ATTITUDE DETERMINATION FROM GPS INTERFEROMETRY APPLIED TO SPIN STABILIZED SATELLITES AT AN ARBITRARY SPIN RATE

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Abstract

Two attitude determination procedures based on GPS carrier phase interferometry for spin stabilized satellites are revisited. Especially their algorithm to solve the integer ambiguity problem is reformulated in order to overcome a restriction to non-slow spin rates. The approach is based on the kinetic constraint due to the spin stabilization. Also a closed-form is presented to solve the signal ambiguity which appears when processing the double-difference of carrier-phase. It is obtained from invariant properties of vectors under finite rotations. Numerical results are presented using simulated data and real data from a ground experiment.

***Key words**:* GPS, Spin Stabilized Satellites, Attitude Determination.

Introduction

On a previous paper[1] a procedure for attitude determination of spin stabilized satellites from GPS interferometry was presented. The approach is based on the single-difference of carrier-phase measured by a pair of antennas linked to a single GPS receiver. Due to operational constraints on the available ground test facilities, a variation of the procedure was later developed[2] based on double-difference of carrier-phase, which is able to process measurements from a pair of antennas linked to distinct receivers. Both procedure versions take profit of the sinusoidal pattern of the observed signal to get attitude information from its amplitude and phase. The concept was thus proved by ground tests[3] carried out at the Instituto Nacional de Pesquisas Espaciais – INPE, in the context of a cooperation agreement between INPE and Universidade Federal do Paraná, UFPR.

The way adopted in Refs. 1-3 to cope with the integer ambiguity, which is always present on carrier-phase observations, is quite simple and efficient but imposes a given upper bound to the satellite spin rate[4]. Unfortunately, for sampling rates offered by current GPS receivers and typical values of antenna baseline length, that requirement represents a strong limitation for practical applications.

In this paper, one removes such limitation provided the spin rate is known within a given uncertainty boundary. A triplet is selected from the data sample and tested with every admissible integer ambiguity solution. The optimal solution is found by checking the goodness of fitting of the triplet-based sinusoidal with respect to the whole sample.

One also presents a new approach to solve the signal ambiguity on the double difference based solution. It is based on two invariant properties of vectors under rotation and it avoids the time consuming numerical search of the original procedure[2].

A proof of the concept algorithm was implemented and tested using both simulation and ground test data. The results are presented and analyzed. The study intends to be a preparatory step to further analysis of the procedure performance using the DLR ground test facilities.

The Integer Ambiguity Resolution Algorithm for Slow Spin Rate

Only the fractional part of the carrier-phase is reliable for attitude determination, not subjected to cycle slips. In this case, the between-antenna, single-difference of GPS carrier-phase observable[5] for the p-th GPS satellite for a pair of antennas $(1,2)$ over a spinning baseline at a given time t may be modeled as:

$$\phi_{1,2}^{p}(t) = \frac{b}{\lambda}\left[A^{p}\cos\omega(t-t_0) + B^{p}\sin\omega(t-t_0)\right] + \tau_{1,2} + N^{p}(t) + v^{p}(t) \ , \qquad (1)$$

$$N^{p} \in Z : \phi_{1,2}^{p} \in \left(-\tfrac{1}{2}, \tfrac{1}{2}\right] ,$$

where ω is the spin rate; t_0 is a reference time; $\tau_{1,2}$ is the time difference between receivers due to clock instability; N^p is the integer ambiguity; v^p is a random white sequence with standard deviation σ; b is the baseline length; λ is the GPS carrier wavelength; and A^p and B^p are coefficients related with the p-th GPS line of sight aspect angle θ^p and its phase angle α^p at t_0:

$$A^{p} = \sin\theta^{p}\cos\alpha^{p} \ , \qquad (2)$$

$$B^{p} = \sin\theta^{p}\sin\alpha^{p} \ . \qquad (3)$$

The first step to estimate the satellite spin-axis attitude and its phase angle is to estimate A^p and B^p from a given data sample $\left\{\phi_{1,2}^{p}(t_1),\cdots,\phi_{1,2}^{p}(t_n)\right\}$ for each GPS satellite. The model equation (1) is affected by four different kind of disturbances: the first one is represented by the term $\tau_{1,2}$ which may be eliminated by connecting both antennas to the same receiver or by taking the double difference $\phi_{1,2}^{p,o}$ between p-th and o-th GPS satellites; the second one is the integer ambiguity N^p, which needs to be resolved; the third one is the random noise v^p, which asks for a statistical approach such as curve fitting, for instance; and the last one is an unknown bias $\delta\omega$ at the spin rate, which must be considered but not necessarily estimated.

