ORBIT AND ATTITUDE DETERMINATION FOR BHASKARA

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ABSTRACT

Bhaskara, an experimental remote sensing satellite, was launched into a 530 km, near circular orbit at an inclination of 50°.7.

An orbit determination program based on R.H. Merson's analytical theory of satellite motion was developed. The post facto nominal position accuracy achieved during the initial period of the mission (using interferrometry data from CNES) was of the order of 500 meters (1σ). Attitude computation was carried out in a near real time mode for attitude control and in a post facto mode for data analysis using the Extended Kalman Filter algorithm. The inorbit accuracy achieved is 2° in the near real time mode and 0°.5 in the post facto mode.

An empirical procedure, which uses Times of Closest Approach (TCA) information available from a one-way Doppler system was developed for updating orbital parameters during the contingency mode of operations. A program employing the method of direct search using simplexes was also evolved for attitude determination for contingency mode of single sensor operation.

Keywords: Orbit, Attitude, Differential Correction, Extended Kalman Filter, Times of Closest Approach, Direct Search, Tracking, Sensors.

1. INTRODUCTION

Bhaskara was launched into a 530 km, near-circular orbit at an inclination of 50°.7, by a Soviet launcher on 7 June 1979, at 10.30 Hrs GMT. The spacecraft, with a spin rate of between 5-8 rpm carried two major payloads. i) two TV cameras for acquiring earth imageries over India and ii) three Satellite Microwave Radiometers (SAMIR) to conduct ocean surface studies. In addition, there are a few secondary payloads for scientific and technological studies. The TV imagery experiment requires the spin axis to be maintained along the orbit normal direction (mode A phase) whereas the SAMIR experiment requires an orbit tangent direction for the spin axis over India (mode B phase). The attitude control is effected in manual or automatic mode through horizon sensor inputs or magnetic bias coils(MBC), respectively.

Orbit and attitude information is required for attitude control, payload data analysis, spacecraft health analysis and for generating tracking schedules. The TV experiment requires the satellite position to be determined within 4 km whereas the other payloads have relaxed requirements. The primary payloads require the attitude to be determined within 1° for data analysis and within 3° for attitude control,

A Satellite Orbit Improvement Program (SOIP), developed based on R.H. Merson's analytical theory, is applicable foralmost all near-earth satellites and is capable of handling a number of types of ground-based satellite observations. The definitive orbit of Bhaskara was computed with SOIP using interferrometry data from CNES(France) during the initial phase of operations. Tone range data from the Indian network was used during the later period. The post facto nominal position accuracy achieved during the early period of the mission was of the order of 500 meters (1σ). Thus the payload requirements were comfortably met during this period.

An empirical procedure to update orbital parameters using the Times of Closest Approach (TCA) information available from a one-way Doppler was used in the contingency mode of operation.

Attitude determination was carried out by two independent programs, NRTATD and PFTARD. The NRTATD computes attitude in a near real time mode for generating commands for attitude control. The PFTATD operates in a post facto mode and is for high accuracy attitude determination for payload data analysis. Both these programs preprocess raw telemetry data, compute an initial attitude estimate and then refine it with the Extended Kalman Filter (EKF) algorithm. To reduce the effect of horizon sensor electronic delays and the earth's CO_2 radiance variation, only the earth pulse width output of the sensor is used for nadir angle computation.

A third program (NRTLIM) employing a method of direct search using simplexes was evolved for contingency mode of single sensor operation.

2. SATELLITE ORBIT IMRPOVEMENT PROGRAM (SOIP)

The orbit model of SOIP uses five basic quantities: e - eccentricity, i - inclination, ω - argument of perigee, Ω - right ascension of ascending node and M - mean anomaly. These are expressed in a polynomial form to approximate the satellite motion as closely as possible. The orbit model considered is given by (Refs. 1,3,5):

$$\begin{aligned} e &= e_o + e_1 (t - t_o) \\ i &= i_o + i_1 (t - t_o) \\ \omega &= \omega_o + \omega_1 (t - t_o) \\ \Omega &= \Omega_o + \Omega_1 (t - t_o) \\ M &= M_o + M_1 (t - t_o) + M_2 (t - t_o)^2 \end{aligned} \quad (1)$$

where e_o, i_o, ω_o, Ω_o and M_o are the mean values of orbital parameters at any epoch, t_o. M_1 is the mean motion and M_2 is an empirical drag parameter. e, i, ω, Ω and M are the base elements at any time t.

