THE DYNAMIC ATTITUDE RECONSTITUTION METHOD

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ABSTRACT

The dynamic attitude reconstitution method makes use of the information contained in the temporal evolution of the Earth colatitude and the Sun-Earth azimuth angles. These angular measurements are readily obtained for spinning satellites equipped with pencil-beam Earth sensors and V-slit Sun sensors in geostationary transfer orbits. Since the derivatives of the Earth colatitude and azimuth angles define a third reference direction, the dynamic method provides reliable results in the case of Sun-Earth (near-) colinearity where conventional methods break down. Furthermore, its proper functioning is not impeded by the loss of Sun colatitude measurements as long as a time pulse for the azimuth angle is available. The present paper summarises the theoretical background of the method and describes the various improvements which have resulted in the operational software now in use to support ESA satellites in geostationary transfer orbit. Extensive software experimentation on the basis of real satellite data (ESA's METEOSAT I) have resulted in a reliable attitude estimation method and increased our confidence in the successful application of the method in future launch support.

Keywords : Attitude Estimation, Sun-Earth Colinearity

1. INTRODUCTION

The geostationary satellites so far launched by ESA have been spin stabilised in the transfer orbit. In fact, spin stabilisation is sometimes also used for non-geostationary spacecraft (e.g. ESA's GIOTTO).

The attitude in spin mode is normally obtained by analysing infrared (IR) Earth and V-slit Sun sensor data which provide us with the angles between the satellite's spin axis and the inertial directions defined by the Sun's and Earth's positions, i.e. the Sun and Earth colatitudes, θ and β. An initial attitude fix may be derived deterministically from a few data samples whereas a few hundred samples are needed in order to reduce the effects of sensor noise to an insignificant level with the aid of stochastic methods. The accuracy of the attitude estimate so obtained depends on unobservable biases in sensor measurements and alignments. It is well-known (e.g. Ref. 1, Ch. 11) that the sensitivity of the attitude estimate to errors becomes unacceptably high when the angle between the Earth and Sun reference directions becomes small (Sun-Earth colinearity). Therefore, a launch window constraint is usually imposed when colinearity may arise, especially around the equinoxes.

In many cases, a launch window constraint of this type is undesirable. Therefore, the dynamic attitude reconstitution method, which is not subjected to colinearity constraints, has been proposed in connection with the launch of ESA's Orbital Test Satellite (OTS), cf. Peyrot (Ref. 2). In addition to eliminating the launch window constraint, the dynamic method has other important advantages which fully justify its inclusion in the operational software. It will produce acceptable results using IR Earth sensor data alone which is extremely important in the case of a (partial) Sun sensor failure.

2. GEOMETRICAL BACKGROUND OF MEASUREMENT EQUATIONS

2.1 Nature of the Measurements

Attitude measurements for a nutation-free spinning satellite are performed by Sun and Earth sensors measuring angular data. As the use of plane IR sensors is restricted to high eccentric orbits (e.g. HEOS A1 and A2, COS B) only the more common pencil-beam IR Earth Sensor is taken into consideration here.

The IR sensor is mounted at an angle of inclination μ with respect to the spacecraft's spin axis. Due to its rotation along with the spacecraft the sensor generates during each satellite revolution one positive and one negative pulse at the Space/Earth (S/E) and Earth/Space (E/S) transitions, respectively. After calibration and validation of the measurements (cf. pagoda effect, Ref. 1, Ch. 9 and Ref. 3) these pulses are transformed in angular measurements such as the Earth chord-length 2κ , Fig. 1.

If the chord is larger than say 75% of the apparent Earth diameter 2ρ, the bias error after calibration is below 0.15° and the random noise below 0.05° (3σ values). Towards the Earth limb the situation rapidly degrades and measurements should be disregarded. The inclination of the IR sensors is selected such that at least one pencil beam scans in a favourable region during the critical attitude reconstitution intervals.

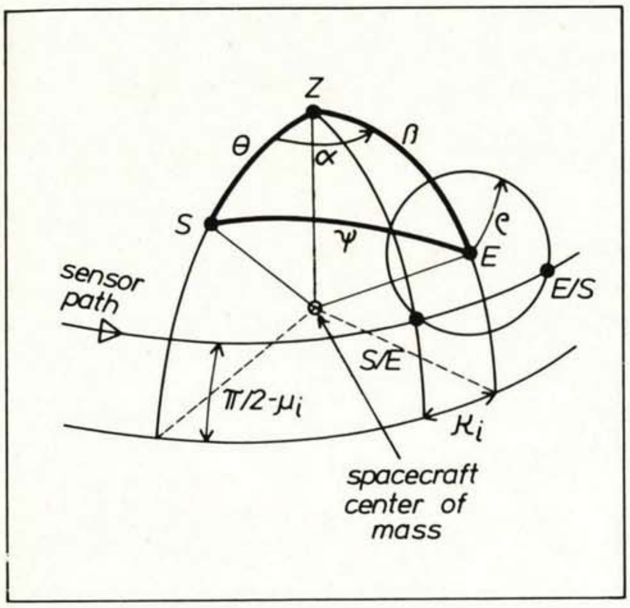

Figure 1. Geometry of Angular Measurements

The Sun colatitude is determined by the time delay between the two pulses in each of the two slits. To a very good approximation, one may assume θ to be constant over an half-hour interval. The V-slit Sun sensors flown on ESA satellites show errors below 0.1° (3σ value) including noise and biases.

