ON PRECISE ORBIT DETERMINATION OF NAVSTAR/GPS SATELLITES AND A RELATED EUROPEAN TRACKING NETWORK

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SUMMARY

European tracking networks for the determination of the orbits of NAVSTAR satellites of the Global Positioning System are designed and simulation studies are carried out. In particular, the computations are concerned with the effects of different error sources (unadjusted measurement and dynamic errors) on the orbit determination using single-difference phase and pseudo-range data at the proposed tracking stations.

As a preliminary result of the feasibility study the orbit can be determined in the 1 m level from each of the four considered networks in Europe allowing relative positioning in the 0.1 ppm range.

Keywords: Global Positioning System (GPS), orbit determination, geodetic positioning, European GPS tracking network

1. INTRODUCTION

Three major reasons force the *geodetic* user of the Global Positioning System (GPS) to study the orbit and its determination, respectively, of the NAVSTAR satellites more in detail.

(i) Whereas the real-time navigation users are satisfied with the readily available "broadcast ephemeris" modulated as so-called navigation message on the basic GPS signals, needs the geodesist and surveyor for precise positioning a higher accuracy of the orbits. Using the rule-of-thumb, see e.g. (Ref.6),

$$\frac{db}{b} = \frac{dr}{\rho} \qquad (1)$$

where b is the baseline length,
ρ is the receiver-satellite distance,
dr is the orbit error, and
db is the error of the baseline-length,

we get with $\rho \doteq 20\,000$ km the following table.

Table 1.

BASELINE ACCURACY AS FUNCTION OF ORBIT ERROR

db/b [ppm]	dr [m]
5	100
1	20
0.5	10
0.1	2

The accuracy of the broadcast ephemerides is presently estimated to be of the order of 40 - 100 m corresponding to 2...5 ppm in baseline determination. Geodesists and surveyors, however, strive for accuracies less than 1 ppm, for geodynamic applications even for 0.1 ppm.

(ii) Where the so-called "codeless receivers" are used, the GPS orbital information has to be provided from an external source. Customers of those instruments might be charged for the additional information.

(iii) Since the Global Positioning System is primarily a military system the availability of precise orbital information could be restricted to certain organizations somewhere in the future.

Above mentioned and other reasons therefore have led the authors to discuss the establishment of a GPS tracking network for Europe which could provide the users the necessary orbits for relative positioning in the 0.1 ppm level.
The paper under consideration summarizes first preliminary results of this study.

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2. ORBIT SENSITIVITY OF NAVSTAR SATELLITES

The forces acting on a GPS NAVSTAR satellite are discussed by several authors, see e.g. (Refs. 1-3, 6, 10, 12). Tables 2 and 3 show the orbit errors of the corresponding forces, which can be divided into *gravitational* forces caused by earth, sun, moon and other planets, their tidal effects (Tab. 2), and into the non-gravitational effects, like solar radiation pressure and albedo radiation (Tab. 3).

Table 2.

NAVSTAR SATELLITE GRAVITY ACCELERATION MODEL AND CORRESPONDING INFLUENCE ON THE ORBIT FOR A ONE-DAY ARC

Force description	Acceleration $[ms^{-2}]$	Orbit error [m]
Earth central body acceleration	0.59	∞
Acceleration due to earth's oblateness (C_{20})	$5\cdot10^{-5}$	10 000
Acceleration due to non-sphericity of the earth (excluding C_{20})	$3\cdot10^{-7}$	200
Attraction by moon	$5\cdot10^{-6}$	3 000
Attraction by sun	$2\cdot10^{-6}$	800
Solid earth tides	$1\cdot10^{-9}$	0.3
Ocean tides	$0.5\cdot10^{-9}$	0.04

The quantities in Table 3 are determined using an area-to-mass ratio of the satellites of $1.24\cdot10^{-2}\ m^2\ kg^{-1}$ and a coefficient for the surface reflectivity of 0.5. The radiation intensity values were taken from Refs. 2, 11. Since the goal of our orbit determination is ± 2 m (corresponding to 0.1 ppm in relative positioning) other effects than those mentioned in Tables 2 and 3 seem to be negligible.