All the coefficients of the polynomials are treated as orbital parameters. The semi-major axis, a, can be derived from M_1 through Kepler's third law. While determining the orbit, some of the orbital parameters are evaluated through theoretical formulae and some are subject to change during the refinement process. In SOIP e_1, i_1, ω_1 and Ω_1 are evaluated through theoretical formulae and e_o, i_o, ω_o, Ω_o, and all the polynomial coefficients of M are refined (Ref. 1).

In the orbit generator of SOIP, only the effects of asphericity of the earth and atmospheric drag are considered. The orbit generator is non-singular at $e \approx o$, $i \approx o$ and at critical inclinations. Compact recurrence relations have been used for zonal harmonics so that these can be considered upto any order. Among the tesseral harmonics only the dominant part of J_{22} effect has been considered. The atmospheric drag effect is included implicitly by introducing the empirical drag parameters (which are floated) in the polynomial coefficients of the mean anomaly (Refs. 1,3,5).

The details of the computation of the satellite co-ordinates and the theoretical observations are given in references 1,5 and 6.

At present SOIP can handle the following types of observations and their various combinations: Range, range rate, direction cosines, right ascension and declination, azimuth and elevation.

SOIP updates the orbital parameters using a differential correction procedure. This method is based upon a comparison of simulated and observed tracking data. The difference between each computed and observed measurement constitutes a measurement residual, and the orbital parameters are refined in an iterative fashion until the weighted sum of the squares of the residuals is minimised.

The singularity due to $e \approx o$ in the differential correction process is avoided by introducing, temporarily, defect-free parameters $e \cos\omega$, $e \sin\omega$, and $M + \omega$, instead of e, ω and M (Refs. 1,5).

On convergence of the differential correction process, a variance-covariance analysis is performed and from this, the standard deviations of the orbital parameters and of the spacecraft position and velocity vectors are computed (Ref. 1).

3. ORBIT COMPUTATIONS OF BHASKARA

The Indian tracking network for Bhaskara, the first of its kind commissioned by ISRO, consists of a VHF tone range system at SHAR and Ahmedabad with a system accuracy of 150 meters. As a back up, a one-way Doppler system of a lesser system accuracy is also being utilised at SHAR to track the spacecraft. During the early phase of operations, for about 45 days after the launch, CNES (France) provided the interferrometry data of Bhaskara from its network of two stations at Kourou (French Guiana) and Pretoria (South Africa). The co-ordinates of all the tracking stations are presented in Table 1.

Table 1.

Ground Station	Geodetic latitude (deg)	East longitude(deg.)	Altitude wrt mean geoid (km)
SHAR	13.173	80.194	-0.008
Ahmedabad	23.010	72.620	0.175
Pretoria	-25.554	28.372	1.191
Kourou	5.251	307.194	0.018

The accuracy to which the orbit can be computed depends to a large extent on the quality and the quantity of the observations. The raw data is a series of experimental observations flawed by noise and systematic errors. Therefore these data have to be smoothed and corrected for environmental effects before feeding to SOIP. For this purpose a separate program was developed, to preprocess the tone range data of Bhaskara provided by the Indian network. The interferrometry data from CNES were already smoothed and system accuracy is of the order of 1' of an arc. As these data were of better quality than those obtained from the Indian network, orbit determinations in the initial period were carried out with these data only. The total number of observations received from CNES was about 600 pairs of direction cosines spread over 45 days with normally 5 pairs of direction cosines per pass and with a daily average of 3 passes (Ref. 2). Data from both north-bound and south-bound passes were present.