An additional angular measurement which results from a combination of Sun sensor and IR Earth sensor outputs is the Sun-Earth azimuth angle α, Fig. 1. The time delays between the Sun pulse and each of the Space/Earth and Earth/Space pulses result in the angular measurements $\alpha-\kappa$ and $\alpha+\kappa$, respectively. It follows immediately that the azimuth angle α is equal to the average of these two measurements. The fact that different measuring devices are involved makes the total possible error on α somewhat larger but in practice below 0.25° provided that the IR pencil beam scans inside the 75% diameter region.

2.2 Measurement Equations

The Earth colatitude β is obtained from the κ_i measurement of the i-th IR pencil beam by considering the spherical triangle Z - S/E - E :

$$\cos\rho = \cos\mu_i \cos\beta + \sin\mu_i \sin\beta \cos\kappa_i \quad (2.1)$$

This relation gives β implicitly as a function of the measured Earth chord-length $2\kappa_i$ with sensor inclination μ_i and apparent Earth radius ρ as known parameters. Alternatively one may consider the spherical triangle Z - S - E :

$$\cos\psi = \cos\theta \cos\beta + \sin\theta \sin\beta \cos\alpha \quad (2.2)$$

Here $\psi = \arccos(E.S)$ with E and S denoting the Earth and Sun unit vectors as seen from the satellite. The positions of the Earth and Sun are obtained from orbit and ephemeris information and can be considered to be exact in the present context. Through Eq. (2.2) the Earth colatitude β is expressed in terms of the angular measurements θ and α with ψ as known parameter. It is emphasized that the accuracy of β will depend on the type of equation or combination of equations used.

Estimation of the unit spin axis vector Z is normally based upon the well-known geometric relationships :

$$Z.S = \cos\theta \quad (2.3)$$
$$Z.E = \cos\beta \quad (2.4)$$
$$Z.(S\times E) = \sin\beta \sin\theta \sin\alpha \quad (2.5)$$

The inclusion of the norm constraint $||Z|| = 1$ by means of Lagrange multipliers has not proved useful in practice (Ref. 4) and is not considered here. The determinant of the coefficients in Eqs. (2.3) to (2.5) can be shown to be equal to:

$$(SxE) . (SxE) = \sin^2\psi \quad (2.6)$$

so that at colinearity of E and S, i.e. for ψ approaching 0° or 180°, the system of Eqs. (2.3) to (2.5) becomes undetermined and the error sensitivity on the estimate Z becomes unacceptably high. In practice it has been found that the required 1° (3σ value) attitude accuracy cannot be reached within 15° from colinearity with a short sample geometric method and within about 10° from colinearity with a least squares or filter method based on at least 30 minutes of data.

When Sun-Earth colinearity occurs at some orbital position corresponding to Earth vector E_1, it is possible to obtain an attitude fix by waiting until the satellite has advanced sufficiently far in its orbit that the second Earth vector E_2 can be taken as an independent reference direction, cf. Hanson and Brown (Ref. 5). In this manner one takes the directions E_1 and E_2 rather than the (near-) colinear E_1 and S_1 as reference vectors.

The dynamic method as implemented in ESOC is a generalisation of this concept as the spread of Earth vectors around an arc of the orbit is characterised by the vector-rate of change $\dot{E} = dE/dt$. Therefore, one can take $\dot{E}$ and $E \times \dot{E}$ as well-defined reference vectors instead of S and S x E. The components of the spin vector Z along the new reference directions $\dot{E}$ and $E \times \dot{E}$ contain the derivatives of the Earth colatitude β and the Sun-Earth azimuth angle α which will be estimated from the measurements themselves. In this manner, the dynamic method takes full advantage of the information contained in the evolution of the measurement angles. Provided that the noise-contaminated data are free of discontinuities (e.g. due to change of sensor coverage) the dynamic method is capable of producing the desired attitude accuracy over a shorter span of data around colinearity than other methods.

3. FORMULATION OF THE DYNAMIC METHOD

The principle of the dynamic method is to make a curve fitting on the actual measurements $\alpha(t)$, $\beta(t)$ as they vary with position around an arc of the orbit. The attitude vector Z is then determined on the basis of the estimates $\hat{\alpha}(t)$, $\hat{\beta}(t)$ as well as the estimated derivatives $\hat{\dot{\alpha}}(t)$, $\hat{\dot{\beta}}(t)$ rather than from the measurements directly. This approach recognises the fact that the reference vectors E(t) lie on a known surface (i.e. a plane) so that a more stable algorithm can be expected than would be the case if this information is not utilised. For the variation of $\alpha(t)$ and $\beta(t)$ a model based on a Kepler orbit has been taken which gives improved accuracy and requires a lower sampling rate than the quadratic function of time proposed by Peyrot (Ref. 2).

3.1 Attitude Vector and Orbital Motion

The equations linking the attitude of the inertially fixed spin vector Z to the measurement β and its derivative are obtained from Eq. (2.4) :

$$Z.E(t) = \cos\beta(t) \qquad (3.1)$$
$$Z.\dot{E}(t) = -\dot{\beta}(t)\sin\beta(t) \qquad (3.2)$$

where $\dot{E} = dE/dt$ denotes the rate of change of the satellite-Earth unit-vector. The equation for the spin axis component along the third reference direction $Ex\dot{E}$ is obtained by considering the changes of E, α and β over an infinitesimal interval of time Δt; from spherical geometry one obtains similarly as in Eq. (2.5) :

$$Z.\{Ex(E+\Delta E)\} = \sin\Delta\alpha \ \sin\beta\ \sin(\beta+\Delta\beta)$$

which in the limit for $\Delta t \rightarrow 0$ becomes :

$$Z.\{E(t)\ x\ \dot{E}(t)\} = \dot{\alpha}(t)\sin^2\beta(t) \qquad (3.3)$$

It is obvious that the Sun vector motion can be neglected in comparison to that of the Earth vector.

