ATTTITUDE RECONSTRUCTION FILTERS: YES OR NO?

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ABSTRACT

We analyse and compare the outcome of six studies dealing with attitude estimation for spinning satellites by means of different filtering approaches. These studies have been sponsored by ESRO/ESA between 1970 and 1980. We highlight the communalities which could explain the poor attitude estimation performance which has been found in almost all cases. It is shown that increased modelling complexity brought about by including either the nutation dynamics or sensor biases, leads to estimation inability in the former case and increased inaccuracy in the latter case. We suggest that the accuracy decrease experienced by supplementary bias estimation is a natural phenomenon rather than a linearisation problem. Supporting evidence is provided by numerical experiments described in the paper.
Keywords: attitude determination, bias estimation, Kalman filtering, recursive filtering, spin axis attitude.

1. INTRODUCTION

Since the introduction of the Kalman filter (Ref. 1) in 1960 a real revolution went on in automatic control theory and engineering. It was immediately recognised that Kalman filtering could also have major merits for many estimation problems which had no automatic control aspect. Quite soon advanced textbooks on estimation only, devoted a number of chapters to filtering, e.g. R. Deutsch in 1965 (Ref. 2). It was therefore not surprising that interest for this new technique arose also in the area of satellite attitude estimation. An early example of this interest was the study of J. L. Farrell completed for NASA in 1966 (Ref. 3). Already in that study, the themes we wish to address in this paper were apparent.

The first theme was the motivation for attitude determination by Kalman filters. Was it not possible, just by moving complexity to ground processing, to mount cheaper and slightly less accurate attitude sensors on a satellite and nevertheless reach the same low estimation error as by using costly and precise sensors? The complexity meant was the introduction of Kalman filters applied to large sequences of measurements. The conclusion of Farrell was not convincing, because many of the cases he treated failed either to converge or did not lead to the expected accuracy levels for the estimates.

The second theme referred to the attempt to derive the full attitude dynamics of spinning and nutating satellites from the sparsely sampled attitude angular measurements. Already Farrell experienced some misfortunes in trying to perform such a challenging estimation. We do not know of later really successful attempts by other authors.

The first disappointments were offset by the ingenuous idea to add measurement biases to the state vector (Ref. 4). By estimating such biases accurately, it was hoped to get rid of the consequences of bias errors which especially in the area of attitude sensing, had emerged as a major hindrance for the improvement of the actual attitude accuracy. This brings us to the point where ESRO - the predecessor organisation of ESA - and the European industry stepped in into a series of studies and attempts to use Kalman filtering not only for automatic control, but also for reaching better attitude estimation accuracies for spin stabilised satellites.

The present paper proposes to give a brief survey of these studies limited to spin stabilisation and to the utilisation of measurements from optical sensors. The attitude we are looking for is essentially the direction of the angular momentum vector. Its orientation and size is fixed in inertial space provided no external torques act on the craft. From theory and experience we know that the torques in question are all negligeable for the time intervals (a few hours) considered for the filtering process. Only for very low earth orbit arcs, the effect of perturbing torques may become visible within the time frames we are interested in. Such cases have not been studied by means of filter estimation and they also bear little practical interest. We can thus exclude the external torques from our considerations.

If a rigid body is made spinning around its largest principal moment of inertia and there are no external torques, then this body will remain spinning around the same axis keeping the inertial direction of that axis as well as the size of the spin rate. This is 'spin stabilisation'. Spin stabilisation is not only a cheap means to orient a spacecraft in a controlled way, it also recovers

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from disturbances in the pure spin motion (nutation), provided liquids or movable parts provide some energy dissipation. Now, all modern spin stabilised satellites have effective nutation dampers. Therefore, nutation is, in practice, only observed for a very short time beyond a manoeuvre involving thrusters (usually much less than 1 hour). If we dwelled somewhat longer on this aspect, it is to demonstrate that for more than 99.9% of the life of a spinning stable satellite, spin axis and angular momentum vector coincide almost perfectly. Thus it is the spin axis, characterised by a right ascension ρ and a declination δ, (i.e. two parameters) which is of practical value. The spin rate is an even more basic attitude parameter whose value can normally be fully decoupled from ρ and δ. Reaching the highest possible accuracy for the (ρ,δ) estimates has been and is still the aim of attitude estimation within the frame of a given sensor hardware and the mission orbit characteristics. Estimation of the full dynamics, or of a set of biases has always been undertaken in the hope to improve upon the accuracy of (ρ,δ). In other words, attitude filtering has been studied to assess the potential accuracy increase one could expect to obtain from the estimation of a more complete model. In estimation theory we say that we introduce more unknowns and in control theory we speak of a state vector extension.

The presentation of our survey requires first the cursory description of attitude dynamics and the attitude sensing which is common and relevant for the discussion of past studies. This is done in section 2. The very basic elements of filtering are repeated in section 3. It serves to define useful notations and to draw the attention to the simplifications which can be made when computing the attitude. These simplifications depend on the approaches which have been analysed in the studies at hand. In section 2 and 3 we are also defining some critical standpoints which are immediately relevant for the case studies and must be considered as an element of the survey rather than a general introduction to attitude and filtering. In section 4 we summarise the pecularities and findings of the case studies which all resulted from ESRO/ESA study contracts. In section 5 we evaluate the common merits and weaknesses of the different filter implementations and compare them with our practical use of filters and other methods. Before to go to the conclusion, section 6 describes the results of a few numerical experiments we made in support to our critical assessment of filter weaknesses.

2. THE ATTITUDE OF SPINNING SATELLITES

2.1 Attitude Measurement Geometry

It is customary to describe the angular momentum orientation by a unit vector which we denote by $\underline{Z}$. It is defined by ρ and δ in an inertial reference frame which we tacitly assume to be the same in all what follows. The 'attitude' $\underline{Z}$ is measured by finding the angles of two inertial directions with respect to $\underline{Z}$. In 5 of the 6 cases analysed, one direction is the sun, characterised by the unit vector $\underline{S}$. The other direction denoted by the unit vector $\underline{E}$ is either the earth direction (albedo and infrared sensors), or a star or planet direction (star mapper sensors). The directions $\underline{Z}$, $\underline{S}$ and $\underline{E}$ build a spherical triangle shown in Figure 1.