When the spin rate is slow enough that assures that the relative motion of the antennas during the sample interval is smaller than a half-wavelength, then the carrier phase difference splits in a discontinuous, two-folded sinusoidal pattern. So, the integer ambiguity can be easily removed[1] by adding to each data the necessary integer amount that makes it as close as possible to the already corrected value of its immediate previous data (see Fig. 1). The first integer ambiguity may be regarded as a third coefficient to be estimated together with A^p and B^p, and is indeed meaningless to the problem.

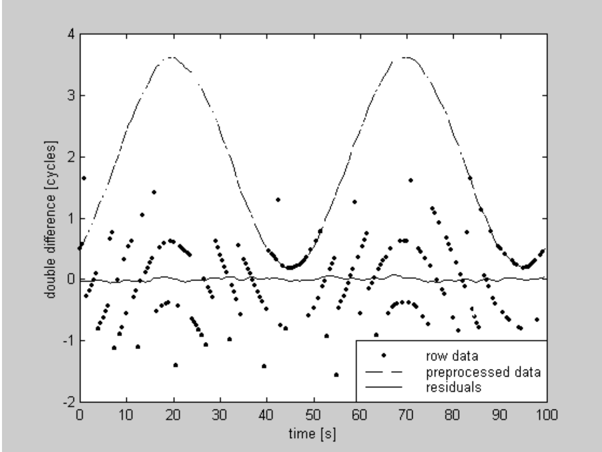

Figure 1: Preprocessing GPS carrier-phase - Ground experiment data, slow spin rate

This algorithm works if the following empirical condition is satisfied[4]:

$$\tau \equiv \omega \Delta t \frac{b}{\lambda} < 0.4 \qquad (4)$$

If Cond. 4 is not fulfilled the split sinusoidal pattern is lost and the algorithm fails to retrieve the original information. This means for instance that for a typical baseline of 1 m and a sampling rate of 1 Hz the spin rate should be smaller than 1rpm. Even considering the state of the art GPS receiver with a sampling rate of 10 Hz and a small baseline of 0.5 m it could be acceptable for some satellites like the Scientific Applications Satellite SACI (designed to spin at 6 rpm), but not for many others, like the environment Data Collecting Satellite SCD1 (currently at 50 rpm) or his twin brother SCD-2 (about 30 rpm). Indeed Cond. 4 restricts considerably the use of this algorithm in real missions.

The Integer Ambiguity Resolution Algorithm for Arbitrary Spin Rate

In this section a new algorithm is presented to solve the integer ambiguity problem for a spinning baseline at arbitrary spin rate. The approach is based on the assumption that the spin rate is known accurately enough to establish a kinetic constraint, which eliminates false solutions. The necessary mathematics derived below is cumbersome but the approach itself is comparatively simple and may be shortly explained as follows. In a first step, the domain of candidate triplets from the data sample is scanned. The triplet that minimizes the residual covariance of the triplet-based curve fitting is selected as the optimum triplet. Then, the domain of admissible integer ambiguity on the optimum triplet in a given range $-N_{\lim} \le N^p(t) \le N_{\lim}$ is scanned. The values that minimize the weighted average of the optimum-triplet-based curve fitting are selected. Once the model is fitted the whole set of integer ambiguity $N^p(t_k)$ is resolved by minimizing the correspondent absolute value of the residual for every sample time t_k.

