ON SPIN MANOEUVRES WITH A SYMMETRIC SATELLITE

F. Janssens

European Space Research and Technology Centre (ESTEC)
Noordwijk, The Netherlands

ABSTRACT

This paper contains some theoretical results and practical formulae to calculate non-ideal spin-manoeuvres with a symmetric satellite. In a non-ideal spin manoeuvre the spacecraft is not rotating uniformly at the start of the manoeuvre and the thruster used is not a pure tangential or spin thruster. Such manoeuvres generate a nutational motion and an attitude change. These aspects can be studied as a special case of the theoretical problem known as "the self-excited rigid body". The nutation is described by Fresnel integrals from which practical formulae giving the maximum nutation and its rate of decrease are extracted. A numerical illustration is given by the spin-up of Meteosat-2. A closed form result for a small attitude change caused by a pure spin up thruster in the presence of initial nutation and by a general thruster on a Kovaleski top (inertia A, A, $\frac{A}{2}$) is mentioned.

Keywords: Spin-up, nutation, self-excited rigid body, Meteosat, Kovaleski.

NOTATIONS

- A : lateral moment of inertia
- C : inertia around the nominal spin axis
- $m_{1,2,3}$: torque components in the principal axis system
- m^* : $m_1 + i m_2$ lateral component of the torque as a complex number
- m_3 : spin component of the torque. Throughout this paper $m_3 > 0$.
- x : $C\omega_3/m_3\tau$ normalised time. Replaces t as independant variable
- x_0 : $C\omega_{30}/m_3\tau$ initial value of x at the start of the manoeuvres
- y : $i\theta e^{-i\varphi}$ intermediate variable for the attitude change
- $C(x), S(x)$: Fresnel-integrals
- $F_2(x)$: $C(x) + i\,S(x)$
- α : intermediate variable that locates the instant of maximal nutation when starting from a uniform spin
- θ_n : nutation angle. Instantaneous angle between the spin-axis and the angular momentum. In terms of the components of the angular velocity $\tan\theta_n = |\omega^*|/\lambda\omega_3$
- θ_0 : nutation angle at $t=0^+$ when starting from rest
- θ_i : initial nutation $(t<0)$ when present
- λ : C/A slenderness ratio. $\lambda > 1$ oblate body (spin axis is stable), $\lambda < 1$ prolate body (spin axis unstable in the presence of energy dissipation)
- $\omega_{1,2,3}$: components of the angular velocity in the principal axis system
- ω^* : $\omega_1 + i\omega_2$ lateral component of the ω-vector as a complex number
- τ : $[\pi C/m_3(1-\lambda)]^{1/2}$ time constant
- μ : $-\left[\frac{1+i}{2} - F_2(x)\right]$
- ψ, θ, φ : Euler angles

1. INTRODUCTION

Spin adjustment manoeuvres are quite common activities in spacecraft operations because different phases in the lifetime of a spin stabilized spacecraft require a different nominal spin rate. Typical examples are: launcher constraints, deployment activities and firing of an apogee boost motor.

In an ideal spin manoeuvre one starts from a uniformly rotating spacecraft and the spin thruster gives only a constant spin up or spin down torque. Under these assumptions the spin manoeuvre causes no nutation and no attitude change. The calculation of the burn duration from the initial and desired spin rate is trivial. In an operational environment one has often to consider spin manoeuvres where one or both of these assumptions are not met. The influence of an initial nutation must be assessed as well as thruster misalignments. Some spacecraft have no pure spin thruster. This is the case with Meteosat-2 for which this study was undertaken. The spin changes are executed with thrusters having a more important lateral as spin-up component.

A reasonable model for non-ideal spin manoeuvres is provided by the theoretical problem known as "The self-excited rigid body" (Ref. 1, 2). Self-excited means that the torque vector has a constant direction in a body-fixed reference frame. We assume that the torque has also a constant magnitude which is equivalent with a constant force assumption for the thruster. The moments of inertia of the spacecraft are also treated as constants, their variation due to out-flowing mass is neglected. Furthermore we neglect internal energy dissipation and consider only symmetric spacecraft.

