DYNAMICS OF AN ACTIVELY CONTROLLED FLEXIBLE EARTH OBSERVATION SATELLITE

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ABSTRACT

The paper presents attitude and flexural dynamics of an Earth-oriented satellite with a rigid main body and two large rectangular flexible Sun-tracking solar panels. It is controlled using three reaction wheels operating on PWPFM logic with a modified Schmitt trigger and attitude sensors. The governing equations being highly nonlinear and coupled, numerical solution is resorted to. The system parameters corresponding to those of the Indian Remote Sensing (IRS) satellite are used for the simulation. After studying the system performance, a modified controller with a 12th order Kalman filter and observer introduced to reduce the effects of the sensor noise and to improve the system characteristics is considered. The effects of sampling and quantization of the sensor output are also studied.

Keywords: Satellite Attitude Dynamics, Flexure, Nonlinear Control, Rotors, Kalman Filter, Sensor Discretization, Simulation

1. INTRODUCTION

Large, light weight spacecraft are being developed to exploit greater potentials of space technology. The satellites are subjected to internal and environmental disturbances. They need to be controlled very accurately to meet stringent mission requirements. Often, reaction wheels are used for three axis control of near Earth satellites.

A number of studies are available on the stability, performance and design of systems carrying internal momentum exchange devices (rotors). Some of these deal with analytical solution of the equations of motion of rigid gyrostats. In an early paper, Roberson (Ref.1) studies the motion of simple rigid gyrostats. Kane (Ref.2) gives a part of the solution (which characterises the motion of the rotor completely) of a rigid uniaxial gyrostat with a driven rotor. Exact analytical solutions are obtained by Cochran et al. (Ref.3). Gething et al. (Ref. 4) present a parametric study on a momentum wheel stabilised satellite to study the effects of a varying degree of flexibility using the root locus techniques. The nonlinear pulse-frequency controller is approximated by a linear model in the analysis. Hughes (Ref.5) analyses a flexible spacecraft with a momentum wheel in the main body. He shows that the unconstrained mode shapes are affected by the wheel whereas the constrained modes are not. Harris (Ref.6) finds that the control-structure interaction is a function of the type of actuator, the control law, the satellite size, the sensor characteristics, and the modal gains, frequencies and structural damping of the appendages. Hughes and Garg (Ref.7) analyse the dynamics of large flexible solar arrays for systems having stored angular momentum. They make use of both the constrained and unconstrained modes in the analysis and define modal gains as indicators of the corrections to be applied due to flexibility. Hablani (Ref.8) studies the dynamics and control of a deformable gyrostat consisting of a rigid core carrying a reaction wheel. His study shows that, as the rotor's momentum builds up, higher frequency modes need to be included in the model for the controller design. In a recent paper, Maharana and Shrivastava (Ref.9) discuss the stability of a large damped flexible spacecraft with stored angular momentum. They make use of the analytical solutions of rigid gyrostats with the modal analysis to study the behaviour of vibration by reducing the equations of vibration to a set of linear ordinary differential equations with time-varying coefficients. The effects of structural flexibility on the nonlinear attitude control is dealt with by Abdel Rehman (Ref.10). He discusses the effects on a pseudo-rate (PSR) modulator and an Integrated-Pulse-Frequency (IPF) modulator. Jet thrusters are used for stabilisation and control. Most of the studies referred above deal with simplified linear systems, which may not always be realistic. Also, in many present and future satellites employing active control, on-off type nonlinear control logics are preferred for optimal use of the available energy on board. Their linearisation can lead to erroneous results. The effects of discretization of the sensor output are also not considered in most of the available analyses.

This paper deals with the attitude dynamics and problems of control-structure interactions in an Earth-imaging spacecraft shown in Fig.1 requiring very high degree of platform stability. The model consists of a central rigid core which

Proceedings of the Second International Symposium on Spacecraft Flight Dynamics, Darmstadt, FR Germany, 20–23 October 1986 (ESA SP-255, Dec. 1986)

houses most of the spacecraft systems, to which two Sun tracking flexible solar panels are attached along the pitch axis. The satellite is assumed to be three-axis stabilised with reaction wheels, one along each of the principal body axes. The control logic for the reaction wheels is of Pulse-Width-Pulse-Frequency-Modulator (PWPFM) type which is basically nonlinear due to the presence of the Schmitt trigger in the controller.