Table 3.

NAVSTAR SATELLITE NON-GRAVITATIONAL MODEL AND CORRESPONDING INFLUENCE ON THE ORBIT FOR A ONE-DAY ARC

Force description	Acceleration $[ms^{-2}]$	Orbit error [m]
Direct solar radiation pressure ($P_{sun} = 1367\ [W m^{-2}]$)	$6\cdot10^{-8}$	200
Albedo pressure ($P_{alb} = 10\ [W m^{-2}]$)	$4\cdot10^{-10}$	0.03

3. ORBIT ESTIMATION MODEL

3.1 Variational Equations

In the following we will briefly outline the orbit estimation model. Readers who are more interested in details are referred to (Refs. 4, 5, 9).

The motion of a satellite can be described by a partial second-order differential equation of the form

$$\ddot{\underline{r}} = \ddot{\underline{r}}(\underline{r}, \dot{\underline{r}}, \underline{q}) \tag{2}$$

where r is the satellite position vector, $\dot{\underline{r}} = \partial\underline{r}/\partial t$ the corresponding velocity vector, and $\ddot{\underline{r}} = \partial^2\underline{r}/\partial t^2$ is the satellite's acceleration. The vector $\underline{q}$ is defined by

$$\underline{q} = \begin{bmatrix} \underline{x}_o \\ \underline{a} \end{bmatrix} \tag{3}$$

where $\underline{x}_o$ represents the initial state vector at time t_o of the considered arc, and which depends again on the satellite's position vector $\underline{r}_o$ and corresponding velocity vector $\dot{\underline{r}}_o$, both at $t = t_o$. All coordinates are referring to an inertial reference frame. The vector $\underline{a}$ is a dynamical parameter vector determined by the solar radiation pressure C_r, (certain) spherical harmonic coefficients C_{nm}, S_{nm} of the earth's gravity field, etc.

$$\underline{a} = \underline{a}(C_r, C_{nm}, S_{nm}, \ldots) \ . \tag{4}$$

Applying optimal suited higher-order predictor-corrector techniques, see e.g. (Ref. 4), we get the position vector $\underline{r}$ and the velocity vector $\dot{\underline{r}}$ of the satellite at time $t > t_o$ starting the integration at t_o and using the initial state $\underline{x}_o$.

Differentiation of (2) with respect to $\underline{q}$ yields the matrix differential equation of partial derivatives

$$\frac{\partial\ddot{\underline{r}}}{\partial\underline{q}} = \frac{\partial\ddot{\underline{r}}}{\partial\underline{r}}\frac{\partial\underline{r}}{\partial\underline{q}} + \frac{\partial\ddot{\underline{r}}}{\partial\dot{\underline{r}}}\frac{\partial\dot{\underline{r}}}{\partial\underline{q}} + \frac{\partial\ddot{\underline{r}}}{\partial\underline{a}} \ . \tag{5}$$

This relation is called the "variational equation of satellite geodesy".

Using matrix notation we get with

$$\underline{A} = \frac{\partial\ddot{\underline{r}}}{\partial\underline{r}}\ , \quad \underline{B} = \frac{\partial\ddot{\underline{r}}}{\partial\dot{\underline{r}}}\ , \quad \underline{C} = \frac{\partial\ddot{\underline{r}}}{\partial\underline{a}}$$
$$\underline{Z} = \frac{\partial\underline{r}}{\partial\underline{q}}\ , \quad \dot{\underline{Z}} = \frac{\partial\dot{\underline{r}}}{\partial\underline{q}}\ , \quad \ddot{\underline{Z}} = \frac{\partial\ddot{\underline{r}}}{\partial\underline{q}} \tag{6}$$

the relation

$$\ddot{\underline{Z}} = \underline{A}\cdot\underline{Z} + \underline{B}\cdot\dot{\underline{Z}} + \underline{C} \tag{7}$$

where the first six columns of matrix C are null-vectors.

