset of the considered 13 physical parameters of the model. It must be specified which parameters have to be identified, their initial values as well as the initial increments and directions vectors. The other parameters are assumed to be known exactly and are also given as inputs.

c) The characteristic elements of the identification process. The decision rule has to be defined. The selected decision criterion is the following : a decision is taken when two conditions are satisfied. The a posteriori probability of the "best" value has to reach some prespecified level, η, and this value of the parameter has to be the best during a continuous period of prespecified length (N time intervals). These values η and N, characterizing the decision rule, have to be given, as well as the reduction coefficient δ.

The programme produces as outputs

a) The evolution of the identification of the unknown parameters. It must be noted that, until the increments fall under the prespecified precision level, no conclusion can be made concerning the values of the parameters.

b) The evolution of the estimated equilibrium configuration and of the eigenfrequencies. As they are directly related to the physical parameters of the model, these informations are provided directly by the dynamical programme.

c) The evolution of the estimation of the state vector and of the error covariance matrix.

Due to the extreme flexibility of the method it is very easy to adapt the programme in order to identify extra-parameters not affecting the dynamical equation of the model but characterizing the measurement devices. These parameters represent, for example, misalignements of the accelerometers and magnetometers, or the components of the earth magnetic field in the $\{\tilde{I}\}$ frame, which are unknown. This possibility has been used as it will be seen in section 4.

4. NUMERICAL RESULTS

There were two phases in the testing of this algorithm. In a first step use was made of measurements provided by numerical simulations and in a second step of real data provided by telemetry. These results are now summarized.

4.1 Numerical simulations.

A first set of runs were carried out to check the validity of the method and the efficiency of the identification algorithm, and to test the precision we could expect, under ideal conditions for the state estimation and the parameters identification. The conditions of the simulations are the following. For a given set of parameters, referred to as the "exact" values, the dynamical programme produces the evolution of the state vector and simulates the outputs. The estimation/identification programme "ignores" several exact values and has to find them by processing the simulated outputs. It is therefore straight forward to check the convergence of the identification algorithm but it is not possible to check the validity of the dynamical model of GEOS. The results of these runs have been published in [2] and are only summarized here. Concerning the identification of the parameters it can be included that

a) The moments of inertia of the central body (I_1, I_2, I_3) cannot be identified simultaneously but the ratios of inertia

$$\frac{I_1 - I_3}{I_2} \quad \text{and} \quad \frac{I_2 - I_3}{I_1}$$

are identifiable with a good accuracy ($5\ 10^{-3}$). These ratios have in fact a direct influence on the dynamical behaviour of GEOS (namely on the eigenfrequencies).

b) The mass of the main body is not identifiable because it is much too large with respect to the tip masses of the cables (270 kg and 0.1 kg).

c) The vertical position of the center of mass is identifiable with a precision of a few millimeters.

d) The lengths of the cables can be identified with a precision of 0.1m (on 20m). Nevertheless the accuracy decreases significantly (up to 0.5m) when the lengths have to be identified simultaneously with the products of inertia.

e) The spin speed is identified with a precision of 10^{-2} rad/sec.

During these systematic study it appeared by experience, that the decision criterion has not to be too severe in order to assure convergence, specially when many parameters have to be identified simultaneously. We selected a critical probability level (η) of 0.55,...., 0.6 and a minimum number of consistent results (N) of 3,...,5. On the other hand the reduction coefficient δ is chosen close to 1.0 (0.8,...,0.9). With these values the convergence is "undamped" with oscillations around the exact value of θ. As the decisions are taken rapidly the risk of a wrong decision increases, but, as more values of θ are tested the risk of convergence to a local minimum of the probability function defined on θ decreases.

These runs were achieved on IBM 370/158. During 900 sec CPU time -800 measurements are processed. The algorithm needs about 400 measurements to converge. If records of this length are not available it is possible to reprocess several times the same data. This possibility was tested successfully.

4.2 In-flight Data.

GEOS I was effectively launched in April 1977. Outputs provided by telemetry are therefore available and are provided by ESOC (European Space Operation Center). They are relative to optical sensors (giving the spin speed), acceleremeters and magnetometers. Accelerometers and magnetometers outputs are unfortunately not always available. Different records of measurements were investigated, corresponding to characteristic periods in GEOS' mission: deployment of short radial booms, of long axial booms, of the two cables... From on phase to the other the physical parameters can be modified and ESA was interested in the monitoring of these parameters, specially the moments of inertia directly related to the stability of the system, the products of inertia related to the tilt of the biased equilibrium with respect to the nominal equilibrium, and the vertical position of the center of mass.

For a first set of data records only accelerometers outputs are available. It can be observed that for these phases the nutation mode is dominant so the two accelerometers signals are purely sinusoïdal. With the hypothesis of nutation motion the information directly related to the physical parameters of the system can be concentrated in five characteristic elements of the output signals : the offset of the two signals (related to the tilt of the equilibrium configuration and therefore to the products of inertia) the frequency of the signals (related to the moments of inertia), the amplitude ratio, and the relative phase between the signals. For any set of physical parameters the dynamical programme provides the values of these 5 characteristic elements. The comparison between the actually observed values of these 5 elements and their values provided by the dynamical programme on basis of the identified values of the physical parameters permits to check the convergence of the algorithm. The conclusion is that the concordance is very good.

For a second set of records only magnetometers outputs are available. These measurements give the components of the earth magnetic field in a frame attached to the central body ($\{\tilde{\hat{X}}^1\}$). Unfortunately the available records correspond to a period where no mode is excited and therefore only spin motion is present, so it is impossible to identify the physical parameters of the system. Nevertheless the sequential decision technique can be implemented in order to identify unknown parameters of the observation process : the inertial components of the magnetic field (in the $\{\tilde{\hat{I}}\}$ frame) and the orientation, with respect to the central body, of the sensitive axes of the magnetometers, which because of electrical phenomena, are not known exactly (the offset with respect to the nominal orientation is identified to be equal to 0.1 rad.). The identification of these output parameters is a preliminary step necessary before the identification of the other physical parameters.

5. CONCLUSIONS

1) The sequential decision technique is particularly adapted for the identification of the parameters of the linear model of GEOS : it preserves the linear structure of the model and constitutes a flexible tool of investigation. It is straight forward to adapt the corresponding programme to take into account particular effects as misalignments of the instrumental devices...

2) An other identification method has been used for GEOS' problem : it is the argumentation of the state vector by the unknown parameters. The resulting non linear filtering problem has been solved by use of the extended Kalman filter ([2]). The two methods can be compared. As concerns their convergence properties, the sequential decision technique is superior when the parameters are far from their nominal values. Conversely, the non linear filtering method produces a higher final accuracy when the uncertainty on the parameters is small. The two techniques are herefore complementary : ideally, one should implement the sequential decision technique first to give a rough estimate of the parameters, and then the non linear filtering to increase accuracy.

3) This partitioning approach can be extended to control problems. Lainiotis proposes a partitioning suboptimal solution for the adaptive control of systems with unknown parameters ([7]). The solution appears as a weighted sum of elementary solutions corresponding to particular values of the unknown parameters. The weighting coefficients are the a posteriori probabilities of the hypothesis and evolve accordingly to eqs (10) and (11). A particular application of this method deals with the control of large space structures ([8]).

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