The observations were analysed in six-day batches with epochs six to seven days apart. Orbit determination was carried out eight times during the above period. The number of observations per orbit determination was 70-80 pairs of direction cosines. On an average about five observations for each orbit determination process were rejected. Though the observational accuracy was taken to be 1' of an arc in each direction cosine, the posteriori accuracy was found to be little worse than this figure. This is due to the uncorrected ionospheric refraction error.

The refined orbit parameters obtained at eight epochs including their standard deviations are listed in Table 2. The overall standard deviation, σ, obtained for the eight epochs varied between 1 and 2.5 with an average of 1.65. That is to say that the posteriori accuracy of the direction cosines is around 1.7' of an arc. The average values of the standard deviations of the orbital parameters for the eight epochs are as follows:

$$\sigma_a = 0.000012 \text{ km}$$
$$\sigma_e = 0.000012$$
$$\sigma_i = 0.0018 \text{ deg.}$$
$$\sigma_\Omega = 0.0011 \text{ deg.}$$
$$\sigma_\omega = 0.42 \text{ deg.}$$
$$\sigma_{M_0+\omega} = 0.0013 \text{ deg.}$$
$$\sigma_{M_1} = 0.0009 \text{ deg/day}$$
$$\sigma_{M_2} = 0.0004 \text{ deg/day}^2$$

A final quantity of interest is the rms position error which is found to be of the order of 500 meters (1σ). The overall positional accuracy required by the experiments was of the order of 4 km which was met comfortably during this period.

Table 2.*

Date	a (kms)	e	i (deg.)	Ω (deg.)	ω (deg.)	$M_0+\omega$ (deg.)	M_1 (deg/day)	M_2 (deg/day^2)
79 JUNE 09	6907.758000	0.002326	50.6671	339.4650	67.620	319.419	5443.3876	0.01620
	10	8	12	11	560	3	7	60
79 JUNE 14	6907.624100	0.002413	50.6671	315.5825	78.570	195.713	5443.5458	0.01450
	10	7	30	8	460	1	7	20
79 JUNE 20	6907.513400	0.002350	50.6723	286.9211	92.120	120.245	5443.6771	0.01405
	20	10	15	10	510	2	14	31
79 JUNE 26	6907.388100	0.002294	50.6691	258.2713	106.443	45.571	5443.6248	0.00709
	7	8	13	10	348	1	4	19
79 JULY 03	6907.321500	0.002140	50.6673	243.3787	119.824	283.845	5443.9034	0.01360
	10	10	17	10	310	1	8	30
79 JULY 07	6907.224000	0.002094	50.6656	215.2702	133.396	114.870	5444.0179	0.01020
	20	14	18	9	419	1	16	60
79 JULY 12	6907.079200	0.001781	50.6621	181.8248	153.066	90.135	5444.1893	0.01268
	9	20	23	9	220	1	6	21
79 JULY 20	6906.954600	0.001350	50.6692	143.5990	178.972	114.644	5444.3370	0.01035
	10	21	28	17	428	1	10	24

* lower line entries give the values of standard deviations.

Figure 1 gives the behaviour of the semi-major axis during the first year of Bhaskara. It can be seen that the semi-major axis has decreased by about 15 km in one year. This was a little higher than the expected decay