Since we do not expect significant velocity-dependent terms for GPS satellites the following simplification is justified.

$$\ddot{\underline{Z}} = \underline{A} \cdot \underline{Z} + \underline{C} \quad (8)$$

This variational equation is integrated by use of numerical techniques which yield the matrix $\underline{Z}$ as a function of time. In detail, it is defined for the time interval (t_o, t), where t_o is the initial epoch and t the final epoch of the integration process. The initial epoch t_o does not coincide, in general, with the time when the observations start.

Let be t_n the time of a satellite observation l. Then the matrix $\underline{Z}(t_n,t_o)$ is needed for the set-up of the design matrix for the orbit adjustment process. From the integration mentioned above the matrices $\underline{Z}(t_n,t_{n-1})$, $\underline{Z}(t_{n-1},t_{n-2})$, ..., $\underline{Z}(t_1,t_o)$ are resulting referring to the specific time intervals. Thus, the matrix $\underline{Z}_n(t_n,t_o)$ is determined applying the chain-rule of differentiation.

$$\underline{Z}(t_n,t_o) = \underline{Z}(t_n,t_{n-1})\,\underline{Z}(t_{n-1},t_{n-2}) \cdots \underline{Z}(t_1,t_o) \quad (9)$$

The partial derivatives of the observations l with respect to the vector $\underline{q}$ are given by

$$\frac{\partial l}{\partial \underline{q}} = \frac{\partial l}{\partial \underline{r}} \frac{\partial \underline{r}}{\partial \underline{q}} \quad (10)$$

3.2 Modelling GPS Observables

In our preliminary study we like to consider two different types of observation techniques for tracking GPS satellites: pseudo-range observables and single-difference (between-station-difference) phase observables. For the simulation it is sufficient to deal with the orbit determination of *one* satellite. This also justifies the single-differenced phases as "observations", and excludes higher-order phase differences (double, triple). The last have to be considered in cases where insufficient time-keeping $(< 10^{-13})$ at receiver's sites takes place, so that those errors cancel widely out.

Pseudo-range observation

The range measurement is described by the relation

$$\rho_i^j = |\underline{r}^j - \underline{r}_i| + c(\delta t^j - \delta t_i + T_i^j + I_i^j) \quad (11)$$

where
- ρ_i^j is the pseudo-range between receiver i and satellite j,
- $\underline{r}^j$ is the position vector of satellite j,
- $\underline{r}_i$ is the position vector of receiver i (ground station),
- δt^j is the satellite synchronization error,
- δt_i is the receiver synchronization error,
- T_i^j is the tropospheric propagation delay,
- I_i^j is the ionospheric propagation delay, and
- c is the speed of light.

The time synchronization errors are modelled by quadratic polynomials.

Receiver clock error model

$$\delta t_i = a_o + a_1(t_i - t_o) + a_2(t_i - t_o)^2 \quad (12)$$

Satellite clock error model

$$\delta t^j = b_o + b_1(t^j - t_o) + b_2(t^j - t_o)^2 \quad (13)$$

a_o, a_1, a_2 and b_o, b_1, b_2 are unknown coefficients. The simple modelling of (12) and (13) can be justified by the assumption that the ground control stations are equipped with Rubidium clocks and/or hydrogen maser standards showing a frequency f stability of about $2 \cdot 10^{-14}$ (Ref. 8). The corresponding clock errors

$$\frac{\Delta t}{t} \doteq \frac{\Delta f}{f} \doteq 2 \cdot 10^{-14} \quad (14)$$

associate a range error of 0.18 m for a single satellite pass, 0.52 m for a one-day arc and 2.59 m for a five-day arc. Since the major part of the clock error can be approximated by a simple drift parameter and an offset, possible lack in sufficient modelling due to (12) and (13) will result in a range error considerable less than 10 cm.