The three reference vectors E, $\dot{E}$ and $Ex\dot{E}$ form by definition a right-handed orthogonal triad. In the case of Kepler motion one may write (Fig. 2):

$$E = E_1 \quad \dot{E} = \dot{\upsilon}E_2 \quad Ex\dot{E} = \dot{\upsilon}E_3 \qquad (3.4)$$

where E_1, E_2, E_3 is an orthonormal triad rotating along with the satellite's orbital motion and $\dot{\upsilon}$ is the orbital rate. The rates of change of $\dot{E}$ and of $Ex\dot{E}$ can readily be expressed in terms of E, $\dot{E}$ and $Ex\dot{E}$:

$$d\dot{E}/dt = (\ddot{\upsilon}/\dot{\upsilon})\ \dot{E} - \dot{\upsilon}^2E \qquad (3.5)$$

$$d(Ex\dot{E})/dt = (\ddot{\upsilon}/\dot{\upsilon})\ (Ex\dot{E}) \qquad (3.6)$$

The orbital rate terms $\dot{\upsilon}$,$\ddot{\upsilon}$ are evaluated from the last orbit determination over the observation interval.

3.2 State and Observation Models

The state vector to be estimated is denoted by x and contains the following components :

$$x = (\gamma, \dot{\gamma}, \alpha, \dot{\alpha})^T \qquad (3.7)$$

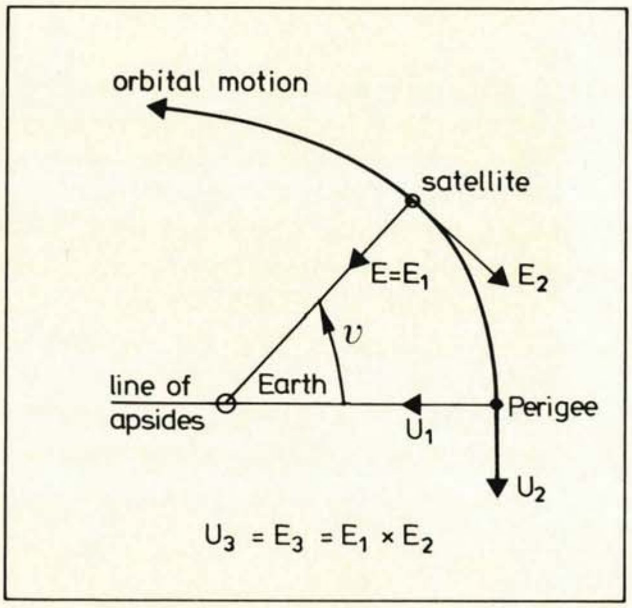

Figure 2. Orbital Reference Frames

where γ is an abbreviation for $\cos\beta$ = Z.E.

The evolution of the state x can be described by the linear matrix equation dx/dt = A(t)x. The elements of the 4x4 matrix A can be collected from Eqs. (3.1) to (3.6).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dot{\upsilon}^2 & \ddot{\upsilon}/\dot{\upsilon} & 0 & 0 \\ \dot{\upsilon} & 0 & 0 & 1 \\ 0 & 0 & 0 & \ddot{\upsilon}/\dot{\upsilon}+2\gamma\dot{\gamma}/(1-\gamma^2) \end{bmatrix} \qquad (3.8)$$

Here, one should note that the coefficients for the $\dot{\alpha}$ equation contain γ and $\dot{\gamma}$. In order to obtain a decoupled linear system the a priori constant values for γ,$\dot{\gamma}$ are taken in matrix A. Since the sampling rate is high relative to the orbital rate $\dot{\upsilon}$ one may take the matrix A constant between samples so that the fundamental matrix ϕ_j for the transition x_{j-1} to x_j (where x_j stands for $x(t_j)$, etc.) can be obtained analytically :

$$\phi_j = \exp\{A(t_j)[t_{j+1}-t_j]\} = \exp\{A_j\tau\} =$$
$$= I + A_j\tau + A_j^2\tau^2/2! + A_j^3\tau^3/3! + \ldots \qquad (3.9)$$

where τ denotes the time-interval $t_{j+1}-t_j$ and I is the identity matrix. In practice, three terms of the series in Eq. (3.9) give sufficient accuracy.