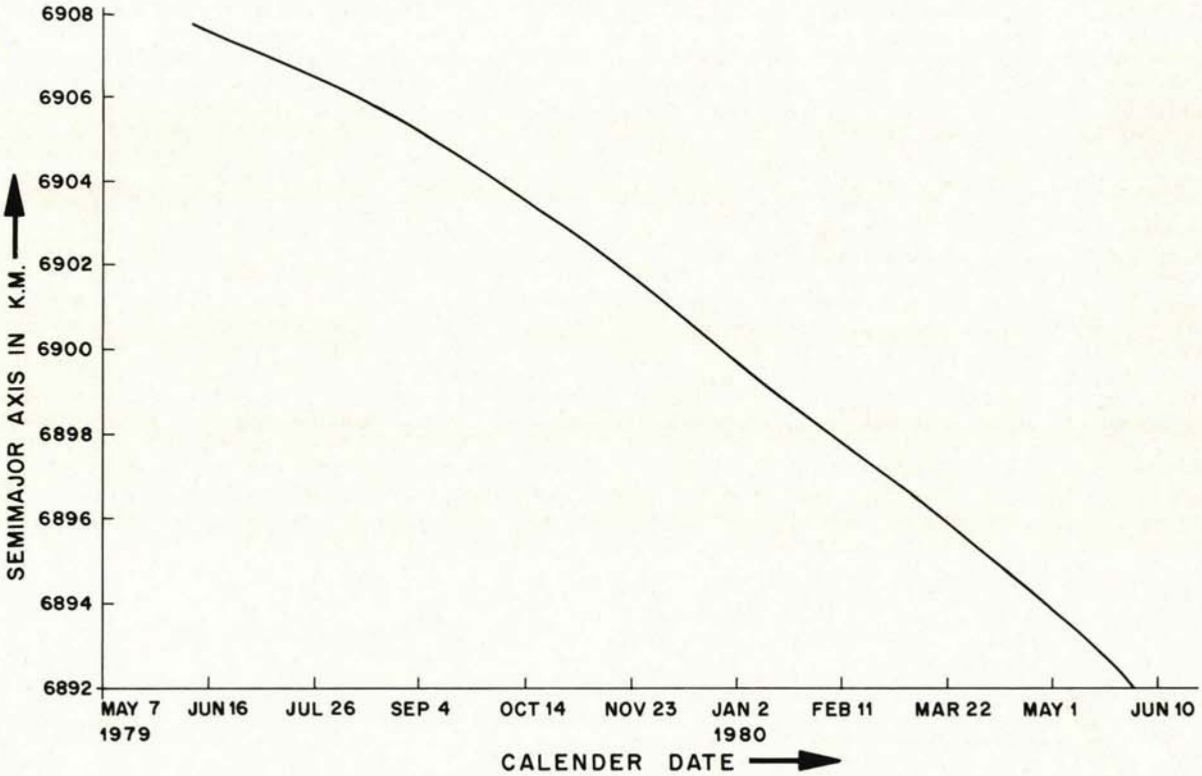

Figure 1. Orbit Decay of Bhaskara

of 12 km and can be attributed to high solar activity during this period. The inclination remained almost constant with small periodic variations. The orbit regression rate was 4°.7/day during the early period and increased to 4°.9/day at the end of the first year.

4. CONTINGENCY MEASURES TO UPDATE THE ORBITAL PARAMETERS OF BHASKARA

During the normal phase of operations, that is 45 days after launch, tracking data from only the Indian network were available for orbit determination. Since this Indian tracking system was being put to use for the first time there were several operational problems. In order to be able to provide atleast approximate orbital information during periods when useful tracking data were not available, an empirical procedure was developed. This procedure was used to correct the coefficients of the polynomial of the mean anomaly M, so as to reduce the along-track error within reasonable limits.

The major perturbations, on the Bhaskara orbit, are due to the asphericity of the earth and atmospheric drag. In the orbit generator the former is modelled adequately. In the absence of orbit determination, the extrapolation of an empirical drag parameter (M_2) introduces a large amount of inaccuracy. Bhaskara, because of its near-circular orbit, experiences uniform drag throughout the orbit and therefore variation in eccentricity due to drag is negligible. The parameters most affected by drag are the polynomial coefficients of the mean anomaly M, which is represented by

$$M = M_o + M_1 t + M_2 t^2 \qquad (2)$$

Any error in the coefficients of the polynomial would cause along-track error which builds up fast with time. Therefore M_o, M_1 and M_2 should be corrected to reduce the along-track error.

The data provided by the one-way Doppler system is not sufficiently accurate to carry out reasonably good orbit determination. But the TCA information provided by this system has an accuracy of better than 1 sec. This information is utilised to update the above parameters using a finite difference scheme.