The atmospheric propagation time delays T_i^j and I_i^j are computed using specific models and dual-frequency measurements. It will be discussed later on.

Single-difference observation

The unlinearized observation equation for single-difference phase observations at epoch t is given by (see also Ref. 5)

$$\psi^j(t) = f^j[\delta_2 t(t) - \delta_1 t(t)] + f^j c^{-1}[\rho_1^j(t) - \rho_2^j(t)] + f^j[T_{1,2}^j(t) + I_{1,2}^j(t)] + N^j \quad (15)$$

where
- f^j is the frequency of the oscillator of satellite j,
- $\delta_1 t$, $\delta_2 t$ are the receiver clock errors at station 1 and 2, modelled by polynomials (12) and (13),
- ρ_1^j, ρ_2^j are the slant ranges between satellite j and receiver 1 and 2, respectively,
- $T_{1,2}^j$ is the tropospheric time delay effect of stations 1 and 2 with respect to satellite j,
- $I_{1,2}^j$ is the corresponding ionospheric time delay,
- c is the speed of light, and
- N^j is the unknown phase ambiguity.

Unknown parameters in the single-difference observation (15) are the position of satellite j, the clock error polynomial coefficients (see (12) and (13)), and the phase ambiguity. In contrast to the pseudo-range observation (11) the satellite clock error does not affect (15).

Let us assume that dual-frequency measurements were carried out so that the ionospheric effects can be sufficiently corrected. The "corrected" single-difference observation $\bar{\psi}$ is formed by a linear combination of the two single-differences ψ_{L_i} , $i = \{1,2\}$.

$$\bar{\psi}^j(t) = \alpha_1 \, \psi^j_{L_1}(t) + \alpha_2 \, \psi^j_{L_2}(t) \quad (16)$$

with

$$\alpha_1 = \frac{f_1^2}{f_1^2 - f_2^2} \quad (17)$$

$$\alpha_2 = \frac{-f_1 f_2}{f_1^2 - f_2^2} \quad (18)$$

where

$$f_1 = 154 \times 10.23 \text{ mHz} = 1575.42 \text{ mHz}$$
$$f_2 = 120 \times 10.23 \text{ mHz} = 1227.60 \text{ mHz} .$$

Observation errors

According to (Ref. 7) the pseudo-range measurement noise can be assumed to be ± 1 m. Although the phase measurement noise is only of the order of millimeters, we will deal in our computations with ± 2 cm in order to be on the save side. As mentioned above we will further assume that the atmospheric effects are corrected by using dual-frequency data. Possible remaining errors are assumed to be of the order of 5 % of the correction.

Solution technique

For the solution of the adjustment process of orbit determination we used the "square-root" formulation, equivalent to the commonly-used procedure of building up normal equations. The reason for that is its advantage of numerical stability and accuracy (Ref. 9). Unknowns in the adjustment process are the satellites' initial state vector, clock errors and phase ambiguities.

4. ORBIT ERROR ANALYSIS

For the simulation various computations were carried out in order to analyse the effect of different error sources and of tracking network geometry on orbit determination.

4.1 Design of Tracking Network

Three different European tracking networks are selected. Each of them varies in size, however, all consist of four tracking stations. As far as it was possible we considered locations where already existing space activities (satellite laser ranging, Very Long Baseline Interferometry - VLBI) take place and precise geocentric coordinates are available. A fourth network contains the stations of network 2 and two additional stations (Grasse, Wettzell), in order to get some statements concerning number of stations versus orbit accuracy from the computations.

The following networks were formed (see also Fig. 1).