The first step of the complete solution of the formulated problem is to obtain expressions for the instantaneous rotation vector $\omega(t)$ in a body fixed frame. Integration of the Euler equations shows that $\omega(t)$ can be expressed in terms of the Fresnel-integrals (Ref. 1, 2). This result is rederived

Proc. Int. Symp. Spacecraft Flight Dynamics, Darmstadt, FRG, 18-22 May 1981 (ESA SP-160, Aug. 1981)

in a simple way and permits a detailed discussion of the evolution of the nutation angle. It is shown that for a spin-up from rest the maximum nutational angle equals the angle between the torque direction and the spin-axis. A practical formula for the nutation induced with an arbitrary thruster and starting from a uniform spin is also given.

To complete the solution of the formulated problem one needs the time history of a suitable set of orientation parameters between the body-fixed and inertial reference frame to calculate the attitude change. The corresponding differential equations contain the previous result for $\omega(t)$ and their structure depends highly on the choice of the orientation parameters. Ref. 3 reduces the case of a pure spin thruster to a Weber equation in orientation parameters related to the Cayley-Klein parameters. In terms of the rotation matrix one obtains a linear first-order matrix differential equation (Ref. 1) for which a converging recurrence formula is given in Ref. 4. The assumption of a small attitude change does not introduce simplifications in each of these mentioned approaches.

Using the Euler angles the remaining differential equations reduce to one linear differential equation under the assumption that the attitude change is small. Analytical results can then be obtained for a pure spin torque in the presence of initial nutation and for a general torque applied to a Kovalski top (inertia's A, A, A/2) which was spinning uniformly.

2. BASIC EQUATIONS FOR THE NUTATION ANGLE

Consider a symmetric rigid body with inertia A about any axis in the principal plane (x,y) through the center of mass and inertia C around the z-axis. The Z-axis is the third principal axis and the nominal spin-axis. From $t \geq 0$ onwards the body is subjected to a constant external torque with components $m_{1,2,3}$ in the principal axis system (x,y,z).

The time-history of the nutation angle $\theta_n(t)$ can be calculated from the components of the angular velocity $\omega_i(t)$. These components are obtained by integrating the classical Euler equations:

$$A\dot{\omega}_1 - (A-C)\,\omega_2\,\omega_3 = m_1 \qquad (1)$$

$$A\dot{\omega}_2 + (A-C)\,\omega_1\,\omega_3 = m_2 \qquad (2)$$

$$C\dot{\omega}_3 = m_3 \qquad (3)$$

with $\omega_i(0) = \omega_{i0}$

Due to the assumed symmetry Eq. 3 is decoupled from Eqs. 1 - 2 and easily integrated:

$$\omega_3(t) = \omega_{30} + \frac{m_3}{C}\,t \qquad (4)$$

So a spin-up always occurs when $m_3 > 0$. This result is not true for an assymetric rigid body. Eq. 4 is now substituted into Eqs. 1 - 2 which are combined into a single linear differential equation in the complex variable $\omega^* = \omega_1 + i\,\omega_2$, the equatorial or lateral component of the angular velocity:

$$\dot{\omega}^* + i(1-\lambda)\,\omega_3(t)\,\omega^* = \frac{m^*}{A} \qquad (5)$$

where $\lambda = C/A$ is the slenderness ratio

$m^* = m_1 + i\,m_2$ the equatorial or lateral part of the torque

and $\omega^*(0) = \omega_1(0) + i\,\omega_2(0) = \omega_0^*$

The general solution of Eq. 5 is:

$$\omega^*(t) = \left\{ \int_0^t e^{\,i(1-\lambda)\int_0^s \omega_3(\sigma)\,d\sigma}\,\frac{m^*}{A}\,ds + \omega_0^* \right\} \times e^{-i(1-\lambda)\int_0^t \omega_3(s)\,ds} \qquad (6)$$

Note that Eq. 6 has been obtained without any transformation of variables in the original Euler equations. To evaluate the integrals remaining in Eq. 6 we first rewrite it as:

$$\omega^*(t) = \left\{ \frac{m^*}{A}\,e^{-i(1-\lambda)\frac{\omega_{30}^2 C}{2m_3}} \int_{\omega_{30}}^{\omega_3} e^{\,i(1-\lambda)\frac{C\omega_3^2}{2m_3}}\,\frac{C}{m_3}\,d\omega_3 + \omega_0^* \right\} \times e^{-i(1-\lambda)\frac{C}{2m_3}(\omega_3^2-\omega_{30}^2)} \qquad (7)$$