5. ATTITUDE SENSORS

Attitude determination, control and payload operations at different phases of the mission are carried out by a set of three types of attitude sensors:

i) one fluxgate triaxial magnetometer
ii) one digital sun sensor and one twin slit sun sensor
iii) two pencil beam infrared horizon crossing indicators (HCI).

In the initial phase, when the arbitrarily oriented spin axis is brought to the orbit normal, the magnetometer and sun sensors are used. In the operation phase the horizon sensors are the prime sensors. The fluxgate magnetometer has a measurement range of ± 45,000 gamma of field intensity, with a digitisation step of 720 gamma. Major errors in the magnetometer were observed to be from temperature variation, subsystem field changes and digitisation. The magnetometer was therefore calibrated against temperature and ground-measured field values of the subsystems were subtracted from readings dependings on their ON/OFF status. The final angular measurement accuracy of the sensor was estimated to be better than 1°.5 (3 σ) including an uncertainty of 0°.5 in the earth's field computation.

The digital sun sensor is an assembly of three identical sensors to provide 0 to 180° coverage of the sun aspect angle with a step size of 0°.5. The twin slit sun sensor consists of a vertical slit and a curved slit which derive two pulses as the sun crosses their meridians. The time interval between these pulses is a linear function of the sun aspect angle. The instant of the sun crossing the vertical slit is also telemetered to provide the azimuth angle of the sun in the satellite body frame. This information, called the sun transit time, should correspond to the centre of the sun disc but the pulse is derived at the leading edge of the 0°.5 diameter sun disc. A correction factor Δt, given by (Ref. 8).

$$\Delta t = \frac{\Delta\theta \times \text{Spin period}}{2} \qquad (3)$$

$$\text{where } \Delta\theta = \frac{0.9999904 - \cos^2 \beta_s}{\sin^2 \beta_s} \qquad (4)$$

and β_s = sun aspect angle,

is applied to the sun transit time.

The two pencil beam horizon sensors are identical in design and work on the 14-16 micron IR radiation of the CO_2 layer of the earth. They are mounted in the same meridian plane of the spacecraft. A sensitivity analysis of measurement errors against mounting angles (Ref. 9) suggested a cant angle of 45° from the satellite equatorial plane. The time interval between the leading and trailing pulses (space-to-earth entry pulse and earth-to-space exist pulse) is measured with a clock of frequency 896 Hz and telemetered in 12 bits for each sensor. A local vertical pulse (LVP), corresponding to the instant the optical axis of the TV camera looking along the nadir line, is also derived from one of the horizon sensors. The earth pulse width is used to compute the nadir angle and the LVP time to get the azimuth angle of earth centre in the spacecraft body frame.

6. NADIR ANGLE COMPUTATION

Figure 2 gives the earth pulse width angle against roll angle for both the horizon sensors canted at ± 45° to the spacecraft equator, for a 525 km circular orbit. Here the roll angle is defined as the nadir angle minus 90°. Thus the nadir angle can be uniquely determined from the earth pulse width output of a single sensor or that of both the sensors. Due to imperfections in sensor optics and delays caused by electronic processing, the leading and trailing pulses from the horizon sensors do not coincide with the edge of the earth disc. The effect of these delays on attitude angle is considerably reduced by computing the nadir angle from the earth pulse width alone. Another source of error for the horizon sensors is the seasonal and positional variations in the IR radiation of the CO_2 layer of the earth. These errors could not be modelled in the mission time frame. But the total error contribution in the earth pulse width output is estimated to be below 0°.2.