The Least-Squares Approach

Both antenna and GPS indexes will be omitted in this section for the sake of clarity. In this way, regardless the application case uses either single or distinct receivers, one may write:

$$\phi(t_k) = f(t_k) + N_k \ , \quad \mathrm{N_k} \in Z \cap [-N_{\lim}, N_{\lim}] : \phi(t_k) \in \left(-\tfrac{1}{2}, \tfrac{1}{2}\right] \ , \tag{5}$$

$$f(t_k) = \frac{b}{\lambda}\left[A\cos\omega(\mathrm{t_k} - t_0) + B\sin\omega(\mathrm{t_k} - t_0)\right] + v_k \ , \tag{6}$$

where ϕ is the fractional observable; f may be thought as a full observable; and $N_{\lim}$ is the integer such that:

$$\frac{b}{\lambda} - \frac{1}{2} < N_{\lim} \le \frac{b}{\lambda} + \frac{1}{2} \ , \tag{7}$$

for the single difference case, and:

$$\frac{2b}{\lambda} - \frac{1}{2} < N_{\lim} \le \frac{2b}{\lambda} + \frac{1}{2} \ , \tag{8}$$

for the double-difference case.

Let f^i and N_k^i be the i-th reference values of the full observable and its integer ambiguity respectively; and c^i the related coefficient vector respectively defined as:

$$f^i(t_k) \equiv f(t_k) + N_i \ , \tag{9}$$

$$c^i \equiv \left\{\frac{b}{\lambda}A \quad \frac{b}{\lambda}B \quad N_i\right\}' \ , \tag{10}$$

$$N_k^i \equiv N_k - N_i \ , \tag{11}$$

where $\bullet'$ denotes transpose. From Eqs. (7) and (9) one may write:

$$f^i(t_k) = \left[\cos\omega(\mathrm{t_k} - t_0) \quad \sin\omega(\mathrm{t_k} - t_0) \quad 1\right]c^i + v_k \ . \tag{12}$$

Let Φ be the fractional observable vector; F^i the full observable vector; η^i the integer ambiguity vector; V the noise vector; and Ψ the base-function matrix defined respectively by:

$$\Phi \equiv \{\phi(t_1) \quad \cdots \quad \phi(t_n)\}' \ , \tag{13}$$

$$F^i \equiv \left\{f^i(t_1) \quad \cdots \quad f^i(t_n)\right\}' \ , \tag{14}$$

$$\eta^i \equiv \left\{N_1^i \quad \cdots \quad N_i^n\right\}' \ , \tag{15}$$

$$V \equiv \{v_1 \quad \cdots \quad v_n\}' \ , \tag{16}$$

$$\Psi = \begin{bmatrix} \cos\omega(\mathrm{t_1} - t_0) & \sin\omega(\mathrm{t_1} - t_0) & 1 \\ \vdots & \vdots & \vdots \\ \cos\omega(\mathrm{t_n} - t_0) & \sin\omega(\mathrm{t_n} - t_0) & 1 \end{bmatrix} . \tag{17}$$

So, Eqs. (5) and (6) may be rewritten in matrix representation:

$$\Phi = F^i + \eta^i \ , \tag{18}$$

$$F^i = \Psi c + V \ . \tag{19}$$

The problem of estimating c^i by the least-squares method given F^i is straightforward and gives:

$$\hat{c}^i = (\Psi'\Psi)^{-1}\Psi' F^i \ , \tag{20}$$

where $\hat{\bullet}$ denotes the least-squares estimate. The estimation error δc^i and its covariance matrix C^i considering the uncertainty in ω due to an unknown bias with standard deviation σ_ω are given by:

$$\delta c^i = (\Psi'\Psi)^{-1}\Psi'\left(\Theta\Psi D\hat{c}^i \frac{\delta\omega}{\omega} + V\right) \ , \tag{21}$$

$$C^i = \xi^i \xi^{i\prime} \frac{\sigma_\omega^2}{\omega^2} + (\Psi'\Psi)^{-1}\sigma^2 \ , \tag{22}$$

$$\xi^i = (\Psi'\Psi)^{-1}\Psi'\Theta\Psi D\hat{c}^i \quad , \tag{23}$$

where Θ is a diagonal matrix and D is a skew-symmetric matrix respectively given by:

$$\Theta = \begin{pmatrix} \omega(t_1 - t_0) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega(t_n - t_0) \end{pmatrix} , \tag{24}$$

$$D = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{25}$$

Unfortunately this solution relies on the knowledge of the full observable vector F^i which due to cycle slips is not observed indeed. If the integer ambiguity were known, F^i could be retrieved from Φ and consequently c^i could be so estimated from Eq. 20.