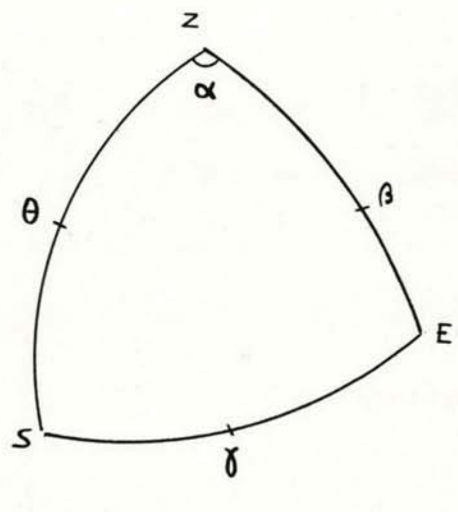

Figure 1. Basic attitude measurement angles of a spinning satellite

The angles $\theta = \arccos(\underline{Z}.\underline{S})$ and $\beta = \arccos(\underline{Z}.\underline{E})$ are (sun-, earth-, or star-) colatitudes which are measured directly or indirectly as we will explain in the next subsection. The vectors $\underline{S}$ and $\underline{E}$ as well as $\gamma = \arccos(\underline{S}\cdot\underline{E})$ are perfectly known. The azimuth angle α represents the rotation angle of the satellite which is necessary to move a given meridian (through $\underline{Z}$) from the intersection with $\underline{S}$ to the intersection with $\underline{E}$. It has usually not been provided on the satellites of the sixties, but nowadays the azimuth angle α is generally measured by all spinning spacecraft. Its importance is obvious from the equation

$$\underline{Z}\cdot(\underline{S}\times\underline{E}) = \sin\alpha\,\sin\beta\,\sin\theta \qquad (1)$$

which together with

$$\underline{Z}\cdot\underline{S} = \cos\theta,\ \underline{Z}\cdot\underline{E} = \cos\beta \qquad (2)$$

allows the unambiguous determination of $\underline{Z}$ provided $\sin\gamma \neq 0$. If we have only the pair of equations of (2) together with $||\underline{Z}|| = 1$, the solution is ambiguous and is only defined if $\underline{Z}$, $\underline{S}$ and $\underline{E}$ are not coplanar, i.e. if $\underline{Z}\cdot(\underline{S}\times\underline{E}) \neq 0$.

2.2 Attitude Measurement Sensing

In our survey, only the sun colatitude θ has once been measured directly by a digital sun sensor, namely in the case of HEOS A2. Usually the sun colatitude is measured by a V-slit sensor. Such a sensor is composed of two independent straight slit (= fan beam) sensors. On the unit sphere a straight slit is equivalent to the arc of a great circle centered around the center of mass of the spacecraft. In all the cases at hand one of these slits has also to play a role in attitude and orbit manoeuvres, and therefore it is mounted parallel to the intended body spin axis, or largest principal moment of inertia. it is called the meridian slit and it is complemented by a skew slit as shown in Figure 2.

Whilst the satellite rotates, one slit - say the skew slit - crosses the sun at time t_{so}, shortly later the meridian slit crosses the sun line at t_{mo}. These times are registered in the telemetry. From the sequence $t_{mo}, t_{m1}\ldots$ we obtain the spin rate and from the differences $t_{mi} - t_{si}$

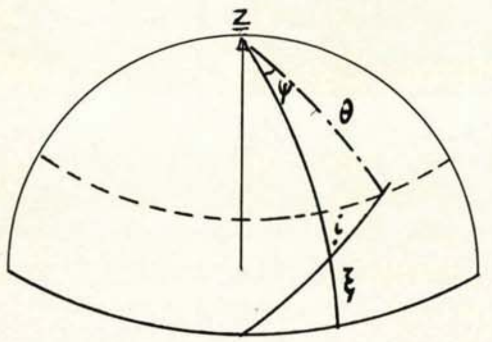

Figure 2. V-slit sensing principle

we derive an azimuth angle ψ which in turn leads to θ by a simple trigonometric relation involving the relative slit inclination i, and the declination ξ of the slit intersection with respect to the spacecraft equator.

Another way to introduce the measurements of a V-slit sensor into the attitude calculation is to stay with the times t_{mi} and t_{si}. If $\underline{N}_m$ and $\underline{N}_s$ are the unit normals on the great circle planes of the meridian and skew slit respectively, then we have $\underline{N}_m . \underline{S} = 0$ and $\underline{N}_s . \underline{S} = 0$ at the times t_{mi} and t_{si}. By utilising this fact and expressing $\underline{N}_m$ and $\underline{N}_s$ as functions of $\underline{Z}$ and time we come to a representation

$$t_{im/s} = f_{m/s}(\underline{Z}, \underline{S}, \varphi, \psi) \qquad (3)$$

where ψ is the spin rate and φ is a reference rotation phase angle at a reference time. This is the way all the studies but one (Ref. 14), introduced V-slit measurements in the filtering.

The ESA-star mapper uses the V-slit principle as well. The albedo sensors on HEOS A2 and COSB were also V-slit sensors. The principle of (3) still applies, but a substantial complexity is introduced by the finite size of the earth and the fact that only the sunlit part of the earth disc triggers the albedo sensors. Details of the sunlit phase sensing geometry can be found in Ref. 5.

It may surprise, but also a V-slit infrared earth sensor configuration has been the subject of an attitude filtering study (Ref. 10) as we will see in section 4. Such a sensor configuration has never flown, however. The satellites SIRIO1/2 carried only infrared meridian fan beams, and not the skew counter part.

Besides the family of V-fan beam sensors we have the infrared pencil beam sensors which, with slight variations, are utilised world wide on most of the satellites which are spinning throughout a ground controlled geostationary transfer orbit. The pencil beam rotates with the satellite and if the spacecraft-spin-axis-earth configuration is favourable (i.e. if there is measurement 'coverage'), the sensor will return a time $t_{S/E}$ when the beam enters the apparent earth disc and again a time $t_{E/S}$ when the sensor makes the transition earth/space. If at that instant the known apparent radius of the earth is R, the measurement equation for filtering is derived from the condition that $\cos R = \underline{P}.\underline{E}$ at the instants $t_{S/E}$ and $t_{E/S}$. The vector $\underline{P}$ is along the optical axis of the sensor and its orientation is a function of time and the spin axis $\underline{Z}$. With simple spherical trigonometry it is also possible to compute β directly. The attitude estimation methods which work with θ and β directly can be given the generic name 'geometrical methods'.

2.3 Attitude Dynamics

In all case studies, there is a simulation part which has been necessary to validate the estimation filter. In these simulations nutation is always an option. Only in three studies (Refs. 10, 12, 14) we find an attempt to estimate the dynamics of an undamped and small nutation. This implies the description of the dynamics in a body frame F_i linked to principal moments of inertia. In that frame the angular momentum vector reads $\underline{H}^T = (A\,\omega_x, B\,\omega_y, C\,\omega_z)$, where A, B and C are the principal inertias. Euler's equation is then

$$\underline{T} = \dot{\underline{H}} + \underline{\omega} \times \underline{H} \qquad (4)$$

where the vector $\underline{T}$ for external torques is to be set to zero as far as our attitude problem is concerned. The essential differences in formulation occur in the next vector equation

$$\underline{\omega} = \underline{V}(\varphi, \theta, \psi, \dot{\varphi}, \dot{\theta}, \dot{\psi}) \qquad (5)$$

which links the angular velocity $\underline{\omega}$ to a set (φ, θ, ψ) of three rotation angles. These angles describe the actual instantaneous orientation of the rigid body or satellite. In two cases (Refs. 10, 12) they are selected as true Euler angles (successive rotations around the z, x, z-axes). In the other study (Ref. 14) they are the Tait-Bryan angles commonly called pitch, roll and yaw (successive rotations around the x, y, z-axes). This is important because all parametrisations of a 3-dimensional rotation by three parameters only, contain singular points (Ref. 6). The case of Refs. 10 and 12 operates directly at, or in the neighbourhood of such a point.