The well-known discrete linear Kalman filter can now be used to estimate the state vector x containing the geometric angles and their derivatives in Eq. (3.7). The following state and observation model for the Kalman filter are taken :

$$\text{State :} \quad x_{j+1} = \phi_j\ x_j + \xi_j \qquad (3.10)$$

$$\text{Observation:} \quad y_j = H_j\ x_j + \eta_j \qquad (3.11)$$

The observation vector y is defined as $y=(\gamma,\alpha)^T$ and the 2x4 observation matrix H_j has non-zero entries only for $H_{11} = H_{23} = 1$. The system noise and observation noise terms ξ and η are assumed to be zero mean, uncorrelated and white noise. The sequential Kalman estimate for x at time t_j is given by (e.g. Bryson and Ho, Ref. 6, Ch. 12):

$$\hat{x}_j = \phi_{j-1}\hat{x}_{j-1} + K_j\,(y_j - H_j\,\phi_{j-1}\hat{x}_{j-1}) \qquad (3.12)$$

where the Kalman gain matrix K_j is determined by the uncertainty in the a priori state estimate and the uncertainty in the new measurement. Introducing the noise covariance matrices :

$$R_j = E(\eta_j\eta_j^T);\; Q_j = E(\xi_j\xi_j^T) \qquad (3.13)$$

one can calculate the a posteriori state covariance matrix by successive propagation using the state transition matrix ϕ_j :

$$M_{j+1} = \phi_j(M_j^{-1} + H_j^T\,R_j^{-1}H_j)^{-1}\,\phi_j^T + Q_j \qquad (3.14)$$

with given M_o (i.e. the initial state covariance).

3.3 Determination of the Spin Vector Attitude

From the Kalman filter estimate $\hat{x}$ the components Z_1, Z_2, Z_3 of the spin vector along the orbital reference directions E_1, E_2, E_3 can be derived directly using the results of Eqs. (3.1) to (3.4):

$$Z_1 = Z\cdot E_1 = \hat{\gamma} \qquad (3.15)$$

$$Z_2 = Z\cdot E_2 = \hat{\dot{\gamma}}/\dot{\upsilon} \qquad (3.16)$$

$$Z_3 = Z\cdot E_3 = \hat{\dot{\alpha}}(1-\hat{\gamma}^2)/\dot{\upsilon} \qquad (3.17)$$

These components can readily be transformed to inertial components by means of the transformation matrix from the orbital to the inertial reference frames. Because of the additional constraint $||Z|| = 1$, three different estimates for the spin vector can be constructed :

i) $\beta\dot{\beta}$ estimate :

$$Z = Z_1E_1+Z_2E_2+Z_3^*E_3$$
$$\text{with } Z_3^* = \sqrt{1-Z_1^2-Z_2^2}\,\text{sgn}(Z_3) \qquad (3.18)$$

ii) $\beta\dot{\alpha}$ estimate :

$$Z = Z_1E_1+Z_2^*E_2+Z_3E_3$$
$$\text{with } Z_2^* = \sqrt{1-Z_1^2-Z_3^2}\,\text{sgn}(Z_2) \qquad (3.19)$$

iii) $\beta\dot{\beta}\dot{\alpha}$ estimate :

$$Z = \{Z_1E_1+Z_2E_2+Z_3E_3\}/\sqrt{Z_1^2+Z_2^2+Z_3^2} \qquad (3.20)$$

Depending on the relative accuracies of the angular measurements α and β one can expect different accuracies for each of the three estimates.

As an overall measure on the quality of the result one may taken the modulus

$$\sqrt{Z_1^2 + Z_2^2 + Z_3^2}.$$

In a later section a way of obtaining a best estimate for the spin vector attitude from a combination of the three estimates of Eqs. (3.18) to (3.20) will be described.

4. OPERATIONAL IMPLEMENTATION AND DIFFICULTIES

The practical implementation of the dynamic method as outlined above posed a number of problems. The operational environment does not permit a lengthy analysis of printouts nor experimentation with sampling rates, data collection intervals and input parameters. On the other hand, an acceptable accuracy of the estimate must be guaranteed. A few of the difficulties encountered and their solutions will be described next.

4.1 The Beta Dot Stability

In the original version $\dot{\beta}$ is computed from the locally most accurate equation amongst (2.1) and (2.2). Since sensor biases are magnified in a different way by different equations the β result will change abruptly when switching from one equation to an other. This naturally leads to large errors in the $\dot{\beta}$ estimate and often results in divergence of the filter. This difficulty has been overcome by calculating β from a linear combination of Eqs. (2.1) and (2.2) which is constructed in such a way that the resulting $\gamma = \cos\beta$ has minimum variance to the first order.

Due to hardware limitations, at most two equations of type (2.1) are available because not more than two pencil beams can be operating simultaneously. Equations of type (2.2) will occur either once or not at all. All of the five possible measurement equations have the form :

$$f_i(\beta) = a_i\cos\beta + b_i\sin\beta - c_i = 0 \;(i=1,..,N) \qquad (4.1)$$

with coefficients a_i, b_i, c_i depending on the actual measurements κ_i, θ and α and the parameters μ_i, ψ and ρ. At each point where more than one measurement relation is available a new function is constructed :

$$g(\beta) = \sum_i \omega_i\, f_i(\beta) \qquad (4.2)$$

with weighting coefficients ω_i. It is clear that the equation $g(\beta) = 0$ has a similar form as Eq. (4.1) with coefficients now also depending on ω_i. The solution of $g(\beta) = 0$ can be determined in an elementary manner and contains the weighting parameters ω_i. The weights ω_i will be selected in such a manner that the variance of $\gamma = \cos\beta$ is minimum given the variances of the actual measurements κ_i, θ and α as well as those of the parameters μ_i, ψ and ρ.