The measurement geometry of the horizon sensor is given in Figure 3. The nadir angle β_E is computed as (Refs. 10,11)

$$\beta_E = \frac{\pi}{2} + \phi \qquad (5)$$

$$\text{where} \quad \phi = \alpha_0 - \mu_0 \qquad (6)$$

$$\alpha_0 = \arctan(\tan\lambda \cos\beta) \qquad (7)$$

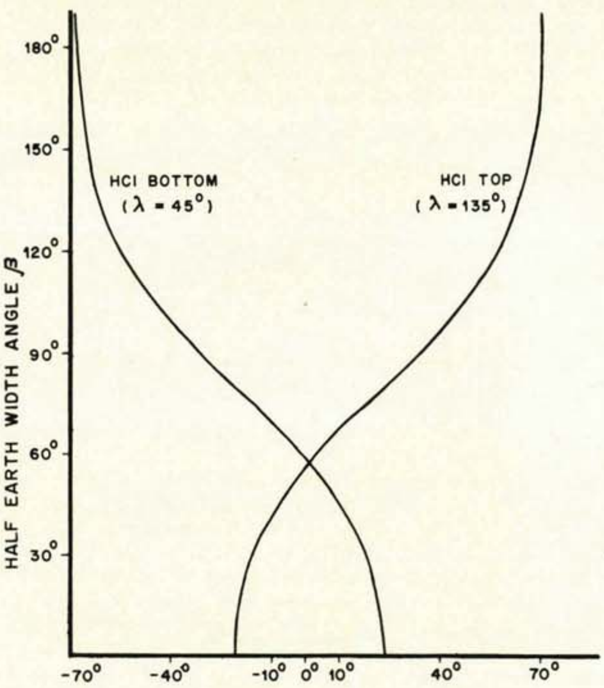

Figure 2. Horizon Sensor Pulse Width against Roll Angle.

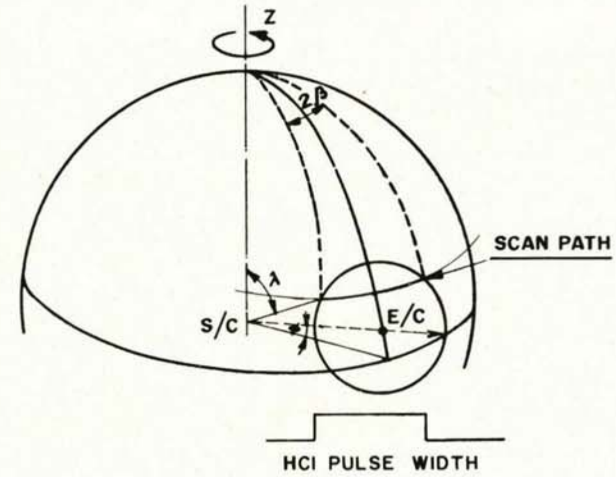

Figure 3. Horizon Sensor Measurement Geometry

$$\mu_0 = \text{arc sin} \left\{ \frac{\cos^2 L}{1 - \sin^2 \lambda \sin^2 \beta} \right\}^{1/2} \qquad (8)$$

$$L = \text{arc sin} \left\{ \frac{Re + Hc}{Re + H} \right\}. \qquad (9)$$

When λ, the angle between the positive spin axis and the sensor axis, is greater than 90°, one takes the negative sign of the square root in Eq.8. In the relation for the earth's subtended angle L (Eq. 9) H is the orbit altitude and Hc is the effective altitude of the CO_2 layer. With two horizon sensors, the roll angle is computed independent of L from

$$\tan \phi = \frac{\sin \lambda_1 \cos \beta_1 - \sin \lambda_2 \cos \beta_2}{\cos \lambda_1 - \cos \lambda_2} \qquad (10)$$

From the orbit performance, the effective altitude of the CO_2 layer was estimated to be 42 km.