We should not omit here to mention the two other essential coordinate systems which are always inherent to an attitude determination process. One is the satellite body coordinate system F_b which is normally defined by the manufacturer and in which all sensor alignments and positions are given. The other is the inertial reference frame F_o in which the orbit and the reference vectors $\underline{S}$ and $\underline{E}$ are known. All studies start with a detailed definition of the relations amongst F_o, F_b and F_i. In our survey they call for no particular comments.

3. ESTIMATION BY FILTERING

3.1 Sequential Estimation

In order to understand clearly the basic estimation potential of a Kalman filter we have to confront it with its historical predecessor, namely Gauss-Markov sequential estimation. This subsection is devoted to the latter. Let us first stress that Gauss-Markov estimation deals with the finding of a time independent vector of unknowns

$\underline{X}$ (rx1). The bracket with a multiplication sign is used to denote the dimension of vectors and matrices. If one has a large set of measurements $\underline{Y}$(nx1) with n > r, then one has to find the best $\underline{X}$ from the matrix equation system

$$H \underline{X} = \underline{Y} + \underline{V} \quad (6)$$

where $\underline{V}$ is a vector of zero mean random errors whose covariance matrix

$$Q = (\underline{V}\underline{V}') \quad (7)$$

is known and of rank n (i.e. invertible). The matrix H (nxr) contains perfectly known coefficients. It was Markov who proved that

$$\underline{X} = (H'Q^{-1}H)^{-1} H'Q^{-1}\underline{Y} \quad (8)$$

is zero bias and minimum variance for a diagonal Q. The prime added to a vector or a matrix denotes transposition. The complete proof including any correlation between the elements of $\underline{V}$ was provided by A. C. Aitken in 1934 (Ref. 7). In all cases

$$R_X = (H'Q^{-1}H)^{-1}$$

represents the covariance matrix of the solution.

Thus if we estimate any constant vector $\underline{X}$, (8) yields the 'very best' result. An other method which claims to be 'optimal' also, must consequently be equivalent to (8), otherwise 'optimal' will have to be preceded by the syllable 'sub'-. If the stochastic vector $\underline{V}$ can be broken down into uncorrelated pieces $\underline{V}_1(n_1x1)$,.....$V_\ell(n_\ell x1)$ with $\sum n_j = n$, then we know that Q is a block diagonal matrix with block matrices Q_i. The matrix H' and the vector Y' can be broken down in a similar way in $H'_1(rxn_1)...H_\ell'(rxn_\ell)$ and $\underline{Y}_1(1xn_1)..\underline{Y}(1xn_\ell)$, respectively. The equation (8) then becomes

$$\underline{X} = (\sum_{i=1}^{\ell} H_i'Q_i^{-1}H_i)^{-1} (\sum_{i=1}^{\ell} H_i'Q_i^{-1}\underline{Y}_i) \quad (9)$$

Should we have stopped to receive measurements at the sample $\underline{Y}_{\ell-1}$, we had found

$$\underline{X}_{\ell-1} = (\sum_{i=1}^{\ell-1} H_i'Q_i^{-1}H_i)^{-1} (\sum_{i=1}^{\ell-1} H_i'Q_i^{-1}\underline{Y}_i)$$
$$= R_{\ell-1} (\sum_{i=1}^{\ell-1} H_i'Q_i^{-1}\underline{Y}_i) \quad (10)$$

where $R_{\ell-1}$ is the covariance matrix for $\underline{X}_{\ell-1}$. Substituting the elements of (10) into (9) yields

$$\underline{X} = (R_{\ell-1}^{-1} + H_\ell'Q^{-1}H_\ell)^{-1}(R_{\ell-1}\underline{X}_{\ell-1} + H_\ell Q_\ell^{-1}\underline{Y}_\ell) \quad (11)$$

This is still a Gauss Markov estimation valid for a 'static' problem. Applied in the form (11) it is often referred to as 'estimation filter', as it allows sequential estimation.

3.2 The Time Discrete Kalman-filter

In this subsection we confine ourselves to a very brief account of the Kalman filter as far as it is relevant for our attitude estimation problem.

The Kalman filter is conceived to handle the state $\underline{X}$ (rx1) of a dynamic system. This means that $\underline{X}$ changes with time according to a known differential equation which can be discretised in time. By dropping any control terms (= free system), the system equation reads

$$\underline{X}_k = T(k, k-1)\ \underline{X}_{k-1} + \underline{W} \quad (12)$$

where $\underline{X}_{k-1}$ is the state at time t_{k-1} and $\underline{X}_k$ is valid for $t_k > t_{k-1}$. The matrix T(k, k-1) (rxr) is the state transition matrix and $\underline{W}$(rx1) is a (maybe incomplete) vector of zero mean white noise. This noise has, in reality, to represent the imperfections of the system (e.g. imperfect control actions) and of its modelling (imperfections going along with the time discretisation and state transition matrix representation). It is therefore called 'system noise' and its covariance matrix

$$R_W = E(\underline{W}\underline{W}')$$

does not need to have full rank.

To the system equation (13) comes a vector observation or measurement equation

$$\underline{Y}_k = H_k\ \underline{X}_k + \underline{V} \quad (13)$$

which contains the $n_k > 1$ measurements or observations made at the time t_k. The zero mean white measurement noise must correspond to an invertible covariance matrix

$$R_V = E(\underline{V}\underline{V}') \quad (14)$$

and $\underline{V}$ and $\underline{W}$ must be uncorrelated. The coefficient matrix $H_k(n_k xr)$ is perfectly known.

In order to come to an estimate also if $n_k < r$, $\underline{X}_{k-1}$ must be available as result of a previous filter step or as an 'a priori' knowledge. If we further wish to achieve optimality, the covariance matrix $R_X(k-1)$ applicable to $\underline{X}_{k-1}$ has to be known. The covariance matrix for (12) as a whole is then

$$R_S = T\ R_X(k-1)T' + R_W \quad (15)$$

If in (11) we identify $R_{\ell-1}$ by R_S and $\underline{X}_{\ell-1}$ by $\underline{X}_{k-1}$ then (11) gives the Kalman filter estimate. The automatic control engineer is not interested in a representation of the shape (11), he rather looks for the equivalent form:

$$\underline{X}_k = T\ \underline{X}_{k-1} + K\{\ \underline{Y}_k - H_k(T\ \underline{X}_{k-1})\}$$

or

$$\underline{X}_k = \underline{X}_{k,p} + K\ (\underline{Y}_k - H_k\ \underline{X}_{k,p}) \quad (16)$$

where $\underline{X}_{k,p}$ is a prediction of the state at t_k, K is the gain matrix and $Y_k - H_k\ \underline{X}_{k,p}$ is the innovation. One realises that the innovation concept is of paramount importance for effecting control at time t_k. Without control task the innovation formulation looses most of its importance.