Eq. 7 is only valid when a spin-up component is present $(m_3 \neq 0)$. The remaining integral cannot be expressed in terms of elementary functions. It is easily put in the following normalised form

$$\int_\alpha^\beta e^{\,i\frac{\pi}{2}x^2}\,dx = \left[F_1(x)\right]_\alpha^\beta = \left[C(x) + i\,S(x)\right]_\alpha^\beta \qquad (8)$$

where

$$C(x) = \int_0^x \cos\frac{\pi}{2}s^2\,ds \qquad (9)$$

$$S(x) = \int_0^x \sin\frac{\pi}{2}s^2\,ds \qquad (10)$$

are the Fresnel-integrals (Ref. 5). The notations are further simplified by using the time constant τ :

$$\tau = \sqrt{\frac{\pi C}{m_3(1-\lambda)}} \qquad (11)$$

and the non-dimensionless variable x

$$x = \frac{C\omega_3}{m_3\tau} = x_0 + \frac{t}{\tau} \qquad (12)$$

$$\omega^*(t) = e^{-i\frac{\pi}{2}x^2}\left\{ \omega_0^*\,e^{\,i\frac{\pi}{2}x_0^2} + \frac{m^*\tau}{A}\left[F_1(x)\right]_{x_0}^{x} \right\} \qquad (13)$$

Now we can use Eqs. 12 and 13 to calculate the nutation angle $\theta_n(t)$. By definition we have:

$$\operatorname{tg}\theta_n(t) = \frac{|\omega^*(t)|}{\lambda\,\omega_3(t)} \qquad (14)$$

where $|\ |$ stands for the module of the complex number.

Using Eqs. 12, 13 we obtain for Eq. 14

$$\operatorname{tg}\theta_n(t) = \frac{1}{x}\left| \frac{A\,\omega_0^*}{m_3\tau}\,e^{\,i\frac{\pi}{2}x_0} + \frac{m^*}{m_0}\left[F_1(x)\right]_{x_0}^{x} \right| \qquad (15)$$

In the next points the meaning of Eq. 15 will be discussed for some special cases of the initial conditions and direction of the torque vector. Table 1 summarises the cases to which Eq. 15 can be specialised. The analysis of the evolution of the nutation angle from Eq. 15 will be more complex when the 2 additive terms are present as there is no simple result for the module of the sum of two complex numbers.

2.1 Spin-up From Rest With a General Thruster (Case III)

Starting from rest the initial conditions are $\omega_0^* = 0$, $\omega_{30} = 0$ or $x_0 = 0$ and Eq. 15 becomes

$$tg\,\theta_n(x) = \frac{|m^*|}{x\, m_3}\, |F_2(x)| \tag{16}$$

The starting value of the nutation angle $\theta_n(0) = \theta_0$ is not given immediately by Eq. 16 as $F_2(0) = 0$ makes Eq. 16 undetermined. The series expansion of $C(x), S(x)$ (Ref. 5, 6) gives

$$\lim_{x \to 0} \left|\frac{F_2(x)}{x}\right| = 1$$

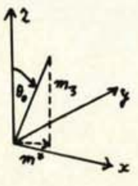

Fig. 1

Therefore

$$tg\,\theta_0 = \lim_{t \to 0} tg\,\theta_n(t) = \frac{|m^*|}{m_0} \tag{17}$$

The nutation angle which is not defined before the thruster is on takes at $t = 0^+$ the value θ_0. θ_0 is the angle between the spin-axis and the torque direction (Fig. 1). This result can also be obtained from Eqs. 1 - 3 when the product terms, which are of second order for small t, are neglected.

Now Eq. 16 can be rewritten as

$$tg\,\theta_n(t) = tg\,\theta_0 \left| \frac{C(x) + i\,S(x)}{x} \right| \tag{18}$$