7. ATTITUDE DETERMINATION

The attitude of the spacecraft is represented by right ascension Ra and declination Dl of the spin axis in the equatorial inertial frame, and the spin angle γ of the TV optical axis. The attitude determination programs, both NRTATD and PFTATD, employ the Extended Kalman Filter algorithm for attitude refinement. An initial estimate for this recursive algorithm is derived from one measurement each of two sensors using the cone intersection method (Ref. 12). The ambiguity in the solutions is resolved by using the azimuth angle measurements. Approximate error in the initial estimate is computed as

$$\sigma_{Ra} = \sigma_{Dl} = \frac{\sigma}{\sqrt{2}}$$

$$\text{where } \sigma^2 = \frac{\sigma_{\beta_1}^2 + \sigma_{\beta_2}^2}{\sin^2 \beta_E}, \qquad (11)$$

where, again, σ_{β_1}, σ_{β_2} are the errors in the two cone angles and β_E is the angle between the axes of the two cones. Before actual attitude computation is done, the telemetered sensor data, stored on magnetic tape, is preprocessed and mixed with orbit data. The output of this preprocessing is a vector of 9 parameters for each selected sensor: inertial reference unit vector, sensor measured azimuth and elevation angles of the same in the body frame, sensor identification flag, time and measurement errors in azimuth and elevation angles.

The refinement of the attitude state vector

$$X = (Ra, Dl, \gamma)^T$$

and its covariance matrix S is done recursively using only one measurement at a time. The recursive formulae for X and S are (Ref. 13)

$$S_{k,k} = (I - H_k M) S_{k,k-1} \qquad (12)$$

$$X_{k,k} = X_{k,k-1} + H_k (Y_k - G(X_{k,k-1})) \qquad (13)$$

where Y_k is the kth observation, G is the predicted observation, M is the partials of the measurement equations and H is the weighting matrix derived from the observation noise and error in predicted measurements. The filtering is terminated when the sum of the variances of the estimates reach below a set threshold value.

In the post facto attitude determination program, PFTATD, which is for payload data analysis, high accuracy in the estimate is required. As a few of the simulation studies revealed, unmodelled bias errors in the sun sensor and the magnetometer can cause divergence of result, especially in near-collinearity cases. Therefore, only the horizon sensor data is used for attitude refinement. About 6 minutes of sensor data (25 degrees of nadir line rotation) is required to achieve 0°.5 (3σ) accuracy in the final estimate. But in NRTATD, which is used for attitude control, a reliable attitude is to be computed even if only limited data is available. Hence all available sensor measurements are used in this program. The convergence limit set in this program is 3° if the horizon sensor is not available and 1°.5 if the horizon sensor is available.

8. INORBIT PERFORMANCE OF ATTITUDE DETERMINATION

Bhaskara was spun upto 4.5 rpm in orbits 11 and 12 and the first attitude determination was carried out in orbit 15 by NRTATD using magnetometer and sun sensor data. The first control command generated from this attitude was executed in orbit 16. Attitude computation and command execution were carried out in all passes over SHAR until the spin axis was brought within 0°.5 of orbit normal in orbit 39. Figure 4 depicts the motion of the spin axis during this period. The horizon sensors acquired earth from orbit 26 onward, but they were put into attitude determination only from orbit 71 after computing the mean effective tangent height of the CO_2 layer.

PFTATD was run from orbit 76 onwards. The results from some early orbits, for both NRTATD and PFTATD, are presented in Table 3. Between orbits 76 and 99 no attitude control was performed and the spacecraft was in the free drift mode. From orbit 100 onwards the spin axis was under constant control. The results from orbit 262 onwards show the westward drift of the spin axis, along with the orbit normal. The continuous decrease in declination is due to the MBC which was continuously on during this period.

Due to the continuous drift of the spin axis, a stability analysis of the attitude results for a fixed attitude could not be done except for orbits 76,84 and 91, where the results match within 0°.5. The accuracy analysis through the Kalman Filter covariance matrix show an accuracy of 0°.5 (3σ) in spin axis direction estimate by the PFTATD program. In the case of NRTATD results, where most of the computations are done with a limited amount of data from all sensors, the accuracy is estimated to be better than 2°.0 (3σ) when the horizon sensors are also used. With only the magnetometer and sun sensor the accuracy of near-real time attitude estimation is 2°.5 (3σ).

Table 3.