After deriving the governing equations of attitude and flexural motion, numerical simulation studies are carried out on both the rigid body model and the flexible model to assess the effects of flexibility on the system performance. The study is also carried out with a modified controller with a 12th order Kalman filter introduced to reduce the effects of sensor noise on the attitude behaviour. The filter acts as an observer also. The observed rate is fedback to reduce the body rates. Another important aspect of the simulation is to show the effects of digital sensor characteristics like sampling and quantization on the system response when the attitude and body rate specifications are very stringent. The study concludes with the analysis of the effects of the nonlinear controller (operating in a pulsed mode) on the flexural motion. Although the simulation is carried out with the parameters of the Indian Remote Sensing Satellite (IRS) which requires the body rates to be less than 3×10^{-4} deg/s, the formulation and findings are applicable to a wide range of satellites.

2. FORMULATION

Consider a spacecraft (Fig.1) with a rigid Earth-oriented central body and two flexible Sun-tracking solar panels in a near-polar circular orbit. The attitude equations of motion are formulated using the Newton-Euler approach. The equations of panel vibration are derived using the Lagrangian formulation. The simplifying assumptions include: (i) the shift in the centre of mass due to the rotation and vibration of flexible panels is neglected, (ii) the solar panels undergo only flexural vibration, (iii) the rotational dynamics of the panels has no effect on the attitude dynamics as the panels are held during payload operation and (iv) the solar panels are idealised as isotropic, homogeneous, rectangular plates. The set of hybrid coordinates (Ref.11) employed includes the attitude variables (discrete coordinates) and generalised coordinates associated with dominant modes of panel vibration. The hybrid coordinates are used due to several

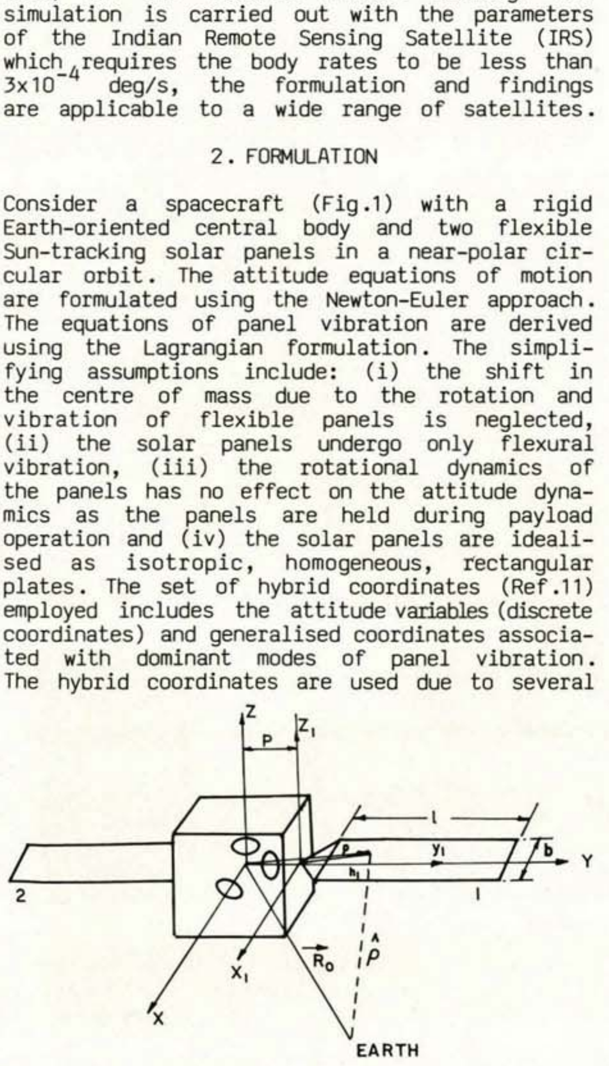

Figure 1. Spacecraft Model

advantages such as (Ref.12): (i) relatively low system order, (ii) versatility of application, (iii) elimination of high frequency components from numerical simulation, and (iv) ease of understanding the physics of the problem. The attitude equations about the roll, pitch and yaw axes are given by Eq.(1), (2) and (3), respectively, as (Ref.13,14):