In Fig. 2. the locus of complex numbers $z(x) = [F_2(x)] / x$ is represented. Looking how the module $|z(x)|$ changes with time (or x) we see that: $|z(x)|$ is maximum for $x = 0$ or $t = 0$. So θ_0 is also the maximal value of the nutation angle during a spin-up from zero. This result gives an upper bound for the nutation angle when the nominal spin-up is with a tangential thruster $(m_3 \neq 0,\ m^* = 0)$ and the influence of misalignments $(m^* \neq 0)$ is evaluated. So, the maximal nutation due to misalignments equals the misalignment angle. Initially the decrease of the nutation angle is monotonic : For $x = 1$ or $t = \tau$ $tg\,\theta_n(\tau) = .9\, tg\,\theta_0$ and for $x = 1.92$, $t \sim 2\tau$ the nutation angle has decreased to $tg\,\theta_n(2\tau) = 0.287\, tg\,\theta_0$. For larger values of x we see that $z(x)$ spirals to zero with curls of decreasing "amplitude" and centered on the first bisector. The evolution of the nutation angle can be described by an average value $\tilde{\theta}_n(x)$ taken on the first bisector and superimposed deviations $\Delta\theta_n(x)$, both $\tilde{\theta}_n(x)$, $\Delta\theta_n(x)$ tend to zero as time goes on.

Using the property that

$$\lim_{x \to \infty} C(x) = \lim_{x \to \infty} S(x) = \frac{1}{2} \tag{19}$$

one obtains easily the following approximate expressions:

$$tg\,\tilde{\theta}_n(t) = tg\,\theta_0\, \frac{\tau}{\sqrt{2}\, t} \tag{20}$$

$$tg\, 2\Delta\theta_n(x) = \frac{.636\; tg\,\theta_0}{x^2 + \frac{tg^2\theta_0}{2}}$$

The decrease of $\Delta\theta_n(x)$ is faster than the decrease of $\tilde{\theta}_n(x)$. For $t = 7.07\,\tau$ $tg\,\tilde{\theta}_n = \frac{tg\,\theta_0}{10}$.

2.2 Spin-up From a Uniform Spin with a General Thruster (Case IV)

At $t = 0$ the satellite is spinning uniformly $(\omega_{30} \neq 0)$ and a thruster with equatorial torque m^* and spin-up torque m_3 is activated. Eq. 15 with $\omega_0^* = 0$ becomes

$$tg\,\theta_n(x) = \frac{tg\,\theta_0}{x}\, |F_2(x) - F_2(x_0)| \tag{21}$$

where $x_0 = \frac{C\omega_{30}}{m_3\,\tau}$

Eq. 21 shows that the maximal nutation caused by this manoeuvre is less than the residual nutation level after a spin-up from zero to ω_{30} . An approximate expression for the maximum of the nutation is derived in Ref. 6 by replacing $F_2(x)$ by its osculating circule at $x = x_0$ one obtains

$$(tg\,\theta_n)_{max} = tg\,\theta_0 \cdot \cos\frac{\alpha}{2} \tag{22}$$

where α is the solution of $tg\,\frac{\alpha}{2} - \frac{\alpha}{2} = \frac{\pi}{2}\, x_0^2$ (23)

For $x_0 \geq 2$ this result can be replaced by

$$(tg\,\theta_n)_{max} = \frac{2}{\pi}\, \frac{1}{x_0^2 + 1}\, tg\,\theta_0 \tag{24}$$

Eq. 24 gives directly $\theta_{n\,max}$ as a function of x_0 and θ_0 , without solving the transcendental equation 23. Table II compares the results of Eqs. 22 - 23 versus Eq. 24.

x_0	$\alpha/2$	$\cos\alpha/2$	$\frac{2}{\pi}\frac{1}{x_0^2+1}$
1	70.34	.336	.318
2	82.63	.128	.127
3	86.34	.064	.064
4	87.85	.037	.037
5	88.60	.024	.024

TABLE II

From Eq. 21 we can also derive a decrease of the average nutation level $\tilde{\theta}_n(x)$ from the limiting value $\lim_{x \to \infty} F_2(x) = \frac{1}{2}(1+i)$

$$tg\,\tilde{\theta}_n(x) = \frac{tg\,\theta_0}{x} \sqrt{\frac{1}{2} + C^2(x_0) + S^2(x_0) - C(x_0) - S(x_0)} \tag{25}$$

This averaged decrease is also inversely proportional to the time.

2.3 A Comment on the General Formula (CASE VI)

Equation 15 gives the evolution of the nutation angle when before the activation of the general thruster $(m^* \neq 0,\ m_3 > 0)$ the satellite is spinning with an initial nutation $tg\,\theta_i = |\omega_0^*| / \lambda\,\omega_{30}$. Numerical applications of Eq. 15 are straightforward when a table of the Fresnel-integrals is available.