The measurements κ_i, θ and α and parameters μ_i, θ and α available at some point of time are collected in the vector λ_k, k=1,...,K . The variation of γ as a function of the variations in the elements of γ is expressed to first order by the differential relation :

$$\sum_i\omega_i\left(\frac{\partial f_i}{\partial\gamma}\right)_{\beta_o}\Delta\gamma = \sum_k\;\sum_i\omega_i\left(\frac{\partial f_i}{\partial\lambda_k}\right)_{\beta_o}\Delta\lambda_k \qquad (4.3)$$

which follows from Eq. (4.2). The subscript β_o indicates that the functions are evaluated at a reference value $\beta = \beta_o$. Introducing the abbreviations :

$$n = \sum_i\omega_i\left(\frac{\partial f_i}{\partial\gamma}\right)_{\beta_o}\;;\; m_k = \sum_i\omega_i\left(\frac{\partial f_i}{\partial\lambda_k}\right)_{\beta_o} \qquad (4.4)$$

one obtains the following first order result for the variances of the result γ expressed in the variances of the measurements and parameters contained in λ:

$$\sigma_\gamma^2 = E\{(\gamma-\bar{\gamma})^2\} = \frac{1}{n^2} \sum_{k,l} m_k m_l \sigma_{kl} \quad (4.5)$$

where $\sigma_{kl} = E\{(\lambda_k-\bar{\lambda}_k)(\lambda_l-\bar{\lambda}_l)\}$ designates the given variances of the elementary measurements and parameters λ_k.

For simplicity it is assumed that the elementary measurements and parameters are uncorrelated, i.e. $\sigma_{kl} = \sigma_{kk}\delta_{kl}$.

An extremum of σ_γ^2 as a function of ω_i is reached when

$$\frac{\partial(\sigma_\gamma^2)}{\partial\omega_i} = 0 \quad (i = 1,\ldots,N) \quad (4.6)$$

Provided that n of Eq. (4.4) is non-zero one finds :

$$\sum_k \left\{\frac{\partial m_k}{\partial\omega_i} - \frac{m_k}{n}\frac{\partial n}{\partial\omega_i}\right\} m_k \sigma_{kk} = 0 \quad (i=1,\ldots,N) \quad (4.7)$$

This system of equations for ω_i, i=1,...,N is homogeneous in the ω_i. Thus, one may select the norm of the ω_i to satisfy the relation :

$$\sum_k m_k^2 \sigma_{kk} = n \quad (4.8)$$

Eq. (4.7) can now be reduced to

$$\sum_k \left(\frac{\partial m_k}{\partial\omega_i}\right) m_k \sigma_{kk} = \frac{\partial n}{\partial\omega_i} \quad (i=1,\ldots,N) \quad (4.9)$$

Returning to the definitions of n and m_k in Eqs. (4.4) one sees that Eqs. (4.9) form a linear system of N equations for ω_i :

$$\sum_{j=1}^{N} c_{ij}\, \omega_j = d_i \quad (i=1,\ldots,N)$$

with

$$c_{ij} = \sum_k \left(\frac{\partial f_i}{\partial\lambda_k}\right)_{\beta_o} \left(\frac{\partial f_j}{\partial\lambda_k}\right)_{\beta_o} \sigma_{kk} \quad ; \quad d_i = \left(\frac{\partial f_i}{\partial\gamma}\right)_{\beta_o} \quad (4.10)$$

After solving for ω_i from this system, the variance of γ follows directly from Eq. (4.5).

The implementation of this technique has resulted in a considerable improvement in the stability of the $\dot{\beta}$ filter performance which must be attributed to the smoothing properties of the linear combination of the measurement equations.

4.2 Filtering Sensitivity

It is well-known that in the presence of modelling errors irreversible divergence of the Kalman filter may result unless adaptive techniques are employed. This situation is aggravated if discontinuities occur because of bad quality telemetry or by begin and end of IR sensor coverage. The estimate of the derivatives will be particularly sensitive to such influences.

To overcome these difficulties divergence of the filter is monitored by testing the modulus of the attitude estimate obtained from Eqs. (3.1)-(3.3): in the case of filter divergence the norm of the unit vector Z will drift away from unity. A complete reinitialisation is undertaken whenever the modulus test is not satisfied within a margin of 5% over a certain number of successive checks. This radical approach may eventually lead to convergence but is certainly not very efficient as a great deal of measurement information is abandoned.

Another difficulty is to find a suitable point to stop the filtering and calculate the attitude vector from the α and β estimates as described in Section 3.2. In the original approach filtering continues until the covariances on the two estimates for $\dot{\alpha}$ and $\dot{\beta}$ reaches a given threshold. This 'snapshot' approach has the disadvantage that the last estimate may not be the best one available. This is particularly true if after a filter divergence the estimation process has been reinitialised.

An additional problem is the choice of a final attitude estimate out of the three possible candidates indicated by $\beta\beta$, $\beta\dot{\alpha}$ and $\beta\beta\dot{\alpha}$ in Section 3.3. Experience with METEOSAT I transfer orbit data shows that the $\beta\dot{\alpha}$ estimate is the most and the $\beta\beta$ estimate the least accurate of the three. Since α is a more direct measurement than β its biases are relatively stable. This influences the accuracy of the $\dot{\alpha}$ estimate in a favourable way. Furthermore, the particular configuration of the spin axis with respect to the orbital plane affects the relative accuracies of the estimates : if the spin axis is near the orbital plane the $\beta\dot{\alpha}$ estimate is more accurate than $\beta\beta$. In the case where the spin axis is close to the orbit normal the $\beta\beta$ estimate is more accurate cf. Peyrot, Ref. 2).