$$
\begin{aligned}
T_x = {} & [I_{xe}+s2\lambda_1\Sigma J_{i1}A_{i1}+s2\lambda_2\Sigma J_{i2}A_{i2}]\dot{\omega}_x-[s\lambda_1\Sigma K_{i1}A_{i1} \\
& +s\lambda_2\Sigma K_{i2}A_{i2}+Ps\lambda_1\Sigma M_{i1}A_{i1}-Ps\lambda_2\Sigma M_{i2}A_{i2}]\dot{\omega}_y \\
& -[c2\lambda_1\Sigma J_{i1}A_{i1}+c2\lambda_2\Sigma J_{i2}A_{i2}-mb^2(s2\lambda_1 \\
& +s2\lambda_2)/24]\dot{\omega}_z+[I_{ze}-I_{ye}]\omega_y\omega_z+mb^2(s2\lambda_1 \\
& +s2\lambda_2)\omega_x\omega_y/24+[s2\lambda_1\Sigma J_{i1}\dot{A}_{i1}+s2\lambda_2\Sigma J_{i2}\dot{A}_{i2}]\omega_x \\
& -2[s\lambda_1\Sigma K_{i1}\dot{A}_{i1}+s\lambda_2\Sigma K_{i2}\dot{A}_{i2}+Ps\lambda_1\Sigma M_{i1}\dot{A}_{i1} \\
& -Ps\lambda_2\Sigma M_{i2}\dot{A}_{i2}]\omega_y-[c2\lambda_1\Sigma J_{i1}\dot{A}_{i1}+c2\lambda_2\Sigma J_{i2}\dot{A}_{i2} \\
& -\Sigma J_{i1}\dot{A}_{i1}-\Sigma J_{i2}\dot{A}_{i2}]\omega_z+Pc\lambda_1\Sigma M_{i1}\ddot{A}_{i1}-Pc\lambda_2\Sigma M_{i2}\ddot{A}_{i2} \\
& +c\lambda_1\Sigma K_{i1}\ddot{A}_{i1}+c\lambda_2\Sigma K_{i2}\ddot{A}_{i2}+\dot{H}_{Rx}+(\omega_y H_{Rz}-\omega_z H_{Ry}), \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
T_y = {} & I_{ye}\dot{\omega}_y-[Ps\lambda_1\Sigma M_{i1}A_{i1}-Ps\lambda_2\Sigma M_{i2}A_{i2}+s\lambda_1\Sigma K_{i1}A_{i1} \\
& +s\lambda_2\Sigma K_{i2}A_{i2}]\dot{\omega}_x-[Pc\lambda_1\Sigma M_{i1}A_{i1}-Pc\lambda_2\Sigma M_{i2}A_{i2} \\
& +c\lambda_1\Sigma K_{i1}A_{i1}+c\lambda_2\Sigma K_{i2}A_{i2}]\dot{\omega}_z+[I_{xe}-I_{ze}]\omega_x\omega_z \\
& -mb^2(s2\lambda_1+s2\lambda_2)(\omega_x^2-\omega_z^2)/24-\Sigma J_{i1}\ddot{A}_{i1}-\Sigma J_{i2}\ddot{A}_{i2} \\
& +\dot{H}_{Ry}+(\omega_z H_{Rx}-\omega_x H_{Rz}), \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
T_z = {} & [I_{ze}+s2\lambda_1\Sigma J_{i1}A_{i1}+s2\lambda_2\Sigma J_{i2}A_{i2}]\dot{\omega}_z-[c\lambda_1\Sigma K_{i1}A_{i1} \\
& +c\lambda_2\Sigma K_{i2}A_{i2}+Pc\lambda_1\Sigma M_{i1}A_{i1}-Pc\lambda_2\Sigma M_{i2}A_{i2}]\dot{\omega}_y \\
& -[c2\lambda_1\Sigma J_{i1}A_{i1}+c2\lambda_2\Sigma J_{i2}A_{i2}-mb^2(s2\lambda_1 \\
& +s2\lambda_2)/24]\dot{\omega}_x+[I_{ye}-I_{xe}]\omega_x\omega_y-mb^2(s2\lambda_1 \\
& +s2\lambda_2)\omega_y\omega_z/24+[s2\lambda_1\Sigma J_{i1}\dot{A}_{i1}+s2\lambda_2\Sigma J_{i2}\dot{A}_{i2}]\omega_z \\
& -2[Pc\lambda_1\Sigma M_{i1}\dot{A}_{i1}-Pc\lambda_2\Sigma M_{i2}\dot{A}_{i2}+c\lambda_1\Sigma K_{i1}\dot{A}_{i1} \\
& +c\lambda_2\Sigma K_{i2}\dot{A}_{i2}]\omega_y-[c2\lambda_1\Sigma J_{i1}\dot{A}_{i1}+c2\lambda_2\Sigma J_{i2}\dot{A}_{i2}+\Sigma J_{i1}\dot{A}_{i1} \\
& +\Sigma J_{i2}\dot{A}_{i2}]\omega_x-s\lambda_1\Sigma K_{i1}\ddot{A}_{i1}-s\lambda_2\Sigma K_{i2}\ddot{A}_{i2}-Ps\lambda_1\Sigma M_{i1}\ddot{A}_{i1} \\
& +Ps\lambda_2\Sigma M_{i2}\ddot{A}_{i2}+\dot{H}_{Rz}+(\omega_x H_{Ry}-\omega_y H_{Rx}), \qquad (3)
\end{aligned}
$$