Network 1. Wettzell (D), Grasse (F), Jodrell Bank (GB), and Kiruna (S)

Network 2. Jodrell Bank (GB), Kiruna (S), Madrid (E), and Dionysos (GR)

Network 3. Hammerfest (N), Canary Islands (E), Iceland (IS), and Dionysos (GR)

Network 4. Network 2 - stations, and Wettzell (D), Grasse (F)

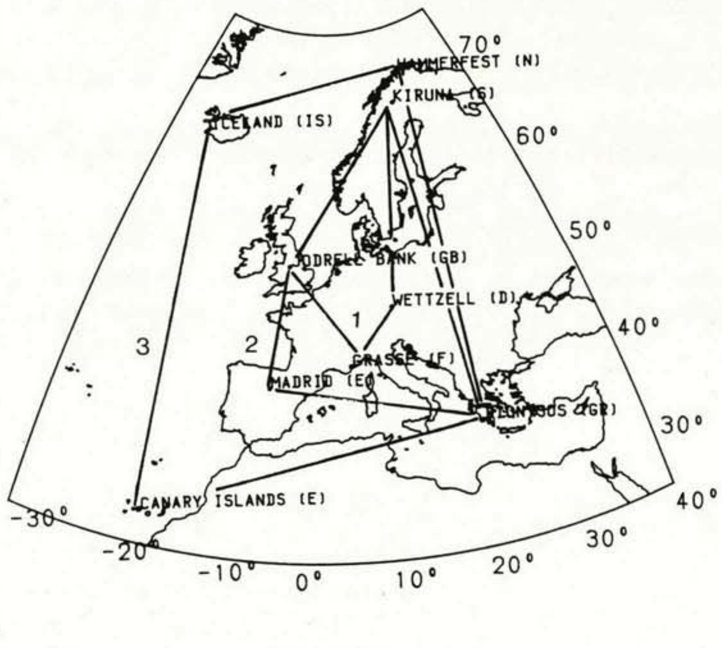

Figure 1. Considered European Tracking Networks in the GPS Orbit Determination Feasibility Study

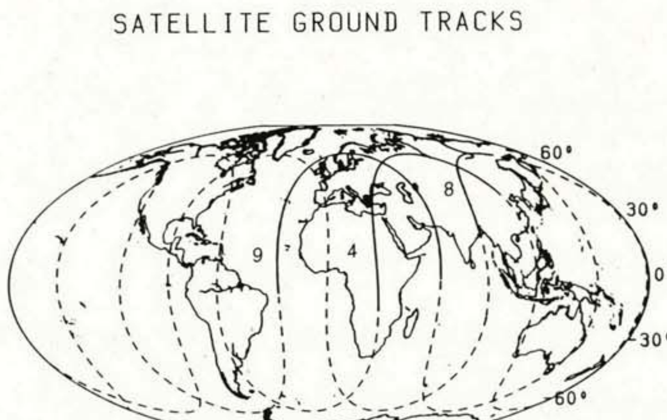

Figure 2. Ground Tracks of Navstar # 4, Navstar # 8 and Navstar # 9

Whereas network 1 and 2 consist of more or less already established space observation stations, is network 3 formed of stations without any space activities at present (except Dionysos, GR). It seems, however, preferable at the first look due to its geometry.

4.2 Effect of Different Error Sources on GPS Orbit Determination

In our simulation we considered the effect of following error sources:

Unadjusted measurement errors

Tracking station coordinate uncertainty:	10 cm in (x,y,z)
Measurement noise: pseudo-ranging	1 m
Measurement noise: single-differences	2 cm
Tropospheric refraction:	5 % of the effect
Ionospheric refraction:	5 % of the effect

Unadjusted dynamic errors

Solar radiation pressure:	5 % of the effect
Gravity field:	5 % of the lumped potential error due to differences in gravity field models (Ref. 11)
Pole position:	0.003 arcsec

4.3 Simulation Techniques

In order to analyse the effect of different error sources on orbit determination we were using the following procedure.

Starting with an initial state vector for the different satellites derived from broadcast ephemeris data we were generating a "perfect errorless" orbit. This was achieved by applying a numerical integration technique and considering the different forces given in tables 2 and 3. For further information about the considered force model the reader is referred to (Ref.6).