Let us return to our attitude problem! Not only do we not have perturbing actions, i.e. we consider free drift attitude, but also we most adequately select the state as a constant vector comprising the angular momentum (the real aim of our estimation), some fixed mounting errors and fixed triggering delays (biases) applicable to the optical sensors and their electronics. In such a filter T(k, k-1) reduces to a unit matrix. Such a system with a stationary state and free of external influences is called an 'autonomous system'. Especially for autonomous systems we could also apply (6) after collection of all measurements and this gives the minimum variance result. But then also (11) must apply to preserve optimality. The Kalman filter reaches exactly the same result if R_w in (15) is identically zero in all filtering steps. This leads to the quite important conclusion: The application of the Kalman filter to an autonomous system is suboptimal if the system noise is not kept zero everywhere.

The automatic control engineer has some mitigating remarks:

1. The estimate will be asymptotically minimum variance;
2. We may need the estimate in real time to control a decoupled process and system noise may be required to ease convergence in the context of linearisation.

For point one we have not seen any proof in the general case and point 2, is giving in to suboptimality as there may be no other choice. At least for ground attitude processing the remark 2 is not a valid excuse because we have the alternative of a Gauss Markov batch calculation.

We can terminate this subsection by mentioning that the time continuous Kalman Bucy filtering has been tried once in the series of studies we survey. The attempt to estimate the nutation dynamics by continuous filtering and by utilising either simulated or flight data failed completely. We can therefore dispense with any introductory considerations.

3.3 Linearisation

Almost all practical estimation problems require linearisation in order to apply any of the previous methods. This is also the case for our attitude problem even if we limit ourselves to autonomous systems. Why is this, if though the equations (1) and (2) are linear in the spin vector components?

The first non linearity arises by the use of a state vector which must be a minimum system representation, because the Kalman filter leaves no room for constraints. The fact that we should estimate Z_x, Z_y and Z_z without the constraint $||\underline{Z}|| = 1$, means a loss of information. In a Gauss Markov batch estimation, constraints can be introduced by means of linearisation of the constraint(s) and an iteration on the final estimate, without endangering the unbiased minimum variance estimate.

The second non linearity applies to both Gauss-Markov and the Kalman filter approach. The measurements utilised in (6) and in (13) should have a cleanly defined covariance matrix. Therefore the control engineer learns that the vector $\underline{Y}$ should be filled by the elementary measurements in the original form they are produced by the system. This is a recipe which only in exceptional cases leads to an unnecessary linearisation, but, in general, it seems quite sound to comply to this engineering rule. In the equations (1) and (2), the earth colatitude β is always a derived angle which implies the solution of a few trigonometric equations which are fed by the elementary measurements. It is not impossible to make a first order approximation of its covariance and correlations, but it is more convenient and straightforward to linearise the pulse time definition equations which are described in section 2.2.

Consequently, both (6) and (13) are the result of linearisation. Let us concentrate on (13) and notice that the exact non linear relation between $\underline{X}_k$ and $\tilde{\underline{Y}}_k$ is described by a vector function $\tilde{\underline{H}}_k(\underline{X}_k) = \tilde{\underline{Y}}_k$. The linearisation is performed like for any differential correction, by computing the sensitivity or measurement matrix H_k, which consists of the elements $(\partial \underline{H}_k)_i/\partial x_p$, where i denotes the i-th component function of $\tilde{\underline{H}}_k$ and p the p-th component of the state. We then have

$$\begin{aligned}\tilde{\underline{Y}}_k - \tilde{\underline{H}}_k(\underline{X}_o) &= \underline{Y}_k \\ &= (\partial \underline{H}_k/\partial \underline{X})_{\underline{X}_o} \underline{X} + \underline{V} \\ &= H_k \underline{X} + \underline{V}\end{aligned} \tag{17}$$

For the automatic control engineers the selection of $\underline{X}_o$ - i.e. the state around which to linearise - has been a matter of long debate. Kalman himself suggested to predict $\underline{X}_o$ 'a priori'. This proved absolutely unsatisfactory in most applications and one decided to utilise $T\,\underline{X}_{k-1}$. But also this 'extension' to Kalman's ideas was not good enough in many cases. In 1966 Denham and Pines (Ref. 8) finally added the ultimate natural improvement which consists to iterate on $\underline{X}_k$ at the k-th filter step. We have spend some time on this aspect in order to stress that from Farrel (Ref. 3) onwards the attitude studies have made use of this 'iterated measurement method' with the exception of the studies Ref. 12 and 13.

The previous argument also shows that the first measurement samples are linearised around a state estimate which is still far from good, and normally one does not return to this early point. Hence, a set of early observations does not contribute to the final estimate in the optimal sense. This is well known! If we are looking for estimation rather than control, any common text book (Ref. 9) will offer a discussion about smoothing filters (i.e. estimation improvement recipes). They concern the optimal combination of repeated forward(time increasing sequence treatment) and backward (time decreasing sequence treatment) filtering. For non autonomous systems these techniques are very valuable. For autonomous systems the numerical inferiority of smoothing filters with respect to linearised Gauss Markov or in other words, weighted differential correction is obvious, because in the latter the linearisation of the whole sensitivity matrix is repeated at each iteration. Thereby, there are no early or late observations in a differential correction. The motivation to use a filter in batch mode for an autonomous system with linearisation problems is therefore hard to understand today.

4. DISCUSSION OF THE STUDIES

4.1 GEOS1 Study A (Ref. 10)

(GEOS1: 1977 - 1978 scientific satellite for geostationary operation)

The purpose of the study was to analyse whether a spin axis attitude reconstitution accuracy of 0.5° could be reached by:

- a digital or V-slit sun sensor with 0.1° standard deviation error denoted by sigma (σ);
- a meridian and skew fan beam (= slit) infrared (IR) sensor; each slit independently gave an earth/space (E/S) and space/earth (S/E) pulse with a sigma of 0.1° equivalent to the pulse time random error. Moreover, an artificial model with three systematic errors has been included in the simulation; these errors were in the range of 0.15° or smaller;
- an 'a priori' attitude which deviated 5° from the true attitude;
- the optional presence of undamped nutation of 0.5° half cone angle;
- the utilisation of a Kalman filter.

Within the framework of these requirements it was attempted to set up a filter whose state vector comprised optionally all the following elements:

- the constant right ascension and declination of the angular momentum vector (α_H, δ_H in notations of Ref. 10);
- three constant Euler angles ψ_o, φ_o and θ which were the initial conditions for an undamped nutation of a symmetric satellite;
- sensor alignment errors: one inclination error for the meridian sun sensor slit and one azimuth (or right ascension) plus one inclination error per other slit, i.e. 7 alignment errors in total;
- the three systematic errors which were simulated were estimated in the form of functions comprising a total of 4 coefficients.

Hence, the full range state had dimension 16. Except for α_H and δ_H, all other state variables were consider parameters which could be selected per group. In the zero nutation case ($\theta = 0$) the reference spin phase $\varphi_o + \psi_o$ was always estimated.