where,

$I_{xe} = I_x+2mP^2+2mPl+mb^2/6+2ml^2/3-mb^2(c^2\lambda_1+c^2\lambda_2)/12,$

$I_{ye} = I_y+mb^2/6,$

$I_{ze} = I_z+2mP^2+2mPl+2ml^2/3+mb^2/6-mb^2(s^2\lambda_1+s^2\lambda_2)/12,$

$\int\phi_{ij}dm = M_{ij}$, $\int x\phi_{ij}dm = J_{ij}$, $\int y\phi_{ij}dm = K_{ij}$,

$s\lambda_j=\sin\lambda_j$, $c\lambda_j=\cos\lambda_j$, $s2\lambda_j=\sin 2\lambda_j$, $c2\lambda_j=\cos 2\lambda_j$,

A_{ij} : i^{th} modal coordinate for j^{th} panel

H_{Rq} : rotor angular momentum (q=x,y,z)

I_q : principal MI of central body

l,b,m : length, breadth and mass of panel

P : distance between C.M. of spacecraft and point of attachment of panel

T_q : torques acting on satellite

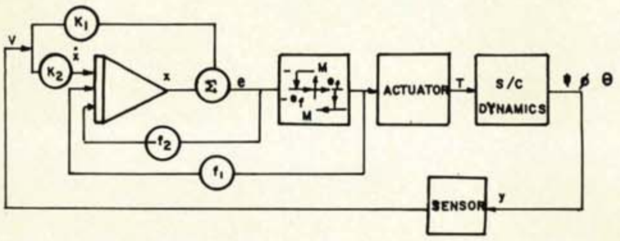

Figure 2. System Block Diagram

ω_q : body rates

λ_j : angular position of j^{th} panel

ϕ_i : i^{th} cantilever mode shape function

Ω_i : i^{th} mode frequency

θ_q : Roll(θ_x), Pitch(θ_y), Yaw(θ_z).

The equation governing the flexural vibration of each solar panel is given by (Ref.13,14),

$$N_r\ddot{A}_{ij}+\Omega_{ij}^2 N_r A_{ij}+(PM_{ij}+K_{ij})c\lambda\dot{\omega}_x-J_{ij}\dot{\omega}_y-(PM_{ij}+K_{ij})s\lambda\dot{\omega}_z$$
$$+(PM_{ij}+K_{ij})s\lambda\omega_x\omega_y+(PM_{ij}+K_{ij})c\lambda\omega_y\omega_z$$
$$+J_{ij}(s2\lambda/2)(\omega_x^2-\omega_z^2)+J_{ij}c2\lambda\omega_x\omega_z = 0, \quad (4)$$

where, $N_r = \int\phi_r^2 dm$, the generalised mass, $i = 1, 2, \ldots n$ = mode number, and $j = 1$ for the right panel and 2 for the left panel. Here, the flexural motion of the panel is assumed to be described by n modes.

The three-axis stabilisation and control are achieved with the help of the reaction wheels. Each reaction wheel is assumed to be controlled independently using a PWPFM (Pulse Width Pulse Frequency Modulated) logic with a Schmitt trigger. The block diagram representation including the controller, the actuator (reaction wheels), attitude sensor and system dynamics is given in Fig.2.

The equations of motion (1)-(4) along with the Euler transformation equations are highly coupled and nonlinear. The nonlinear control logic adds to the complexity. It is impossible to find even an approximate analytical solution without loss of the basic characteristics of the system.

3. SIMULATION AND RESULTS

3.1 System response

The coupled, nonlinear equations are solved numerically using the modified Runge-Kutta-Gill's Method (Ref.15). The system parameters of the Indian Remote Sensing Satellite (IRS) used in the simulation are listed in Table 1. For this satellite, the attitude accuracy requirement is specified to be ±0.1 deg. in pitch, roll and yaw, and the body rates are to be limited to 3×10^{-4} deg/s to provide high quality imagery. For the simulations initial disturbance in the form of non-zero initial conditions in the satellite attitude angle and body rate are assumed. No environmental torque is taken into account since the control torque magnitude is fairly high as compared to these disturbances and the simulation is carried out over a short duration. The controlled attitude and body rate responses for the rigid body dynamical model (assuming the solar panels also to be rigid) are given in Fig.3. The initial conditions chosen are 0.2 deg in attitude and 3×10^{-3} deg/s in body rate about all the three-axes. The system undergoes many cycles of oscillation before settling down within the accuracy specifications, thereby showing that the damping is poor. The damping is about 5% in the beginning and reduces as time elapses. The frequency of attitude oscillation also reduces with time. This is due to the nonlinearity of the closed loop system. There is a bias of about 0.02° in the pitch motion in the steady state. This fact has been verified with the results of the analog simulation of the decoupled pitch equation. The step changes in body rate response (Fig.3b) are due to the triggering of the controller.