The spin axis attitude and its phase angle may be estimated from the first two elements of c^i for single[1] and double[2] difference cases. The whole problem is therefore depending on the integer ambiguity resolution.

The Triplet-Based Solution

Instead of trying to find out the entire vector η^i whose domain contains $(2N_{\lim}+1)^{n-1}$ unknown elements (for $N_i^i = 0, \forall i$), it is more reasonable to start by guessing only two of their values.

Let $\tilde{\eta}^{i,j,k}$ be the reduced integer ambiguity vector associated with the triplet (i, j, k):

$$\tilde{\eta}^{i,j,k} = \{0 \quad N_j^i \quad N_k^i\}' \quad , \tag{26}$$

and $H^{i,j,k} \in \Re^{3 x n}$ a related sparse matrix with element (r, s) given by:

$$H_{r,s}^{i,j,k} = \delta_{1,r}\delta_{i,s} + \delta_{2,r}\delta_{j,s} + \delta_{3,r}\delta_{k,s} \quad , \tag{27}$$

where $\delta_{i,j}$ is the Kröenecker symbol. The matrix $H^{i,j,k}$ is designed to extract a triplet (i, j, k) from an arbitrary vector on $\Re^n$.

In view of Eqs. 15, 26-27 one may write:

$$\tilde{\eta}^{i,j,k} = H^{i,j,k}\eta^i \quad , \tag{28}$$

$$H^{i,j,k}\Phi - \tilde{\eta}^{i,j,k} = H^{i,j,k}\Psi c^i + H^{i,j,k}V \quad . \tag{29}$$

The (i, j, k) triplet-based least-squares estimate of c^i is given by:

$$\hat{c}^{i,j,k} = \left(H^{i,j,k}\Psi\right)^{-1}\left[H^{i,j,k}\Phi - \tilde{\eta}^{i,j,k}\right] \quad , \tag{30}$$

and the estimation error is:

$$\delta c^{i,j,k} = \left(H^{i,j,k}\Psi\right)^{-1} H^{i,j,k}\left(\Theta\Psi D\hat{c}^{i,j,k}\,\frac{\delta\omega}{\omega} + V\right) \tag{31}$$

Residual Analysis

Once $c^{i,j,k}$ is estimated, the (i, j, k) triplet-based estimate of F^i may be retrieved:

$$\hat{F}^{i,j,k} = \Psi\hat{c}^{i,j,k} \quad , \tag{32}$$

with its associate residuals:

$$\begin{aligned} &\rho^{i,j,k} = \Phi - \left(\hat{F}^{i,j,k} + \hat{\eta}^{i,j,k}\right) , \\ &\hat{\eta}^{i,j,k} \in Z^n : \rho_l^{i,j,k} \in \left(-\tfrac{1}{2}, \tfrac{1}{2}\right] \ \forall l \in \{1, \ldots, n\} . \end{aligned} \tag{33}$$

The residual uncertainty is:

$$\delta\rho^{i,j,k} = G^{i,j,k}\left(\Theta\Psi D\hat{c}^{i,j,k}\,\frac{\delta\omega}{\omega} + V\right) \quad , \tag{34}$$

$$G^{i,j,k} = I - \Psi\left(H^{i,j,k}\Psi\right)^{-1} H^{i,j,k} \quad , \tag{35}$$

where I is the identity matrix on $\Re^3$. Therefore the residual covariance matrix is:

$$R^{i,j,k} = G^{i,j,k}\left[\Theta\Psi D\hat{c}^{i,j,k}\hat{c}^{i,j,k\,\prime} D'\Psi'\Theta\frac{\sigma_\omega^2}{\omega^2} + I\sigma^2\right]G^{i,j,k\,\prime} \tag{36}$$

The optimal triplet should minimize the trace of $R^{i,j,k}$. Nevertheless, at this point $\hat{c}^{i,j,k}$ is still unknown. So, a sub-optimum triplet is selected which minimizes the following superior limit of $R^{i,j,k}$:

$$\mathrm{tr}\left\{G^{i,j,k}\left[\Theta\Psi DD'\Psi'\Theta\frac{b^2\sigma_\omega^2}{\lambda^2\omega^2}+I\sigma^2\right]G^{i,j,k'}\right\}\geq \mathrm{tr}\left\{\mathrm{R}^{\mathrm{i,j,k}}\right\} \quad , \tag{37}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix.