It may be interesting to note that the state transition matrix T was unity also in the case of a treatment of the dynamics. The latter entered only in the sensitivity matrix H. We thus faced an autonomous system (with a time invariant state). The filter failed in all cases where $\theta \neq 0$ and where one has attempted to find φ_o and ψ_o, i.e. to estimate the dynamics. The inclusion of a non zero nutation in the simulation and the exclusion of the dynamics from the estimation gave acceptable results in a number of test cases. The conclusion of the study starts with the following paragraph (translated from French):

- The essentially non-linear nature of the measurements does not allow the direct application of the Kalman filter to the attitude reconstitution of spinning satellites except in the absence of alignment and systematic errors. Even then, the theoretical variance computed by the Kalman filter method yields too optimistic results for the estimation error. In the general case, where one has to estimate a large number of parameters, this method (i.e. filtering) leads to the divergence of the estimation error.

In the study, the cause of all evil was sought in the non-linearity and not in observability, which was thought to be no problem or at least not to affect the α_H, δ_H estimations. Other parameters than α_H, δ_H were of no real importance! The weapon against the difficulties was the utilisation of 'system noise' and smoothing techniques which consisted in a weighted recycling of the filter.

4.2 GEOS1 Study B (Ref. 11)

At the time of specification of the requirements for this study the design of the real spacecraft was crystalising. Now, one had to find out whether a spin axis attitude reconstitution accuracy of 1° could be obtained in the geostationary transfer orbit by:

- a V-slit sun sensor with a random error on each slit triggering of 0.03°(σ) and an azimuth systematic triggering error of less than 0.1°;
- two pencil beam IR sensors (dual beam IR = DBIR) at ±6° from the satellite equator; the random errors for S/E and E/S could be selected in the range 0.03° to 0.12° for both pulse types separately and their biases could go up to 0.08° for both pulses separately;
- knowing the alignments within 0.1°;
- sampling triggering time differences only, e.g. the duration from the sun meridian pulse to S/E of the upper telescope was one measurement in the telemetry;
- the utilisation of a Kalmal filter.

The sensors, the accuracies and even the nature of measurement sampling differed from the GEOS1 study A. The sampling of triggering time 'differences' made any attempt to observe ψ_o, φ_o or $\psi_o + \varphi_o$ impossible and thereby any estimation of the dynamics faced unobservability. Hence, from the outset one considered a fully constant state vector even without reference to nutation. It contained:

- the attitude vector α_H, φ_H;
- the 7 sensor alignment errors defined as before;
- 6 systematic errors defining simple functions of orbital position and attitude.

This brought us to a total of 15 state variables for the complete model. Here again, either alignment, or biases or both groups as a whole could be excluded from the filtering process. The level of measurement errors and biases was much smaller than in study A and thereby convergence of the attitude estimate within a 1° error cone was usually obtained if the measurement equations were properly iterated. It is very interesting to note, that the final attitude accuracies could not be reached if the measurements did not include the azimuth angle connecting earth pulses with sun pulses. This is equivalent to the angle α in equation (1) which is thereby confirmed in its importance.

The conclusions given in the GEOS study B claimed that the final accuracy on the attitude was only reached if the 13 auxiliary state variables were estimated as well. Reaching 'the accuracy' also meant that a realistic covariance of the final estimate was obtained. This criterion was left unclear in the study report and we suspect that the study authors have given more weight to the combined criterion "accuracy better than 1° and meaningful covariance matrix", than to "higher attitude accuracy (e.g. 0.3° error instead of 0.85°) but accompanied by a non significant covariance matrix". In section 5 and 6 this suspicion will be confirmed.

4.3 HEOS A2 Study (Ref. 12)

(HEOS A2: 1972 - 1975 scientific satellite on a very excentric and high inclination orbit)

The purpose of this study was to see whether the nutation dynamics together with the angular momentum orientation could be reconstructed by:

- using real satellite data with a residual nutation of 1° or less (nutation damper of HEOS A2 failed);
- using the sun colatitude of a digital sensor with a quantisation of 0.5°;
- using the crossing time of a meridian sun sensing slit;
- using the S/E crossing times of a meridian and inclined fan beam albedo sensor (= V-slit). Transition S/E occurred at the albedo discontinuity taking place either at an illuminated limit, the terminator or an illumination horn. The latter measurements were extremely inaccurate and had to be rejected. The others had empiric errors of up to 0.8°, which were due to slowly varying biases at the illumination boundaries;
- sampling the measurements every 64 secs once for a spin period of 6 secs;
- considering three different principal inertias.

If we compare these conditions with the GEOS study A, we see that this time the context is much less favorable with respect to the type and the accuracy of the observations. In the HEOS A2 study one had to look for a working estimation scheme. Therefore, one compared different methods.

The state was defined as a constant vector ($\sigma, \tau, \psi_o, \dot{\psi}, \theta_a, \varphi_o, \dot{\varphi}$) for a differential correction. The angles σ, τ were defining the angular momentum orientation in the Ref. 12 terminology. The other angles except θ_a were initial Euler angles and their constant derivatives (first order approximation) while θ_a was a constant intervening in the definition of the time varying θ angle. The differential correction program did not fail completely as it could be made working on simulated data in very favorable conditions. In realistic conditions, σ, τ, $\dot{\psi}$ and $\dot{\varphi}$ converged in an acceptable way, θ_a converged to a much too small value, but ψ_o and φ_o did not converge. The sum $\psi_o + \varphi_o$ converged in some cases. This fact was identified too late in the study and pointed to the singularity in the Euler angle representation when θ is zero or very small as explained in section 2.2.

For the Kalman filter a 'dynamic' state representation was selected. It comprised the three components of the time derivative of the velocity vector $\dot{\underline{\omega}}$ and further the Euler angle derivates $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$ not considered as constants. A time discrete filter and a time continuous filter was set up, but both diverged in all trials which were undertaken in the context of the study.

The most interesting fact came from the attempt to build a time discrete filter for the autonomous system also treated by the differential correction. What worked partially for the differential correction failed now completely for the Kalman filter, and this in all cases with simulated or real data. Only the differential correction method could partly resist the adverse effect of the almost unobservability of φ_o and ψ_o or φ and ψ which was largely due to an unfortunate selection of the rotation parametrisation. It also seems that a proper iteration of the measurements as suggested by Denham and Pines (Ref. 8) has not been applied. Combined with the low sampling rate this shortcoming has added in making the filtering absolutely inadequate in all the cases studied.

4.4 The COS-B Study (Ref. 13)

(COS-B: 1975 - 1982 scientific satellite on a very excentric polar orbit)

When industry selected the COS-B attitude measurement sensors, it also opted for the extended Kalman filter as the obligatory ground attitude estimation method. Some contractual incentives were linked to the inflight performance of the sensors and the ground attitude reconstitution. Facing the inflight reality, industry did not obtain the full attitude related incentive notwithstanding a lengthy investigation by all parties involved. We do not want to reopen that debate, but we only mention it as the background which led to the COS-B study contract we refer to here. The purpose of the study was to replace the 'official COS-B filter' by a better alternative, having in mind that the 'official filter' was conceived before flight whereas the study could build upon experience gained from inflight data. The COS-B attitude measurement characteristics comprised:

- the utilisation of a V-slit sun sensor
- the utilisation of two V-beam albedo sensors one canted 15° above, the other canted 15° below the satellite equator; the skew slits had opposite symmetric directions (± 30° wrt the meridian fan beam);
- the measurements were triggering time differences equal to:
  - . the rotation time separating the sun meridian slit pulse from the sun skew slit pulse;
  - . the rotation time from the sun meridian pulse to the earth limb contact pulse (the limb triggering was always selected independently from the fact that it might be at S/E or E/S) of the meridian albedo slit;
  - . the same angle for the oblique slit;
  - . the repetition of both albedo sensings for the redundant sensor unit.