The tracking network stations together with the "true" orbit were then used to calculate pseudo-range and single-difference observations. Noisy measurements were derived from these "true" observations by using a random number generator and an appropriate atmospheric model.

The effect of unadjusted dynamic errors was modelled by applying a 5 % change to the solar radiation pressure constant and a 0.003 arcsec change to the pole position. The effect of insufficient knowledge of the earth's gravity field was considered by using different earth models for generating the "true" orbit and determining the orbit by simulated measurements.

The tables presented in the following contain the root mean square errors for the along, cross and radial component of the orbit.

4.4 Analysis Results

For the tracking networks described in chapter 4.1 and NAVSTAR # 4 satellite the effect of unadjusted measurement errors as well as dynamic errors on its orbit were computed for a four-hour arc. The corresponding ground track of NAVSTAR # 4 is graphically shown in Figure 2. The results of the computations are summarized in Tables 4 and 5.

For each network and each error component the best results are obtained by combining pseudo-range and single-difference observations. Whereas measurement noise and uncertainty of station position have relative small influence on orbit accuracy the residual error of atmospheric corrections affects satellite position accuracy on the 30 cm level. In spite of the geometric superiority of network 3 the results of it are worse in comparison to network 2, if single-difference observations are used. This is due to the fact that the number of single-differences is much smaller for network 3 than for network 2.

The orbit accuracy improvement by adding two more tracking stations in network 4 is very small. The results are given in Table 6.

The effect of dynamic errors is given in Table 5, which is nearly identical for all considered networks. The magnitude of these influences is small in comparison to the assumed atmospheric effects.

In order to analyse the effect for different satellite passes we considered two additional satellites and computed rms-errors for different error sources. The results are given in Table 7. The ground tracks of all satellite passes are graphically described in Figure 2.

Since we supposed that the rms-error derived from the adjustment process does not represent the actual orbit error we defined a different error estimate which was given in Table 6. These values were derived by comparing the orbital plane components of "true" and "computed" orbit for each measurement epoch corresponding to the "true" initial orbital plane state vector. Therefore they are representing the ability of our model to reproduce the "true orbit" from noisy measurements much better than the rms-values do.

Table 6 shows that the accuracy of the computed orbit in the considered case is of the order of 1 m.

Table 5.

EFFECT OF UNADJUSTED DYNAMIC ERRORS ON NAVSTAR # 4 ORBIT FOR A FOUR-HOURS ARC IN CENTIMETERS (RESULTS ARE NEARLY IDENTICAL FOR ALL CONSIDERED NETWORKS)

Error source	RMS error [cm]		
	Radial	Along	Cross
Gravity field	2	1	3
Pole position	15	10	8
Solar radiation	1	1	1
Total	16	11	10

Table 4.

EFFECT OF UNADJUSTED MEASUREMENT ERRORS ON NAVSTAR # 4 ORBIT FOR A FOUR-HOURS ARC IN CENTIMETERS (RMS-VALUES)

Error source	Network 1			Network 2			Network 3		
	Radial	Along	Cross	Radial	Along	Cross	Radial	Along	Cross
Pseudo-range data:	1139 observations			1142 observations			1073 observations		
Measurement noise	799	333	537	623	317	448	427	163	301
Station position	81	34	54	63	23	45	43	17	31
Tropospheric refraction	232	97	156	183	64	132	126	48	89
Ionospheric refraction	397	154	248	294	102	212	189	72	134
Total	898	374	603	704	245	507	480	183	339
Single-difference data:	785 observations			735 observations			575 observations		
Measurement noise	3	1	2	2	1	2	2	1	2
Station position	18	9	15	13	6	12	17	7	14
Tropospheric refraction	51	24	43	35	18	32	49	21	42
Ionospheric refraction	69	33	58	46	23	41	48	20	41
Total	82	39	69	56	28	50	67	28	57
Pseudo-range and single-difference data:									
Measurement noise	2	1	2	2	1	2	2	1	2
Station position	10	4	8	8	3	6	9	3	7
Tropospheric refraction	26	12	20	22	9	18	26	10	22
Ionospheric refraction	38	17	29	27	12	22	25	10	21
Total	46	20	35	33	14	28	36	14	39

Table 6.