I_{xx}	I_{yy}	I_{zz}	m	l	b	Ω
990 Kgm^2	390 Kgm^2	990 Kgm^2	24 Kg	4.7 m	1.1 m	0.5 Hz

Table 1. System Parameters

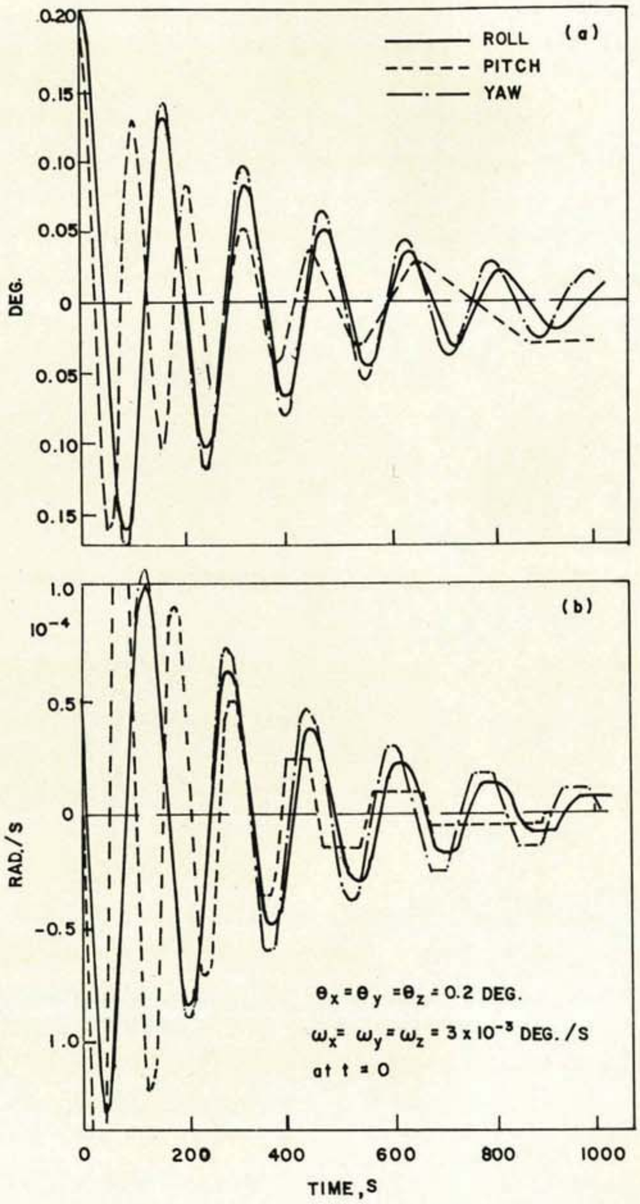

Figure 3. Response of Rigid Satellite : (a) Attitude Angles, (b) Body Rates

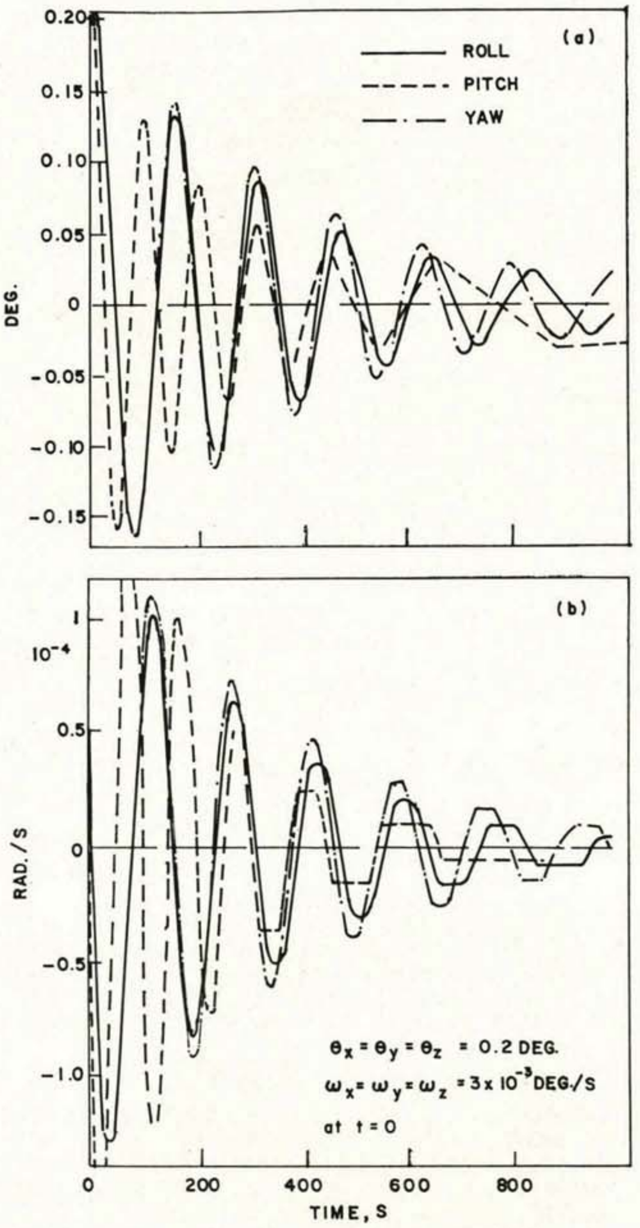

Figure 4. Response of Satellite with Flexible Panels : (a) Attitude Angles, (b) Body Rates

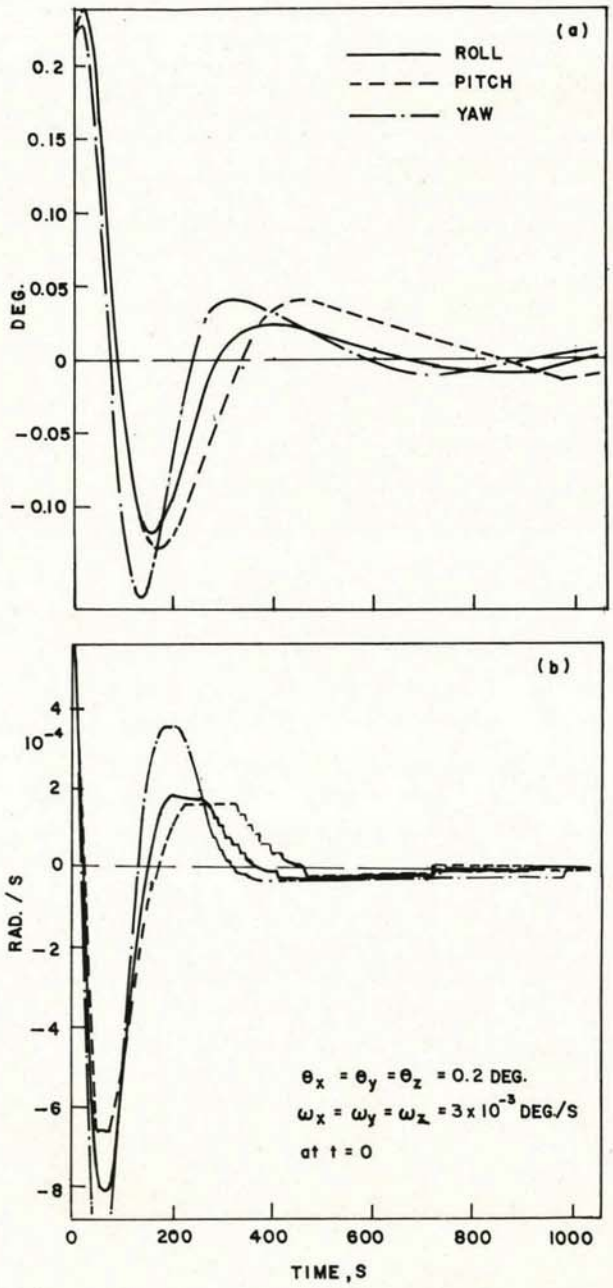

Figure 5. System Response with Kalman Observer : (a) Attitude Angles, (b) Body Rates

Fig.4 gives the attitude and body rate response for the model with the flexible panels for the same initial conditions as in the earlier case. The panel flexibility is represented by the first cantilever mode only. This is found to be sufficient in the present case as the controller and structural frequencies are well separated. The response is observed to be similar to that for the rigid body model (Fig.3) with minor variation in the roll and yaw motion. This implies that, for the system (IRS) parameters chosen, the panel flexibility doesnot affect the attitude behaviour significantly. It is also observed that the amplitude of tip motion is quite small and the frequency of vibration corresponds to the first cantilinear mode of the panel.