The 'official' or 'operational filter' tried to estimate an eight dimensional constant state comprising two attitude angles and a bias for each fan beam except the sun meridian slit, because it was the reference for the triggering differences. The measurement equations were not iterated.

The study authors made a thorough investigation of the error residuals they could derive from the real albedo sensor measurements. It appeared that the biases had a drifting nature and were comprised between ±1°. This was due to the variability of the earth albedo in the area of sensor contact. Therefore an adaptive mechanism was selected for a new filter formulation. It was essentially taken from the Ref. 16 where it had been termed as the j-adaptive estimator. Its principle was to track the bias errors U_i by subjecting them to an error model $\dot{U}_i$ = constant. At each filtering step $\dot{U}_i$ was estimated but the covariance related to it was left unmodified so that the different bias errors could evolve as independend random walks. The size of the state vectorincreased from 8 to 14 and the state transition matrix was no longer a unit matrix. It was verified that the programming part was correct by solving successfully the cases without simulated errors and without offset in the initial attitude. If the initial attitude was offset by typically 2^o, most cases converged, but unfortunately to the wrong attitudes. The addition of system noise could delay the convergence at wish but the attitude estimate remained incorrect. The study authors showed that observability was at the root of the problem. As a consequence, the j-adaptive filter was given up.

4.5 The Star Mapper Study (Ref. 14)

The ESA star mapper sensor was designed in the middle of the seventies, at a time when it was not yet known which should be the first user spacecraft. The first satellite which carried the star mapper in slightly adapted version was GIOTTO, which encountered Halley's comet in March 1986.

In 1975 a study contract was led in order to investigate the attitude reconstitution potential hidden in a star mapper able to detect the brightest stars (a few hundred with a magnitude in the order of 3 at most). It was remarkable that one attempted to work without any sun reference, not even a sun meridian slit was foreseen. The star mapper itself worked on the V-slit principle as mentioned earlier. The problem was now that more than one star could cause triggering and the equivalent of some pattern recognition was necessary. As this concerned preprocessing, not involving estimation filtering, we will assume that such questions were solved.

The study authors made the following options:
- to consider the dynamic motion of a nutating asymmetric spacecraft (nutation up to 0.5^o);
- to consider a state vector comprising:
  . the dynamic state with ω_z, ω_x/ω_z, ω_y/ω_z and three Tait-Brian angles ψ, φ and θ;
  . eight constant system parameters, 2 of them related to inertia ratios, the others describing the alignments;
- to implement the 'square root' filtering;
- to introduce a divergence detection mechanism based on the size of the innovation and its predicted range; when divergence was found the state covariance matrix was adapted;
- to represent measurement pulses as intersections of the star mapper slits and the star position in the sensor coordinate system.

Notwithstanding this very high degree of originality and the introduction of a powerful adaptive technique, the filter only worked with very low nutation (a few arcminutes) and starting within a few arcminutes from the true attitude. If the system parameters were not estimated one could accept levels of measurement noise with a standard deviation of σ_t = 0.2 msec on the triggering times (i.e. 0.01° phase angle) at most. What was even more worrying, was that a run without estimating the system parameters - started of with exact 'a priori' values - diverged with σ_t = 0.55 msec (i.e. 0.033° phase angle).

In a follow up study (Ref. 19) the study authors programmed a geometrical method. The additional presence of a V-slit sun sensor was assumed and made the geometrical treatment absolutely robust. Accuracies of better than 1 arcmin for the angular momentum vector were reported for a determination based on data corrupted by a nutation of 15 arcmin half cone angle.

In Ref. 17, cases were described where the adaptive square root filter was fed with the initial conditions from the batch program and where the nutation was 30 arcminutes large. Nice convergence on all parameters was displayed, but the picture of the assumed simulation errors was incomplete. Therefore, the results of Ref. 17 did not represent a change with respect to Ref. 14, namely that the filter was not stable when facing realistic conditions.

4.6 The METEOSAT 1 Study (Ref. 15)

(METEOSAT 1: 1977 - 1985, geostationary meteorological satellite)

The adaptation mechanism which had been developed during the star mapper study motivated a next study. The METEOSAT 1 transfer orbit attitude data were selected with the intention to estimate both the attitude and a variable systematic error to be attributed to each of the four IR pencil beam sensors. We may recall that METEOSAT 1 had much resemblance with the earlier GOES satellites and had the following relevant properties:

- there were two redundant V-beam sun sensors (only 1 was utilised)
- there were 4 IR pencil beam sensors at $\pm 4^o$, $+23^o$ and -50^o from the spacecraft equator;
- the triggering times of all sensing events was recorded separately in the telemetry;
- the satellite was subject to active nutation damping in the transfer orbit, this means that the spacecraft was spinning around its smallest principal moment of inertia and that its diverging nutation had to be reduced actively.

At the outset of the study it was decided to consider:

- geometrically computed sun colatitudes;
- earth chord measurements resulting from the time separating S/E from E/S for a given pencil beam;
- no sun-earth azimuth related measurements (cf. α in equation (1)) as measurements;
- a constant mean spin axis or angular momentum by the state variables φ, θ (study notations);
- to attribute a bias to each pencil beam which had to represent the systematic error on the chord measurement. As there were four pencil beams, the state vector had dimension 6.

As in the star mapper study a square root information filter formalism was utilised. The divergence detection was again based on the behaviour of measurement residuals. The attempt to adapt both the measurement covariance and the state covariance on the basis of the innovation divergence detection, failed. The state covariance adaptation could be retained but for the measurement covariance adaptation an independent noise estimation was set up. The measurement covariance estimation was empirical and based on the slope analysis of consecutive observations.

The attitude accuracies obtained by using METEOSAT life data were good to moderate (less than 1° difference with post AMF attitude assessments) and the filter behaviour was stable. The biases obtained did, maybe, bear little resemblance with the expected physical behaviour of the sensors, but the order of magnitude was right. It could be concluded that this filter had some potential for use in an operational environment. This study was also summarised in Ref. 18.

5. EVALUATION

5.1 Practical Applications

From the previous studies only 2 qualified for a potential utilisation in operations.