Using Eq. 19 $tg\,\theta_1(x)\cdot x$ becomes constant as time goes on:

$$\left| \frac{A\,\omega_0^*}{m_3\,\tau}\, e^{i\frac{\pi}{2}x_0} + tg\,\theta_0\left(\frac{1+i}{2} - F_2(x_0)\right) \right| \qquad (26)$$

where the x-axis is chosen coinciding with m^*. When Eq. 26 is maximal the initial phasing, ω_0^* was the worst possible. Eq. 26 can be written

$$\left| \frac{A\,\omega_0^*}{m_3\,\tau}\, \mu'(x_0) - tg\,\theta_0\, \mu(x_0) \right| \qquad (27)$$

and as $F_2(x)$ tends to circle around $(\frac{1}{2}, \frac{1}{2})$, $\mu'(x_0)$ becomes perpendicular to $\mu(x_0)$ as x_0 increases. To make the two terms of Eq. 27 co-linear ω_0^* becomes perpendicular to $\mu'(x_0)$ which means that the thrust direction is perpendicular to ω_0^*. Eq. 27 becomes

$$\left| x_0\, tg\,\theta_i + tg\,\theta_0\, |\mu(x_0)| \right|$$

from which the maximum nutation level can be calculated it is always smaller as the nutation cal- in 2.2. augmented by θ_i.

3. ATTITUDE CHANGE

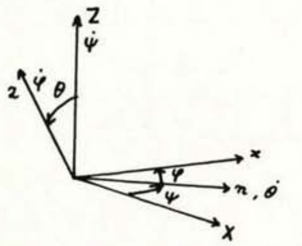

Fig. 3

The determination of the principal axes frame with respect to an inertial frame completes the solution of the self-excited rigid body problem. This part is much more difficult than the calculations of the nutation angle. In fact a complete analytical solution has not yet been found. We will use the Euler angles (Fig. 3) to define the orientation of the body-fixed frame in inertial space. At $t=0$ the connection between the two frames is given by the initial values $\psi_s, \theta_s, \varphi_s$. During the manoeuvre one expects eventually a stabilisation of the spin-axis in inertial space. It was indeed, shown that the nutation angle goes to zero which implies in turn that the influence of the equatorial torque (m^*) on the angular momentum averages out over a nutation period. The limiting motion $(t\to\infty)$ is a uniform spin about some unknown direction: ψ_f, θ_f. The azimut ψ_f of this direction can have any value in the range $0, 2\pi$ but it is not completely absurd to consider the small angle approximation on θ_f and θ.

The differential equations that define the evolution of the Euler angles are (Ref. 3, 6):

$$\omega_1(t) = \sin\theta\,\sin\varphi\,\dot\psi + \cos\varphi\,\dot\theta \qquad (28)$$

$$\omega_2(t) = \sin\theta\,\cos\varphi\,\dot\psi - \sin\varphi\,\dot\theta \qquad (29)$$

$$\omega_3(t) = \cos\theta\,\dot\psi + \dot\varphi \qquad (30)$$

Introducing the small angle approximation on θ we can use Eq. 30 to eliminate $\dot\psi$ in Eqs. 28, 29:

$$\omega_1(t) = \dot\theta\cos\varphi - \theta\sin\varphi\,\dot\varphi + \omega_3(t)\,\theta\sin\varphi \qquad (31)$$

$$\omega_2(t) = -\dot\theta\sin\varphi - \theta\cos\varphi\,\dot\varphi + \omega_3(t)\,\theta\cos\varphi \qquad (32)$$

With the complex variable

$$y = y_1 + i\,y_2 = \theta\sin\varphi + i\,\theta\cos\varphi = i\,\theta\, e^{-i\varphi} \qquad (33)$$

Eqs. 31 and 32 are combined into:

$$\dot y + i\,\omega_3(t)\, y = i\,\omega^*(t) \qquad (34)$$

$$y(0) = i\,\theta_s\, e^{-i\varphi_s} \qquad (35)$$

which is also a linear first order differential equation with as general solution

$$y(t) = e^{-iP(t)}\left\{\int_0^t e^{iP(s)}\, i\,\omega^*(s)\, ds + y(0)\right\} \qquad (36)$$

where $P(t) = \int_0^t \omega_3(s)\, ds = \frac{m_3\tau^2}{2C}(x^2 - x_0^2) = \frac{\pi}{2}\frac{x^2 - x_0^2}{1-\lambda}$ (37)