5. LEAST-SQUARES ESTIMATE OF ATTITUDE

Because of the difficulties outlined above the following modifications have been introduced :

i) to include the Sun sensor measurements whenever they are available

ii) to build a sequential weighted least-squares estimate of the attitude vector based on the output of the Kalman filter and the Sun measurements.

The usefulness of incorporating the Sun colatitude measurement θ is clear as it provides in general another independent reference direction for the attitude vector. Also in the colinearity situation a significant advantage can be expected as the covariances on θ are lower than those on β so that the uncertainty on the attitude estimate can be reduced.

In the application of the least squares filter the output of the Kalman filter (ie. $\alpha, \beta, \dot{\alpha}$ and $\dot{\beta}$) are interpreted as observations. Along with the real Sun colatitude measurements they are fed into a weighted least-squares filter for the estimation of the attitude vector Z.

Figure 3 provides a schematic block diagram from which the complete estimation process can readily be visualised. In order to keep the least squares estimates of Z as accurate as possible bad data

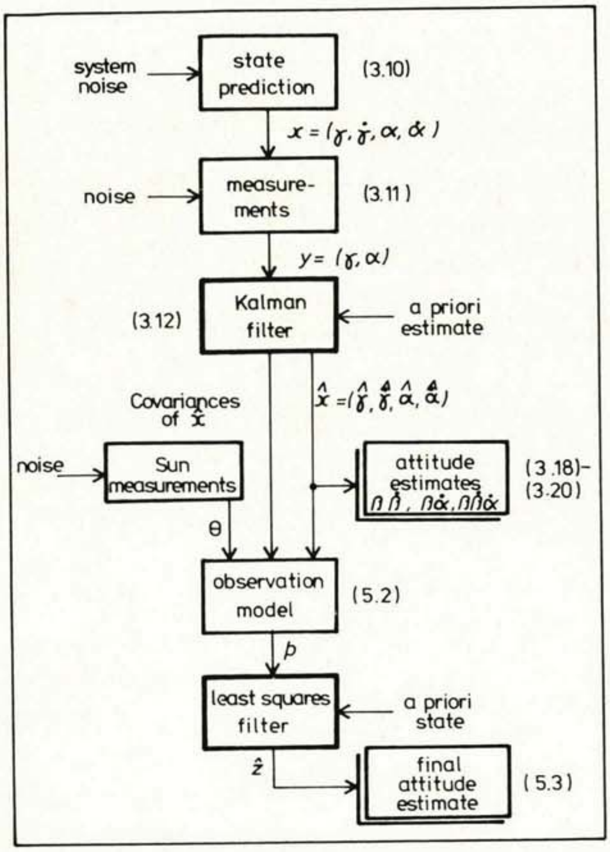

Figure 3 Overview of Attitude Determination Process

(according to the modulus criterion) are rejected. Provided that the Kalman filter converges over some interval the least-squares estimate will properly reflect the actual attitude orientation. Although the Kalman filter may diverge at a later stage the least squares estimate remains unchanged and waits until good input arrives again.

5.1 Weighted Least-Squares Estimation

The spin vector attitude Z is to be estimated from the five 'observations' α, β, $\dot{\alpha}$, $\dot{\beta}$ and θ. Because of geometrical reasons it is convenient to estimate the three components z_1, z_2 and z_3 of Z along the apsidal inertial reference frame U_1, U_2, U_3 (Fig. 2). The inertial spin vector components follow then after a straight-forward coordinate transformation involving the orbital elements i, ω and Ω.

The four different measurement equations for the attitude vector Z are :

$$
\begin{aligned}
Z \cdot S &= \cos\theta = (s_1, s_2, s_3)(z_1, z_2, z_3)^T \\
Z \cdot E &= \cos\beta = (e_1, e_2, 0)(z_1, z_2, z_3)^T \\
Z \cdot \dot{E} &= -\dot{\beta}\sin\beta = (\dot{e}_1, \dot{e}_2, 0)(z_1, z_2, z_3)^T \\
Z \cdot (E \times \dot{E}) &= \dot{\alpha}\sin^2\beta = (0, 0, \dot{\upsilon})(z_1, z_2, z_3)^T
\end{aligned}
\quad (5.1)
$$

Components with respect to the apsidal reference frame are written in small letters. The satellite - Sun vector components are s_1, s_2, s_3 and e_1, e_2, 0 denote the satellite - Earth vector components along the U_1, U_2, U_3 axes. In abbreviated standard form Eqs. (5.1) are written as :

$$p = Bz + v \quad (5.2)$$

where p is the m-dimensional ($m \leq 4$) observation vector, i.e. the right-hand sides of Eqs. (5.1). The mx3 observation matrix B can be obtained readily from Eqs. (5.1). The m-vector v designates the mean-zero noise in the 'observations' p. For a Sun measurement θ the covariance of the noise follows directly from the inaccuracy of the Sun sensor, i.e. σ_θ. The β, $\dot{\beta}$, $\dot{\alpha}$ observations are obtained from the Kalman filter, cf. Fig. 3. Therefore, it appears most reasonable to take the a posteriori state covariance of the Kalman estimate for β, $\dot{\beta}$ and $\dot{\alpha}$, Eq. (3.14) as the noise covariance for these observations. The least-squares estimation is initiated only after the Kalman filter has stabilised sufficiently, i.e. when the modulus test is satisfied (section 4.2). The mean value of the three attitude estimates ($\beta\dot{\beta}$, $\beta\dot{\alpha}$, $\beta\dot{\beta}\dot{\alpha}$) available at that time is taken as the initial estimate for the least-squares filter. A relatively large covariance matrix P_o is attributed to this a priori estimate.