The cost function can be finally defined as the weighted square average of the residuals of the selected triplet:

$$J\left(N_j^i, N_k^i\right)=\rho^{i,j,k'}\left(R^{i,j,k}\right)^{-1}\rho^{i,j,k} \quad , \tag{38}$$

The entire domain of $(2N_{\mathrm{lim}}+1)^2$ integer possibilities for the pair $\left(N_j^i, N_k^i\right)$ is searched and their respective cost functions are evaluated. The optimum solution corresponds to the minimum value of the cost function, whose expected value is given by the number of degrees of freedom of the problem: $n-3$

Some Practical Aspects

In practice it was observed that the optimal solution always corresponds to symmetric triplets with respect to t_i. This means that only the triplets of the form:

$$\left(i, \quad i+l, \quad i-l\right)\ ,\ l\in\left[1,\min\{\mathrm{i}-1,\mathrm{n}-\mathrm{i}\}\right],\ i\in\left[2, n-1\right] \tag{39}$$

need to be scanned. This reduces the size of the search domain from

$$S_0=\tfrac{1}{6}n(n-1)(n-2) \quad , \tag{40}$$

for the full set of possible triplets to

$$S_*=\begin{cases}\tfrac{1}{4}n(n-2) & \text{, if } n \text{ is even}\\ \tfrac{1}{4}(n-1)^2 & \text{, if } n \text{ is odd.}\end{cases} \tag{41}$$

Also some possible pairs $\left(N_j^i, N_k^i\right)$ are mutually exclusive. This eliminates about 25% of the search domain. Therefore, the total number of required cost function evaluations is nearly given by:

$$\mathrm{T}=\tfrac{3}{4}\left(2N_{\mathrm{lim}}+1\right)^2 \quad . \tag{42}$$

From Eqs.41-42, the number of candidate solutions to be tested grows with the square of the sample size and the baseline length. This may represent a considerable time consuming task especially for on-board applications, although still feasible in view of currently available microprocessor technology. In this concern one should note that the algorithm is supposed to be needed only to initialize the process, when no a priori information is available. Afterwards the estimated coefficients could be used to extrapolate $f(t)$ and thus propagate the integer ambiguity in a recurrent way.

The Signal Ambiguity Resolution

The p-th GPS line of sight unit vectors in the rotating body frame coordinates W^p obeys[2]:

$$W^p=W^o+\Delta W^p \quad , \tag{43}$$

$$\Delta W^p=\begin{Bmatrix}A^{p,o}\\ B^{p,o}\\ \varsigma\sqrt{2\left(1-\cos\psi^p\right)-\left(A^{p,o}\right)^2-\left(B^{p,o}\right)^2}\end{Bmatrix} \quad , \tag{44}$$

where $\varsigma^p\in\{-1,+1\}$ is a signal ambiguity; and ψ^p is such that:

$$U^{p'}U^o=\cos\psi^p \quad , \tag{45}$$

where U^p is the p-th GPS line of sight unit vectors in the reference frame. Attitude may be determined from W^p and U^p using, for instance the algorithm QUEST[6]. W^p could be called a GPS “pseudo-attitude” observation vector. Since they are unit vectors, in view of Eq. 42, W^o must obey:

$$\Delta W^{p'}W^o=-\left(1-\cos\psi^p\right),\ \forall p \quad . \tag{46}$$

Therefore, given $A^{p,o}, B^{p,o}$ and ψ^p, W^o is the solution of the linear system 46.

The signal ambiguity could be resolved empirically by checking the unit vector condition for all 2^n possible signal combinations[2]. In this section an analytical, closed form solution for the signal ambiguity is presented though. The method is based on the properties that scalar products and determinants remain invariant under rotations. At least 4 GPS satellites are required to be at sight.