The first algorithm resulted from the GEOS case B study and for simplicity we term it the GEOS filter. Adapted to the operational environment in ESOC, the GEOS filter was completely reprogrammed in 1974/75. In parallel to the filter a differential correction was set up using exactly the same measurement equations as those of the filter. The filter was once applied in a batch program without system noise and once in a near real time program with system noise. The batch filter was extensively tested and it was found that the absolute accuracy of the attitude estimates was detrimentally influenced when including bias estimation. We are still today in GTO operations utilising the program in parallel to differential correction (without bias estimation). Since ten years we are used to see at most a few hundreds of a degree difference between both attitude estimates gained from the same data interval. The author remembers two occasions: once in tests and once in operations, where the filter gave a markedly worse result. Each time an outlayer measurement has been identified in the preprocessed data batch.

The second algorithm - namely the METEOSAT 1 study filter - was installed and adapted to the operational environment at the end of the study. It was subjected to a number of tests based on simulated data having quite realistic characteristics. The filter was stable but the absolute attitude accuracies obtained were systematically worse than those obtained by the GEOS filter and the geometric methods. The errors in attitude were some two to four times larger. It was then attempted to include the azimuth information (equivalent to α-angle in equation (1)) into the filter, which required the addition of one bias per pencil beam. The results improved but could nevertheless not compete with the estimates obtained by the other methods we utilise.

The filter did therefore not pass its qualification for a utilisation in real transfer orbit operations.

It must be noted that the state covariance realism has never been a criterion to select an attitude estimation method for ground operations.

5.2 Lessons

The results of the six studies summarised before, learn us that:

a) estimation of the nutation dynamics on the basis of optical sensing - even with densily sampled accurate star sensor events - did not appear feasible by filtering. This is derived from the studies GEOS case A, HEOS A2 and STAR MAPPER. The irony of it is that nobody is in fact interested in these dynamics, one only thought that the dynamic modelling should improve the angular momentum estimation accuracy;
b) the more biases one wishes to estimate the lesser the actual stability of the filter and the lesser the actual attitude accuracy!
c) the more complex a model is made (and as long as its filtering is stable) the more realistic the state covariance becomes. Unfortunately, in operations we do not want an accurate covariance, but an accurate attitude!

These facts, which we just have enumerated must have some fundamental background and each study author has put the accent on an aspect he thought to be the reason. In fact, observability and non-linearity is a common concern, but our question is whether we do not amplify the sensitivity with respect:

- to non-linearity by utilising a sequential processing
- to observability by introducing more system parameters to estimate.

Concerning non-linearity and filtering the answer is yes, and it has already been given in section 3.3. The question about observability is more difficult and will be discussed in the next section.

The problem of mismodelling can only be considered for the METEOSAT1 study where essentially flight data were used. In all other cases the same team made a simulator and the corresponding estimator leaving nearly no room for shortcomings in the estimator modelling.

6. NUMERICAL EXPERIMENTS

6.1 Problem Statement

What we are missing in all the studies is the relative appraisal of attitude accuracies by a comparison with other methods. Its absence is explained by the hope one had to find realistic covariances as a result of a good modelling. With such realistic covariances one should then make absolute accuracy statements, i.e. requiring no comparison. This has revealed itself to be a dream whose realisation is still as remote as 15 years ago.

The alternative to a covariance assessment consists in a statistical assessment. The latter consists in making something like 50 different estimations on the same case by making only very small random variations in hidden biases, noise levels and initial conditions. The mean performance of two methods is then judged on their mean estimation error and the standard deviation of these errors. Though such an empirical 'estimation bias assessment' and empirical covariance analysis is very computer intensive, we have undertaken it in a simple case to clarify the quantitative influence of linearisation by asking the following questions.

Question 1: what is the relative value of a 'geometrical method' consisting of solving the three equations (1) and (2) for each measurement sample and taking the mean of the isolated results.

Question 2: what is the relative value of a 'weighted geometrical method' using the covariance information (to the first order) and considering the normality constraint on the estimated vector. (Gauss-Markov, but not linearised).

For both questions the weighted differential correction (Gauss-Markov) is the reference. We have seen that theoretically a sequential filter without system noise must give the same result. This is fully verified in practise (cf. the GEOS filter) provided no linearisation problems preclude the convergence of the filtering.

We come to a third and crucial question:

Question 3: what is the relative performance of a differential correction if we add the estimation of biases in cases where they are either actually absent or small?

The answer to question 3 will equally apply to the filter (of an autonomous system) with the difference that in sequential estimation any performance degradation can be worsened via the required step by step linearisation.

6.2 Comparison of Methods

The simple 'geometrical method' needs no further explanation of how it works. Before to introduce the weighted methods we have to tell what we simulate and with which errors.

We define the spin axis with the right ascension $\rho = 0^{\circ}$ and declination $\delta = 5^{\circ}$ so that we can represent $\underline{Z}$ as $(\cos\rho\cos\delta, \sin\rho\cos\delta, \sin\delta)$ in the differential correction without danger for the parametrisation singularity at $\delta = 90^{\circ}$. The sun vector is kept constant with r.a. (right ascension) at 45° and d.c. (declination) at 0°. In order to simulate (very artificially) a satellite motion the unit vector $\underline{E}$ is moving approximately in a plane with low d.c. For the $\underline{E}$-vector motion we have 10 approximately equidistant points between r.a. 55°, d.c. 0.8° at the start and r.a. 59.5° and d.c. 8.0° at the end. At each of these ten intermediate points ten samples of the measurements θ, β, α are taken. These measurements are assumed to be mutually uncorrelated. The random errors added to θ, α have a rectangular distribution of $\pm 0.25^{\circ}$ and $\pm 0.35^{\circ}$ width respectively, while β is subject to a normal error with standard deviation $\sigma = 0.2^{\circ}$.

We have submitted this simulated case 50 times to the attitude estimation methods. To enforce slight realistic differences amongst these cases, we have started each of the separate estimations by adding mutually different randomly selected biases to θ, β, α. They follow all three a normal distribution with equal standard deviation set to 0.04° w.r.t the sequence of 50 trials. The random number sequences used in each of the 50 simulated cases are totally different and independent.

The weighted differential correction, computes ρ and δ on the basis of the separate measurements presented as

$$\theta = \arccos(\underline{Z} \cdot \underline{S}) \tag{18}$$

$$\beta = \arccos(\underline{Z} \cdot \underline{E}) \tag{19}$$

$$\alpha = \arcsin\{\underline{Z} \cdot (\underline{S} \times \underline{E})\} \times (1-(\underline{Z} \cdot \underline{S})^2)^{-1/2} (1-(\underline{Z} \cdot \underline{E})^2)^{-1/2} \tag{20}$$

The weight matrix being diagonal, there are no problems in setting up the differential correction.

The weighted geometrical method works on the basis of the three equations of (1) and (2) to which a non-diagonal (first order approximation) covariance matrix Q_o is applied. The construction of the inverse covariance matrix for the solution is performed step by step to yield $\sum (H_i' Q_o^{-1} H_i)$ as well as the computation of $\sum (H_i' Q_o^{-1} \underline{Y}_i)$. If $\underline{X}$ is the unconstrained solution then the constrained solution with a total covariance matrix R is

$$\underline{X}_c = R\{0.5[1 - ||\underline{X}||^2]\underline{X}'R\underline{X} + \sum H_i' Q_o^{-1} \underline{Y}_i\} \tag{21}$$

This is to be computed at the end of an estimation only.