The sequential weighted least-squares estimate for the attitude vector in the apsidal frame is derived from the well-known result (cf. Bryson and Ho, Ref. 6, Ch. 12) :

$$\hat{z}_k = \hat{z}_{k-1} + P_{k-1} B_k^T R_k^{-1} (p_k - B_k \hat{z}_{k-1}) \quad (k=1,2,\ldots)$$

with

$$P_k = (P_{k-1}^{-1} + B_k^T R_k^{-1} B_k)^{-1} \quad (5.2)$$

R_k stands for the covariance matrix of the observation noise v_k and P_k is the covariance matrix of the error in the estimate $\hat{z}_k - z$.

It is emphasized that only those β, $\dot{\beta}$, $\dot{\alpha}$ observations which have passed the modulus test are considered in the least-squares estimation. Thus bad data are excluded and stability of the least-squares estimate over longer intervals can be expected. Also the inclusion of the Sun measurements has a stabilising effect on the estimate as their noise level is lower than that of the artificially constructed β measurements and the indirectly derived $\dot{\alpha}$ and $\dot{\beta}$ observations.

6. DISCUSSION OF RESULTS

The dynamic method has been tested extensively by means of the actual telemetry data of ESA's METEOSAT I satellite launched on 23rd November 1977. Although various program versions have been used at different times for clarity only the two main versions are discussed here. Version 1 refers to the program producing the three attitude estimates $\beta\dot{\beta}$, $\beta\dot{\alpha}$ and $\beta\dot{\beta}\dot{\alpha}$ of Sections 3 and 4. Version 2 contains the least-squares attitude estimate discussed in Section 5.

As is probably the case with all attitude estimation programs a great deal of experimentation is required before the right configuration (e.g.

system and measurement noise levels; initial state and covariance estimates) is established. Because of its sensitivity to discontinuities in the received telemetry data the dynamic method requires some extra care with respect to the selection of the filtering interval (e.g. no or very few sensor switches; no IR data near the Earth's limb) and the choice of a suitable sampling rate. Sometimes a change in sampling rate by only 25% makes the difference between a stable and a diverging attitude estimate. Also a proper balance of the various kinds of measurement is important.

The best interval for a posteriori testing of an attitude reconstitution method is the free drift interval just before Apogee Motor Firing (AMF). This is because the actual attitude can be derived to an accuracy of less than 0.1 degree from the Δv direction achieved during AMF. The Δv direction follows directly from orbit determinations before and after AMF.

6.1 Results of Version 1

A few of the results obtained for the $\beta\dot{\beta}$, $\beta\dot{\alpha}$ and $\beta\dot{\beta}\dot{\alpha}$ estimates are presented in table 1. All entries refer to the last free drift period before AMF, i.e. 23rd November 1977 from 13 hr 39 min to 19 min. The angular errors listed are taken with respect to the actual attitude, i.e. $\alpha = 353.2^\circ$ and $\delta = -22.6^\circ$ with an error less than 0.1°. It is confirmed in Table 1 that the $\beta\dot{\beta}$ estimate is always the least accurate whereas $\beta\dot{\alpha}$ is usually the best.

Mean Time of Interval	Duration of Interval	Right Ascen-	Decli-nation	Angular Error	Type
16hr;19min	58 min	352.86° 353.00 352.87	-22.08° -22.96 -22.44	.64° .35 .35	$\beta\dot{\beta}$ $\beta\dot{\alpha}$ $\beta\dot{\beta}\dot{\alpha}$
16hr;19min	59 min	352.94 353.03 352.95	-22.40 -22.99 -22.64	.33 .37 .21	$\beta\dot{\beta}$ $\beta\dot{\alpha}$ $\beta\dot{\beta}\dot{\alpha}$
16hr;56min	131 min	353.22 353.21 353.21	-23.10 -22.79 -22.98	.46 .15 .34	$\beta\dot{\beta}$ $\beta\dot{\alpha}$ $\beta\dot{\beta}\dot{\alpha}$
16hr;56min	131 min	353.15 353.14 353.14	-23.26 -22.80 -23.07	.61 .16 .43	$\beta\dot{\beta}$ $\beta\dot{\alpha}$ $\beta\dot{\beta}\dot{\alpha}$

Table 1 Results of Version 1 over Last Free Drift Period in Transfer Orbit of METEOSAT I

The first three entries in Table 1 are obtained with a four times higher sampling rate than the others. It is seen that a higher sampling rate does not necessarily lead to better results. In the third and fourth set of three entries in Table 1 high and low noise levels on μ_i, α and θ have been assumed, respectively. It appears that these noise levels are not very significant.

6.2 Results of Version 2

The performance of the dynamic method has been evaluated by means of a comparison with the results from a Kalman filter based on the Denham-Pines method. The filter includes the estimation of sensor misalignments as well as iterations on the non-linear measurement equations. Experience has shown that this filter is very reliable in routine attitude determinations.

Figure 4 shows the evolution of the estimates for right ascension and declination obtained by the two methods based on actual telemetry data of METEOSAT I over the third free-drift interval, i.e. 23rd November 1977 from 5 hr 19 min to 13 hr 38 min. Apart from some convergence difficulties in the beginning (caused by sensor switches) the agreement between the results is excellent. In the second half of the interval shown in Fig. 5 the angular difference between the two estimated attitude directions is less than 0.3 degrees with a minimum of 0.11 degrees.