For the attitude change $\theta = |y(t)|$ Eq. 36 becomes:

$$\theta(x) = \left| \tau\int_{x_0}^{x} e^{i\frac{\pi}{2}\frac{x^2 - x_0^2}{1-\lambda}}\, \omega^*(x)\, dx + \theta_s\, e^{-i\varphi_s} \right| \qquad (38)$$

To eliminate the remaining integral the expression $\omega^*(x)$ corresponding to the particular case considered has to be used.

3.1 A Pure Spin-up Torque on a Spacecraft with Initial Nutation

When $m^* = 0$, $\theta_i = \frac{|\omega_0^*|}{\lambda\,\omega_{30}}$ we have (Ref. 6):

$$\omega^* = \omega_0^*\, e^{-i\frac{\pi}{2}(x^2 - x_0^2)} \qquad (39)$$

and Eq. 38 becomes:

$$\theta(x) = \left| \tau\,\omega_0^*\sqrt{\frac{1-\lambda}{\lambda}}\; e^{-i\frac{\pi}{2}x_0^2\frac{\lambda}{1-\lambda}} \left[F_2\left(\frac{x\sqrt{\lambda}}{\sqrt{1-\lambda}}\right)\right]_{x_0}^{x} + \theta_s\, e^{-i\varphi_s} \right| \qquad (40)$$

Defining θ_s, φ_s such that all constant terms in Eq. 40 dissapear:

$$\theta_s = \tau\,|\omega_0^*|\sqrt{\frac{1-\lambda}{\lambda}}\; \left|F_2\left(x_0\sqrt{\frac{\lambda}{1-\lambda}}\right)\right|$$

$$\theta(x) = \tau\,|\omega_0^*|\sqrt{\frac{1-\lambda}{\lambda}}\; \left|F_2\left(x\sqrt{\frac{\lambda}{1-\lambda}}\right)\right| \qquad (41)$$

and $\theta_F = \lim\limits_{x\to\infty} \theta(x) = |\omega_0^*|\,\tau_1$ (42)

with $\tau_1 = \sqrt{\frac{\pi}{2}}\sqrt{\frac{A}{m_3}}$ as the relevant time constant. (43)

Notice that this initial orientation of the inertial frame is not on the angular momentum. The limiting value of the attitude change is after some calculations found to be:

$$|\theta_F + \theta_i| \qquad (44)$$

3.2 Spin-up from a Uniform Rotation with a General Thruster

For this case ω^* is given by:

$$\omega^* = \frac{m^* \tau}{A} e^{-i\frac{\pi}{2}x^2}\left[F_2(x) - F_2(x_0)\right] \quad (45)$$

and the remaining integral in Eq. 38 denoted $J(x)$ becomes:

$$J(x) = \left\{\int_{x_0}^{x} e^{i\frac{\pi}{2}\frac{x^2\lambda}{1-\lambda}} F_2(x)\,dx - \sqrt{\frac{1-\lambda}{\lambda}}\, F_2(x_0)\left[F_2(y)\right]_{x_0\sqrt{\frac{\lambda}{1-\lambda}}}^{x\sqrt{\frac{\lambda}{1-\lambda}}}\right\} \times \frac{m^*\tau^2}{A} e^{-i\frac{\pi}{2}\frac{x_0^2}{1-\lambda}\lambda} \quad (46)$$

When $\lambda = 0.5$ (top of Kovaleski) Eq. 46 reduces to:

$$J(x) = \frac{m^*\tau^2}{2A} e^{-i\pi x_0^2}\left[F_2(x) - F_2(x_0)\right]^2$$

and $$\theta(x) = \left| \frac{|m^*|\,\tau^2}{2A}\left[F_2(x) - F_2(x_0)\right]^2 + \theta_s e^{-i\varphi_s}\right| \quad (47)$$

No closed form of Eq. 46 for other values of λ was found. The equations 43, 46 constitute a good starting point for further study of the attitude changes taking place during spin manoeuvres.