By the scalar product invariant property, neglecting the effect of uncertainty one may write:

$$\left(U^{p}-U^{o}\right)'\left(U^{q}-U^{o}\right)=\Delta W^{p}\Delta W^{q} \quad , \tag{47}$$

which in view of Eq. 44 yields:

$$\varsigma^{q}=\varsigma^{p}\operatorname{sign}\left(\left(U^{p}-U^{o}\right)'\left(U^{q}-U^{o}\right)-\sum_{i=1}^{2}\Delta W_{i}^{p}\Delta W_{i}^{q}\right) \tag{48}$$

By the determinant invariant property one may write:

$$\det\left(U^{p}-U^{o}\quad U^{q}-U^{o}\quad U^{r}-U^{o}\right)= \det\left(\Delta W^{p}\quad \Delta W^{q}\quad \Delta W^{r}\right) \quad , \tag{49}$$

which in view of Eqs. 44,47 yells:

$$\varsigma^{p}=\frac{\det\left(U^{p}-U^{o}\quad U^{q}-U^{o}\quad U^{r}-U^{o}\right)}{\det\left(\Delta W^{p+}\quad \Delta W^{q+}\quad \Delta W^{r+}\right)} \quad , \tag{50}$$

where the superscript $+$ denotes a variable with $\varsigma=+1$. This completes the algorithm.

Numerical Results

The proposed algorithms for both integer ambiguity resolution and signal ambiguity resolution were implemented and tested. From simulation tests it was found that the procedure is able to estimate the correct value of the integer ambiguity with 95% of confidence level if the following empiric condition applies:

$$\Delta\tau \equiv \frac{b}{\lambda} n\sigma_{\omega}\Delta t < 0.4 \quad . \tag{51}$$

The condition is similar to Cond. 4 and means that the total motion of the antennas during the sample period, away from their predicted position must be smaller than the carrier-phase half-wavelength.

For the double difference case, data from a ground experiment[3] using distinct receivers were processed. A summary of the results is given in Table 1 and a representative case is shown in Fig.2 (compare with Fig. 1). The ability to resolve the integer ambiguity even when Cond. 4 was not satisfied was clearly demonstrated. The algorithm was able to retrieve the integer ambiguity even with ten times less data then the original sample shown in Fig. 1.

One should mention that the aim of UFPR & INPE's first GPS experiment campaign was to investigate the feasibility of spin-axis attitude determination from GPS interferometry in the primary level of proof of concept. There were not any concern about accuracy related aspects, which were reported secondarily only.

Table 1: Summary of Attitude Errors of ground experiment with τ>0.4

GPS Frequency	Attitude Error [arc min] x-axis	y-axis	z-axis	τ
L1	53	60	7.1	2
L2	-31	46	-1.3	1.5

For the single difference case data simulating the antennas linked to a single GPS receiver were processed. The results illustrated in Fig. 3 are similar to those obtained when the spin rate was in agreement with Cond. 4. The attitude determination procedure for this case includes a Kalman filter and is fully described in Ref. 1. The purpose of including it here is only to show the validity of the integer ambiguity resolution procedure for arbitrary spin rates.

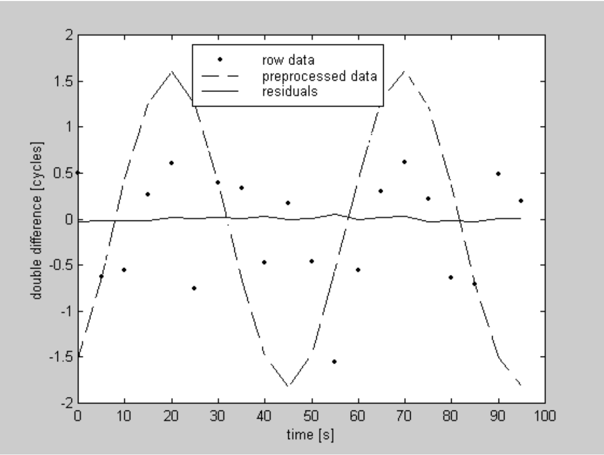

Figure 2: Preprocessing GPS carrier-phase - Ground experiment data, arbitrary spin rate

Conclusions

An algorithm to solve the integer ambiguity problem for a spinning baseline with arbitrary spin rate was proposed. The algorithm was first tested using simulated data from two antennas linked to a single GPS receiver. Later on it was tested using real GPS data from two antennas linked to distinct receivers on a ground experiment.