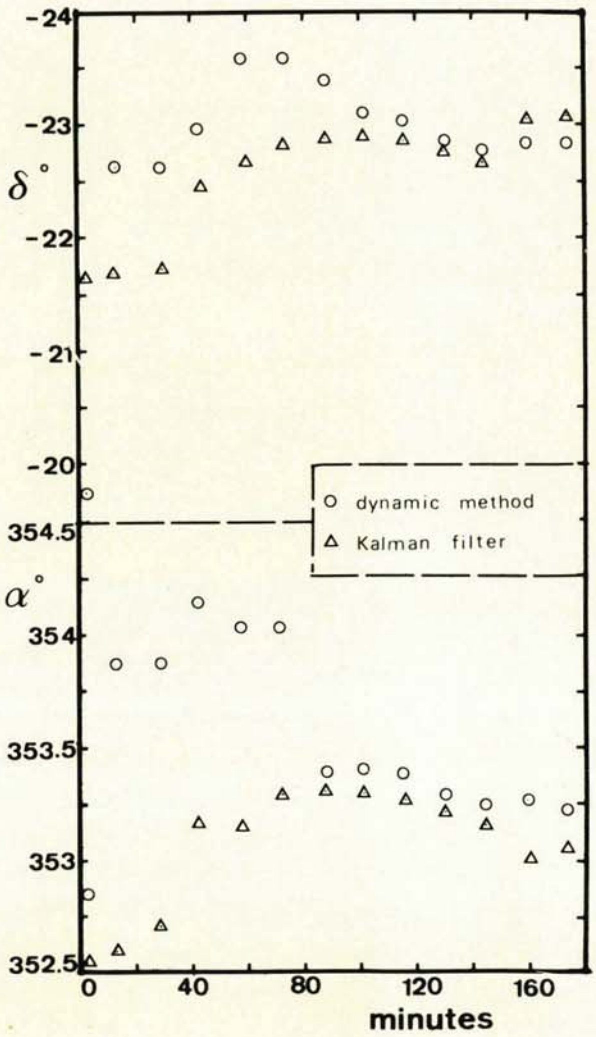

Figure 4 Comparison of Results from Dynamic Method and Kalman Filter

The major justification for the development of the dynamic method is the avoidance of divergence near Sun-Earth colinearity. Therefore, it is of interest to compare the results of the dynamic method and Kalman filter in such a case. Near-colinearity of 173 degrees occurs in the last free drift period before AMF where the attitude is known to an accuracy of less than 0.1° from a posteriori Δv analysis. Figure 5 shows the evolution of the angular error η of the attitude estimates obtained by the Kalman filter and the dynamic method (one run including the Sun measurements and one without θ information). It is seen that the inclusion of the Sun measurements accelerates the convergence and improves the stability of the estimate. It is emphasized that the initial estimate for the dynamic method runs was off by 18 degrees. Within eight minutes the error is reduced to less than one degree. The attitude uncertainty limit of 0.1 degree is reached after about one hour of (relatively low) sampling. Because of the colinearity the convergence of the Kalman filter stagnates and eventually diverges to an angular error of more than six degrees. In fact, it takes over three hours before the Kalman filter estimate comes within 0.5 degree of the actual attitude again.

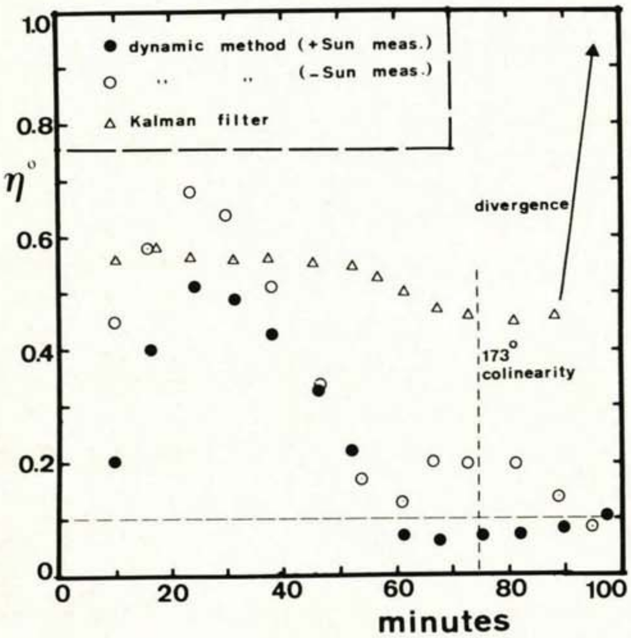

Figure 5 Comparison of Results of Dynamic Method and Kalman Filter in Near-Colinearity Case

7. CONCLUSIONS

A dynamic attitude reconstitution method which takes full advantage of the information contained in the rates of change of the Sun and Earth colatitudes has been formulated. The colatitudes and their derivatives are estimated by means of a Kalman filter using an observation model derived from a Kepler orbit. Three different attitude estimates ($\beta\dot{\beta}$, $\beta\dot{\alpha}$ and $\beta\beta\dot{\alpha}$) follow directly from the Kalman estimates. A single attitude estimate is obtained by a weighted least-squares combination of the Kalman filter outputs and the Sun measurements. The method has been evaluated by means of actual METEOSAT I telemetry data. It is shown that in the case of a near-colinearity of Sun and Earth vectors the attitude estimate remains stable whereas conventional methods diverge.

8. REFERENCES

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