3.2 Performance of controller with Kalman Observer

One of the major difficulties in achieving the precise pointing stability arises from the noise in the attitude sensor. It also adds to the problem of control-structure interactions. A filter is often introduced in the feedback control loop to overcome this problem. A simple filter, however, is not adequate. For the system under consideration here, simulation is carried out by incorporating a 12th order Kalman filter which also acts as an observer to get the best attitude and rate estimate in the presence of sensor noise, momentum coupling and bias torques. The sensor noise is introduced through a random number generator. With the initial conditions remaining unaltered, the attitude and body rate response for the flexible model are given in Fig.5. It is seen that the system settles down quickly without under-going many cycles of oscillation (it may be compared with Fig.3 and 4). The body rates also come down quickly and their

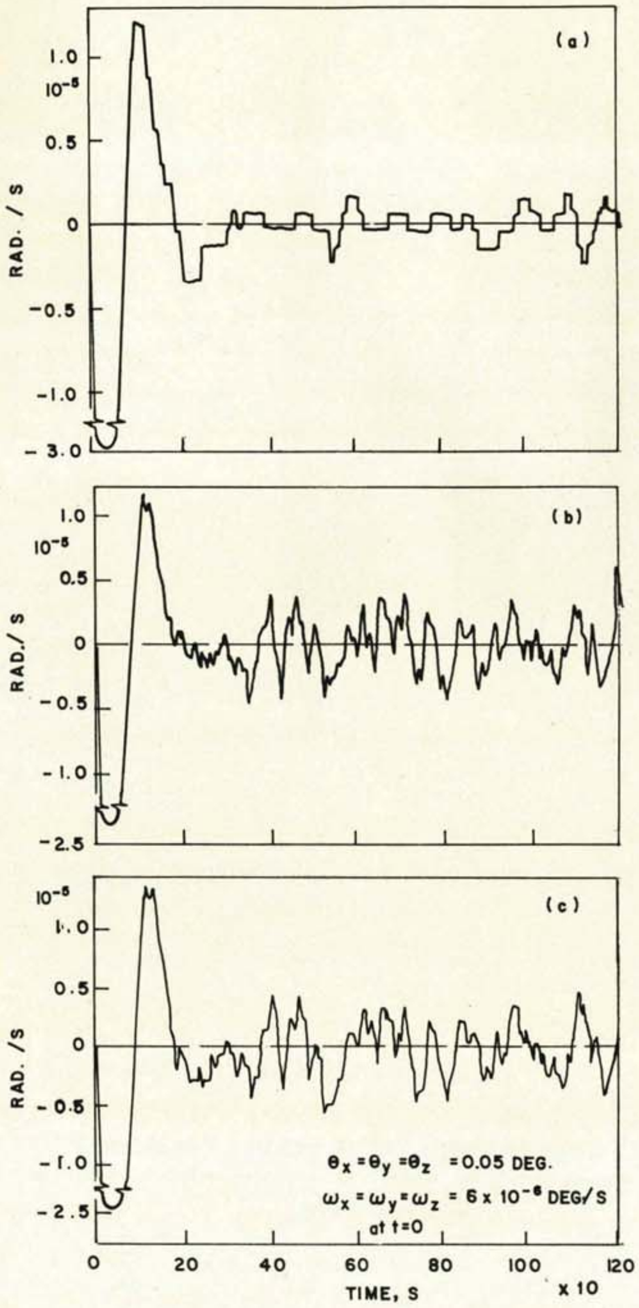

Figure 6. Roll Rate Response : (a) Rigid Satellite with Continuous Sensor Output, (b) Rigid Satellite with Discretized Sensor Output, (c) Satellite with Flexible Panels and Discretized Sensor Output.

magnitudes are within the specified limits in the steady state (3×10^{-4} deg/s). The damping increases considerably (around 40%). This is due to the introduction of the rate feedback from the Kalman observer. In the present case, the effect of flexibility is not found to be significant.

3.3 Effect of sampling and quantization in the sensor output

In the above analysis, the sensor output is assumed to be available continuously. In reality, however, only the quantized sensor output at discrete time intervals is available for the feedback. It is, therefore, necessary to consider the effects of sampling and quantization on the control system performance. For this study, an initial error of 0.05 deg. in the attitude and 6×10^{-6} deg/s in the body rate about all the three axes is assumed. This small error is chosen to study the steady state behaviour. The sampling and quantization does not seem to aggravate the attitude response beyond the accuracy limits. However, the body rate is affected adversely by the discretization, going beyond the specifications at times. Fig.6a and 6b compare the roll rate for the rigid body model with continuous and discretized sensor output. The discretization is also found to have a degrading effect on the flexible model (Fig.6c).