4. NUMERICAL APPLICATION TO METEOSAT-2

The formulae derived in point 2 have been used to calculate the nutation during the spin-up from 10 to 100 RPM of Meteosat-2 which will be launched in June 81. Meteosat-2 has no pure spin thrusters. Spin-up manoeuvres can be executed with either of the 2 thrusters called R_1 and V_2. Both of these thrusters generate a large equatorial torque (Table III) and the inertia figures of the Meteosat-2 are: $\lambda = .39$ $B = 211\ kgm^2$

	$\lvert m^* \rvert$ (Nm)	m_3 (Nm)	$tg\,\theta_0$	θ_0	τ (sec)
R_1	19.59	1.92	10.203	84.40°	$\tau_R =$ 23.79
V_2	2.67	0.744	3.588	74.43°	$\tau_V =$ 38.22

TABLE III - Thruster data

ω_{30} (RPM)	x_0	Man.duration	Max.nut.	End Nut.
10	4.837	1035"=17'16"	14.91°	0.8°
5	2.419	1092"=18'13"	43.47°	1.58°
0	0	1151"=19'11"	84.40°	8.54°

R_1 $(x_f = 48.87)$

ω_{30} (RPM)	x_0	Man.duration	Max.nut.	End Nut.
10	7.770	2673"=44'33"	2.13°	0.10°
5	3.885	2821"=47' 1"	8.08°	0.22°
0	0	2970"=49'30"	74.43°	1.87°

V_2 $(x_f = 77.70)$

TABLE IV - Spin-up to 100 RPM with R_1, V_2

Table IV shows that for initial spin-rates between 5 to 10 RPM the final level of the nutation is quite acceptable. The major difference is the maximum nutation level which occurs at the beginning of the manoeuvre. Ref. 6 contains more numerical results and a comparison of these results against integration on an analogue computer. The agreement is good. A major difficulty during the simulation is the large range of the variables involved, a scaling of the variables which covers the range 0 to 100 RPM allows not a sufficient precision on the output variables. Moreover the simulations are time consuming, they had to be done in the real time option to avoid the influence of internal filtering which showed up in the fast option. If the same problem is treated with a digital computer a variable step integration method must be chosen to avoid excessive run times.

Finally one may not forget the two most important assumptions under which these results are derived:

1) Internal energy dissipation is neglected. As Meteosat-2 is unstable the dissipation of energy tends to increase the nutation. At a certain spin-rate and nutation level, this effect can cancel the spin-up component of a given thruster. If this happens one has first to reduce the nutation by pulsed thrusting before the same thruster can spin the spacecraft further on. With the dissipation data of Meteosat-1 this problem does not arise with either of the 2 thrusters (R_1, V_2) considered.

2) The spacecraft is symmetric. Although Meteosat is an almost symmetric spacecraft there is a fundamental difference in the behaviour of symmetric and asymmetric rigid bodies. For symmetric bodies $m_3 > 0$ is sufficient to establish any desired value of the spin-rate, independent of $|\omega_0^*|, m^*$. For asymmetric bodies, the so-called separation plane defines two regions where the average spin-rate is or about the nominal axis or about the transverse principal axis. When the initial rotation ω_0^* is about the transverse axis $(\theta_i = 90°)$ it is not trivial if a thruster with m^* such that $|\omega^*|$ decreases and $m_3 > 0$ will succeed in establishing a spin-rate around the desired spin-axis. This problem of flat-spin recovery (Refs. 7, 8) does not exist for a symmetric rigid body and for a nearly symmetric rigid body it is always present for high-enough initial transverse rates. It was checked that irrespective of the sign of the flat spin and initial transverse rates of 4 RPM (corresponding to 10 RPM about the nominal axis) and a nominal orientation of the inertia ellipsoid the recovery took always place just by firing the R_1 or V_2 thruster. The formulae presented in this paper can be used for a nearly-symmetric body as long as the initial spin rate is about the nominal spin-axis (which excludes Case V of Table I).

REFERENCES

1. Leimanis E 1965, *The general problem of the motion of coupled rigid bodies about a fixed point*, Springer-Verlag.

2. Magnus 1971, *Kreisel-Theorie und anwendungen*, Springer-Verlag.

3. Luré L 1968, *Mécanique analytique*, Masson et Cie.

4. Bellman R 1960, *Introduction to matrix analysis*, McGraw-Hill.

5. Abramowitz, *Handbook of mathematical functions*, Dover.

6. EWP 1247 (ESTEC) 1980, Nuation of a symmetric rigid body under continuous thrusting. Application to Meteosat 2, by F. Janssens.