COMPARISON OF RMS AND "TRUE" ERROR FOR NETWORK 4 AND ONE PASS OF NAVSTAR # 4
(1724 PSEUDO-RANGES AND 1273 SINGLE-DIFFERENCES)
RMS = ROOT MEAN SQUARE ERROR FROM ADJUSTMENT PROCESS
"TRUE"ERROR = $\pm\ (\Sigma\varepsilon\varepsilon/n)^{0.5}$ WITH
ε = "TRUE"-COMPUTED

Error source	RMS error [cm]			"TRUE" error [cm]		
	Radial	Along	Cross	Radial	Along	Cross
Measurement noise	2	1	2	7	4	2
Station position	7	3	6	15	8	25
Tropospheric refraction	18	8	15	14	11	46
Ionospheric refraction	26	11	21	119	101	40
Total	29	13	24	20	9	173

Table 7.

EFFECT OF UNADJUSTED MEASUREMENT ERRORS ON NAVSTAR # 8 AND # 9 FOR ONE PASS IN CENTIMETERS USING TRACKING NETWORK 2 (RMS-ERROR)

Error source	Orbit NAVSTAR # 8			Orbit NAVSTAR # 9		
	Radial	Along	Cross	Radial	Along	Cross
Measurement noise	2	3	4	0	0	1
Station position	10	10	14	2	1	4
Tropospheric correction	43	45	62	4	2	8
Ionospheric correction	10	16	23	2	1	4
Total	46	48	66	5	2	10
Pseudo-ranges	565			1161		
Single-differences	238			822		

Figure 3. Actual status of stations in Europe where through ground laser and VLBI techniques precise geocentric coordinates are available.

5. CONCLUSIONS AND FURTHER ACTIVITIES

Looking onto the results in Tables 4 to 7 the goal of determining a GPS orbit better than 2 m as a prerequisite for relative positioning in the 0.1 ppm level can be considered as achievable. Moreover, by combining between-station-to-station phase differences (single-differences in phase) and pseudo-range observations the orbit can be determined in the 1 m level using anyone of the considered European tracking networks. Differences in accuracy of the orbit between the four networks are so small, that no considerable advantages for a certain tracking network can be given, except the reason of presently already available space instrumentation, and, consequently, precise geocentric coordinate information.

Thus, Fig. 3 shows the actual status of stations in Europe where through ground laser and VLBI techniques precise geocentric coordinates within the 10 cm - level are available. Two GPS orbit determination tests are still planned for this year - so far no further break-down of one of the few GPS satellites is happening. During the establishment of a precise zero-order threedimensional network with average side lengths of about 100 km in West Germany the stations *Onsala (S)*, *Kootwijk (N)*, *Wettzell (D)*, *Lustbuehel-Graz (AUS)* and *Zimmerwald (CH)* will permanently record the selected four GPS satellites using Texas Instruments TI 4100 receivers and precise frequency standards. Although the area covered by these stations is extremely small, we hope to determine (may be better to improve) the orbits to such an extent to permit positioning in the 10^{-7} (0.1 ppm) range.

A second European campaign is concerned with the observation of more or less a traverse of stations between England and Austria.

Also here an orbit determination of GPS satellites is anticipated. Final goal of those studies should - at least in the opinion of the authors - result in the establishment of a European GPS tracking network which could provide European users of the U.S. military GPS/NAVSTAR satellites the necessary precise orbit information.

In the meantime also experiences of a GPS orbit determination test by U.S. non-government institutions are available, see, e.g., Refs. 13-15.

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