3.4 Effect of nonlinear controller on the structural motion

A typical vibration pattern of the panel (tip motion) is shown in Fig.7. It is seen that the amplitude of vibration pattern is random in nature. This is due to the action of the nonlinear controller. To understand this, consider the simple linearised vibration equation given by

$$N_r(\ddot{A}_{ij}+2\xi\Omega_{ij}\dot{A}_{ij}+\Omega_{ij}^2A_{ij}) = -P_1c\lambda\dot{\omega}_x+P_1s\lambda\dot{\omega}_z,$$

where, ξ = the structural damping (assumed to be 2% in the present case) and $P_1 = PM_{ij}+K_{ij}$. The right hand terms represent the forcing function arising out of attitude motion. Owing to the controller triggering (Pulsing), the body accelerations ($\dot{\omega}_x$ and $\dot{\omega}_z$) undergo sudden variation. The variations depend on the triggering of the roll and yaw controllers and their relative phase. For the nonlinear system, these are all unpredictable. This leads to the randomness in the vibration which, fortunately in the present case, are of very small amplitude.

4. CONCLUDING REMARKS

The attitude and vibration equations for a generic model representing a wide range of present and future satellites are derived using the Newton-Euler and Lagrangian approaches. The coupled nonlinear system equations along with the nonlinear controller are simulated numerically. It is shown that the introduction of rate feedback (derived from the Kalman observer) improves the system performance considerably. For the specific spacecraft considered here, there seems to be no problem due to control-structure interaction in the normal mode of operation. The introduction of sampling and quantization in the sensor output is shown to have a significant adverse effect on the system performance. In the present case, the effect of discretization is more prominent than the flexibility of the appendages. The nonlinearity of the attitude controller introduces randomness in the vibration of the panels. This may be of concern for more flexible spacecraft or for spacecraft with sensors mounted on the flexible appendages.

Acknowledgement : The study was supported by ISRO-IISc Space Technology Cell. The authors wish to thank Dr. P.K. Maharana and Dr. K.S. Chandra of the ISRO Satellite Centre for their suggestions and discussions during this study.

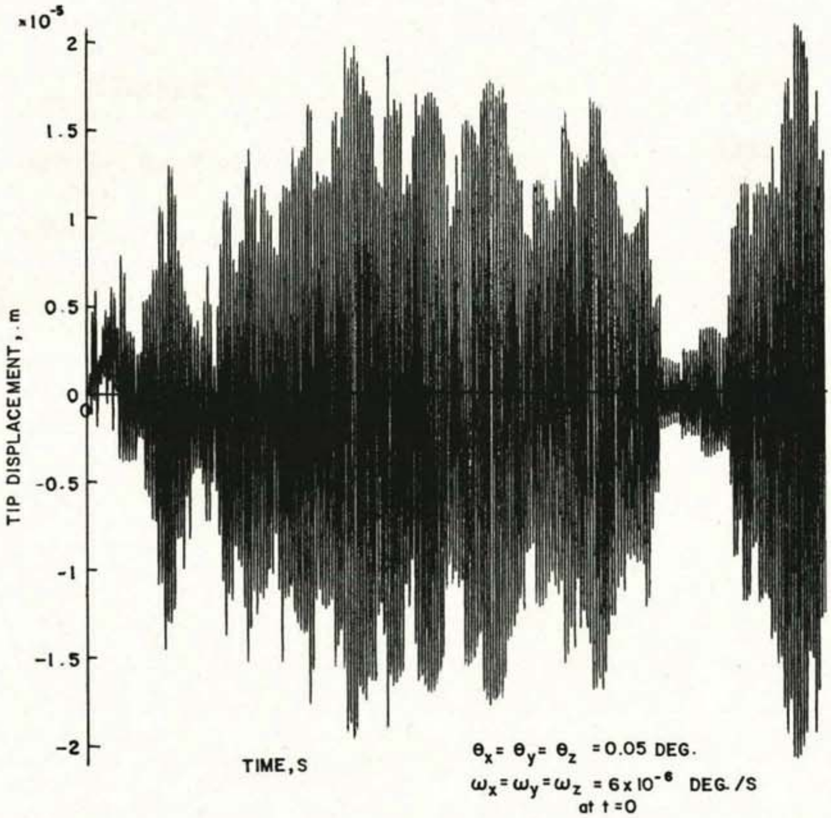

Figure 7. Tip Vibration of Flexible Panels

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