7. EWP 807 (ESTEC) 1974, Flat spin dynamics of Meteosat, by M. Kluiters.

8. Kluiters M 1976, Flat spin recovery of a spinning satellite, *AIAA Guidance and control conference, San Diego 1976.*

	ω_0^*	ω_{30}	m^*		
I	0	V	0	. ideal spin-up from ω_{30} onwards	$\theta_n(t) = 0$
II	$\neq 0$	$\neq 0$	0	. spin-up with a tangential thruster in the presence of initial nutation θ_i	$tg\,\theta_n(x) = tg\,\theta_i\,\frac{x_i}{x}$
III	0	0	$\neq 0$	. spin-up from rest with a general thruster	$tg\,\theta_n(x) = tg\,\theta_0\,\frac{\lvert F_2(x)\rvert}{x}$ $\quad tg\,\theta_0 = \frac{\lvert m^{*}\rvert}{m_3}$
IV	0	$\neq 0$	$\neq 0$	. spin-up from ω_{30} onwards with (no initial nutation	$tg\,\theta_n(x) = tg\,\theta_0\,\frac{\lvert F_2(x) - F_2(x_0)\rvert}{x}$
V	$\neq 0$	0	$\neq 0$	. spin-up from a flat-spin condition	$tg\,\theta_n(x) = \frac{1}{x}\left\lvert \frac{A\omega_0^*}{m_1\tau} + \frac{m^*}{m_3}\,F_2(x)\right\rvert$
VI	$\neq 0$	$\neq 0$	$\neq 0$	. general case	$tg\,\theta_n(x) = \frac{1}{x}\left\lvert \frac{A\omega_0^*}{m_1\tau}\,e^{i\frac{\pi}{2}x_0} + \frac{m^*}{m_3}\left(F_2(x) - F_2(x_0)\right)\right\rvert$

TABLE 1

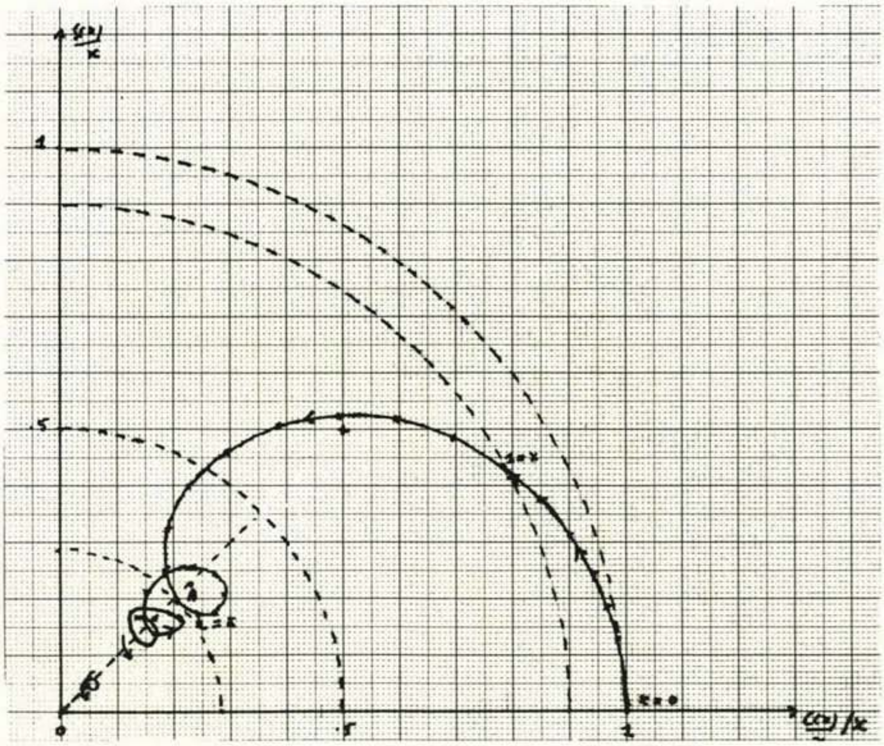

FIGURE 2 - Plot of $\frac{F_2(x)}{x} = \frac{C(x)}{x} + i\,\frac{S(x)}{x}$