Orbit No.	Near Real Time Attitude		Post facto Attitude	
	Ra	Dl	Ra	Dl
76	236.38	36.53	235.54	35.10
84	234.07	35.20	235.11	35.19
91	236.79	35.49	235.27	34.84
262	172.16	40.57	172.22	40.34
270	170.54	40.66	170.04	39.98
279	166.18	39.79	166.36	39.50
300	160.27	41.52	158.69	39.09
314	153.62	38.45	153.68	38.56
324	150.18	37.71	150.08	37.87
329	148.09	37.88	147.98	37.77
339	144.34	37.51	144.50	37.55

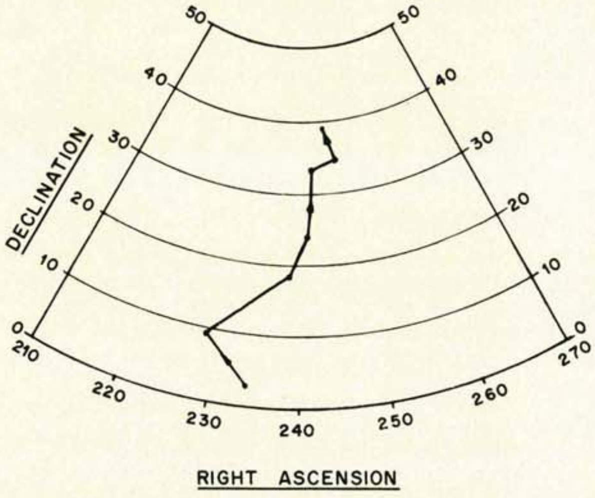

Figure 4. Attitude Acquisition

9. CONTINGENCY MODE OF ATTITUDE DETERMINATION

The more critical of the two attitude program is NRTATD, which has a built-in provision to compute attitude from any two sensors even in the absence of azimuth measurements. A contingency of a single sensor mode also was thought of, but NRTATD cannot compute an initial estimate in this case. So a separate program (NRTLIM), has been developed to compute attitude to an accuracy within 3 degrees using data from either magnetometer or horizon sensor.

This program generates a measurement equation

$$\bar{P} \cdot \bar{W} = \cos\Theta \qquad (14)$$

for each available sensor measurement, where $\bar{P}$ is a unit inertial reference vector, $\bar{W}$ is unit spin axis vector and Θ is the corresponding aspect angle measurement. A redundant set of linear equations thus generated out of all available sensor measurements (single or multiple sensors) is solved in a least square way with the constraint

$$\|\bar{W}\| = 1 \qquad (15)$$

on the solution vector. The method employed for solving the set of equations is a direct search using simplexes (Ref. 14). Here a search for this solution is carried out on the surface of a unit sphere starting from an arbitrarily chosen point. The starting point selected is (1/3, 1/3, 1/3). This program could give accurate results using 2 minutes of continuous magnetometer data or 5 minutes of horizon sensor data. The program gave sufficiently accurate results even with less data whereas a simple least square method, with the constraint (Eq. 15) removed, failed to give any satisfactory result. In this program no azimuth measurement is used.

10. CONCLUSION

SOIP, which was specially developed for Bhaskara, is successfully being utilised for the purpose of orbit determination. During the early phase of the mission the satellite position was determined to an accuracy of 500 meters (1σ) using CNES interferrometry data. In the contingency modes when orbit determination could not be carried out the empirical procedure developed to limit the along-track error served as a useful tool. Indeed it was possible to limit the along-track error within 10 km utilising this procedure.

The orbit decay rate was higher than the expected value, and the orbit decayed by 15 km during the first year.

During the early period the mission requirements were comfortably met and reasonably well during the later period.

The inorbit accuracies achieved in attitude computation are 2° (3σ) in the near-real time mode and 0°.5 (3σ) in the post facto mode. The earth's IR radiance calibration of the horizon sensors was not used to achieve the above accuracies. The method of direct search with a unit norm constraint on attitude vector is more stable than an ordinary least square in an environment of limited data. The performance of the programs was satisfactory and the mission requirements were comfortably met.

11. ACKNOWLEDGEMENT

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