The result is given in table 1. In the first column we indicate the initial value of the r.a. of the earth vector which was moved in steps of 2 degrees, to obtain ten cases. We give the mean angular error of the differential correction attitude in the columns 'error' and its standard deviations in column 3. The relative performance of the geometrical method and constrained geometrical method are obtained by making the ratio with the differential correction figures. A relative performance larger than 1 is a performance worse than that of the differential correction.

The reader realises that differential correction and the constrained weighted geometric method give equivalent results from a statistical point of view. Thereby it appears that linearisation is hardly a drawback!

Initial r.a. of $\underline{E}$	Absolute Perf.		Relative Performance			
	Diff.	Corr.	Geom.		Constrained Geom.	
	error		error		error	
55°	.119°	.078°	1.304	1.212	.966	.961
57°	.103°	.061°	1.470	1.156	.993	.994
59°	.092°	.061°	1.382	1.252	.975	.956
61°	.076°	.045°	1.444	1.120	.985	.964
63°	.073°	.041°	1.384	1.007	1.015	1.012
65°	.073°	.041°	1.474	1.459	.971	.983
67°	.072°	.049°	1.304	0.992	1.000	.988
69°	.064°	.035°	1.422	1.291	.996	.975
71°	.059°	.032°	1.472	1.398	1.009	1.023
73°	.061°	.035°	1.307	1.078	.998	1.010
mean	-	-	1.396	1.197	.991	.987

Table 1 Relative performance of different estimation approaches w.r.t. linearisation

6.3 Introduction of Biases

It is quite difficult to make some general considerations on the observability of a system with and without biases by looking at the rank of $|H_1' T_1', H_2' T_2', \dots H_1' T_1|$ comprising the transition matrices T_i and the measurement matrices H_i. This is due to the fact that the system modelling differences are very free in their possible alternatives. We will therefore make some heuristic considerations which make it plausible that the attempt to find more biases - whose value is expected to be low - leads to less overall accuracy.

Consider a linear bias b_s applicable to the sun colatitude meaurement of the previous subsection or

$$\theta = \arccos (\underline{Z} \cdot \underline{S}) + b_s \qquad (22)$$

If we leave β and α without bias, all the θ-measurements will be mobilised to find b_s. As $\underline{S}$ does not move, they will not contribute to the knowledge of $\underline{Z}$. On the contrary, $\underline{Z}$ derived from the equations (19) and (20) will in fact help to eliminate $\underline{Z}$ from (22) in order to find b_s. Thus $\underline{Z}$ will only profit from the sequence of $\underline{E}$ and $\underline{S} \times \underline{E}$ vectors. If there is not much variation for $\underline{E}$, it may even be that $\underline{Z}$ is more or less coplanar to $\underline{E}$ and $\underline{S} \times \underline{E}$ and then the problem becomes singular, there where without bias modelling it was not.

This is an extreme example, where it is obvious that the introduction of b_s is detrimental. But also for variable configurations it appears that some information is used up to find the biases, i.e. the information is 'diluted'! There is a problem dependent threshold from where biases start to become important in their own right.

If we knew that b_s was at least 5°, we could not live without equation (22), but in a global treatment the equation was used up anyway. The information dilution is not taking place when the treatment of b_s is decoupled, because b_s originates from somewhere else and goes then into a compound or calibrated measurement $\theta - b_s$. In that case (22) again contributes to $\underline{Z}$.

To illustrate this phenomenon we have repeated the example of the previous subsection. The standard deviation for the random erors on θ, β, α is 0.15° and they have a bias of .05, .13° and -.22°, respectively. The normal error on these biases - which varies over a set of 50 test estimations - is retained and has a standard deviation of 0.03°. To improve observability the $\underline{E}$-vector r.a. varies over 10°. To save some central processing time the E-vector variation is performed in only 5 steps and at each step six samples of three measurements are taken. We perform a differential correction without bias estimation, one with the estimation of the bias b_s, one differential correction with the 2 biases estimated (on θ and β) and one with three biases estimated. The iterations of the differential correction estimations all start at 5° from the error free attitude. The bias estimation cases are fed with the attitude which results from the estimation without bias. This initial condition is always closer than .7° to the error free attitude. Convergence has occurred in all cases.

Table 2 displays the result in much the same way as in table 1. The bias estimates are normally not representative of the true biases and their mean error size increases dramatically when going from one to three biases. The attitude estimates are still valid in the 3-bias case, but the attitude accuracy may go down by a factor of almost 10. We feel that this is quite sufficient to reconsider fundamentally the value of estimating small biases and small nutations with or without filtering.

Initial r.a. of $\underline{E}$	Absolute Performance		Relative Performance					
	no bias estimation		1 bias estimation		2 bias estimation		3 bias estimation	
	error		error		error		error	
55°	.554°	.010°	1.60	4.01	1.04	3.65	2.53	5.36
57°	.512°	.096°	1.46	4.42	1.03	4.84	2.61	6.56
61°	.422°	.078°	1.52	5.97	1.59	5.61	3.40	8.06
63°	.388°	.052°	2.16	6.82	1.95	10.33	4.00	15.63
67°	.357°	.062°	2.05	7.64	1.98	8.43	4.26	11.03
69°	.333°	.059°	2.15	7.55	2.50	10.42	4.86	12.93
71°	.329°	.059°	2.23	8.36	2.41	9.55	5.03	13.62
73°	.296°	.056°	2.14	7.70	2.15	9.99	4.99	12.28
mean	-	-	1.92	6.53	1.77	7.50	3.88	10.46

Table 2 Comparison of attitude estimation accuracies with and without biases

We have been successful in demonstrating that the empirical facts uncovered in this subsection are based on a basic property of estimation (Ref. 20).

7. CONCLUSION

The results of our comparative analysis can be broken down in three categories of applicability, namely:
- attitude determination for spinning satellites
- estimation performance of particular recursive schemes
- comparative estimation performance of particular modelling techniques.

In the area 'attitude determination' we have to report that the estimation of nutation dynamics based on optical sensing has repeatedly failed, also when employing different sensors and applying different estimation methods. The feasibility of such an estimation has thereby become very questionable.

In four of the six studies analysed, we have faced autonomous dynamic systems, i.e. no control and a time invariant state description. We have shown that - in the context of linearisation - the recursive treatment of such a particular system is numerically inferior with respect to a batch weighted differential correction. To this we add that the unnecessary recursive treatment may call for the illegal use of system noise, thereby enforcing suboptimality.

The more interesting finding is related to modelling techniques. There is convincing evidence from the studies analysed and from numerical experiments reported in this paper, that the addition of time invariant biases to recursive or batch estimation schemes degrades the estimation performance. This remains true also if these biases represent a perfect process model. We refer to Ref. 20 for the theoretical confirmation of this phenomenon.

The question addressed in the title of this paper can now be answered. For the 'post factum' refined attitude determination of spinning satellites the answer is 'no' for the filters. The reply 'yes' must be reserved for real time estimation requirements and automatic control.

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