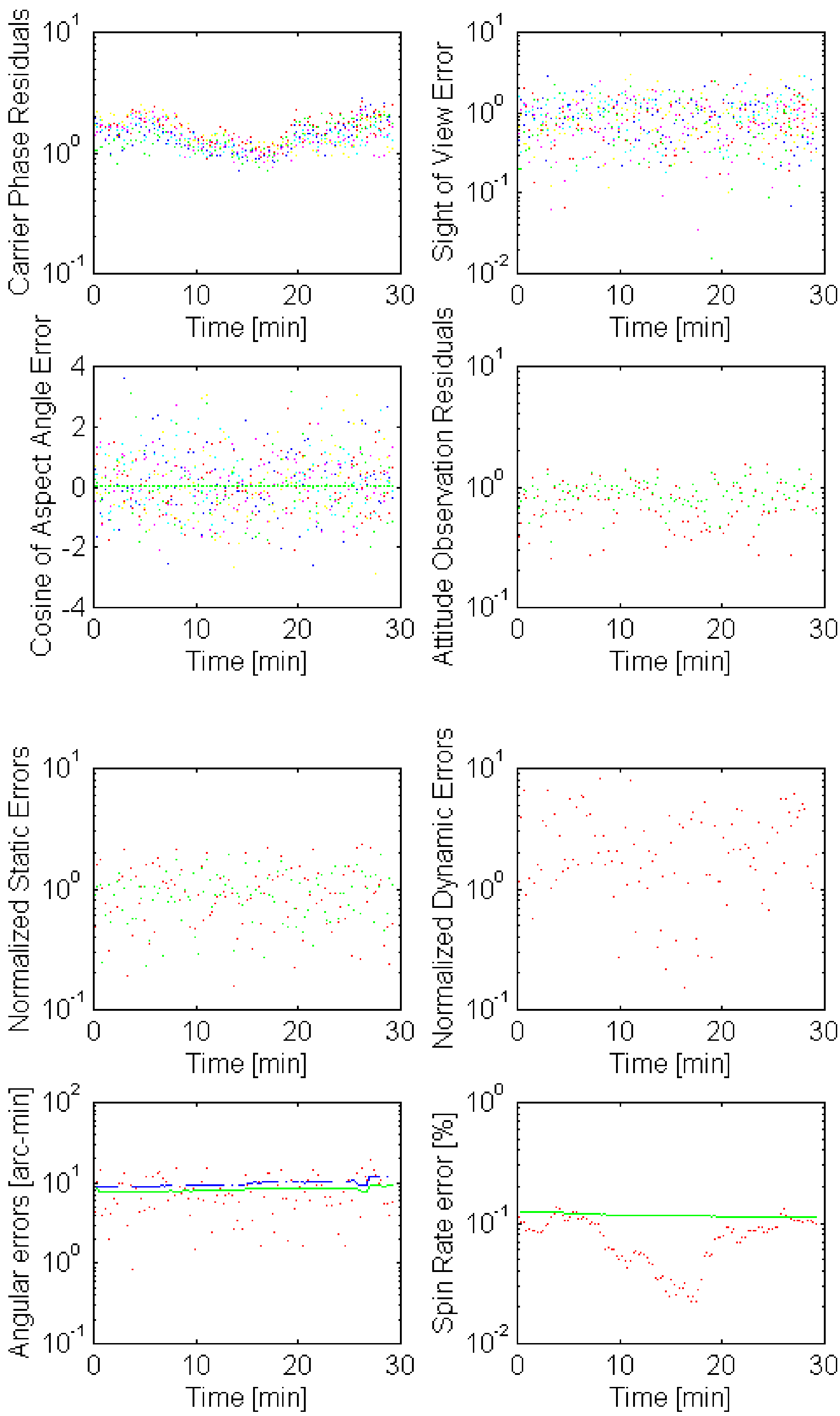

Figure 3: Summary of Procedure Performance: Simulated Data

The results show that the algorithm overcame successfully the spin rate constraint reported by previous works. A spin rate accuracy constraint applies instead.

A closed form to quickly resolve the signal ambiguity present in the double difference application case was also tested successfully.

These contributions intend to represent a step forward towards future space applications of the algorithm. As a next step, accuracy and operational aspects shall be carefully addressed with the aid of the DLR test facilities, in the frame of the Brazilian-German Government Agreement for Scientific and Technological Cooperation.

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