COARSE ORBIT DETERMINATION AS A DIVIDEND FROM ATTITUDE SENSORS FOR SPIN-STABILIZED SATELLITES

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Abstract

A batch least-squares procedure for coarse orbit and attitude determination of LEO spin stabilized satellites is presented. The observations are the axial and radial components of the geomagnetic field and the Sun angle only. The spin axis is considered inertial during sample periods of few tens of minutes. The state propagation and its transition matrix evaluation are carried out by their two-body problem closed-form. The geomagnetic field model considers the full set of IGRF coefficients, and its partial derivatives are evaluated numerically. The initial state is obtained from the geomagnetic field intensity range observed during up to one orbit. Two orbit inclination cases were simulated: 25^o and 98^o. The results are compatible with those obtained by other magnetometer-based procedures for three-axis stabilized satellites. Results from in-flight data of Data Collecting Satellite SCD-1 presented position errors about 100 Km.

Key words: Orbit determination, magnetometer, spin stabilized satellites.

Introduction

This paper presents a coarse orbit and attitude determination procedure for low Earth orbit (LEO), spin stabilized satellites, using attitude heritage hardware, namely a three-axis magnetometer and a solar sensor. Conventional spacecraft orbit determination has been performed often on ground using radar-like observations (range, range-rate and angular measurements) with typical accuracy from hundreds to a few meters. More recently, GPS receivers have been used for on board orbit and attitude determination as well as for on ground orbit determination with up to sub-meter accuracy. Such high performance equipment is however not necessary for a class of missions requiring 50km to 100km of accuracy only to assure tracking by ground stations. Furthermore, it may not be affordable for many of them considering power consumption, weight, size and cost related aspects. In such cases a coarse orbit determination based on attitude sensor observations may represent an attractive alternative.

The feasibility of on board orbit determination of LEO satellites from magnetometer observations has already been successfully demonstrated by several authors[1-7]. Some of the several proposed algorithms are attitude independent, using only the total geomagnetic field intensity[1,3], while others use its three components alone[1] or together with attitude[4-7] or attitude rate[2] information. There are both deterministic[6] and non deterministic[1-5] approaches, and the accuracy of reported results from real data range from several kilometers to tens or even hundreds of kilometers, mainly depending on magnetometer accuracy level, orbit inclination and altitude, and model compatibility with actual application. Besides the orbit elements, the state vector of non deterministic algorithms may include attitude, sensor bias, and drag related parameters, and may take from one orbit revolution to one day to converge from a very crude initial guess with error magnitude of thousands of kilometers. An initial guess is however clearly necessary to non-deterministic algorithms and a systematic way to obtain it is a question that has not been addressed yet. Also, all those reported results refer to three-axis stabilized satellites, even if they could as well be applied to a spin stabilized satellite with minor modifications, if any.

This paper follows and extends those previous ideas specially for applications on LEO spin stabilized satellites like Brazilian environmental satellites Data Communication Satellite SCD-1 and SCD-2, and Scientific Applications Satellite SACI. The approach explores the spin axis stabilization in order to retrieve more information from the magnetometer measurements with a comparatively weak requirement of attitude estimation. A rough procedure to initialize the algorithm is also presented.

A proof of concept algorithm was implemented using MATLAB, with specific code parts in FORTRAN. Preliminary results from in flight data of SCD-1 have shown an error magnitude of 100km when using attitude sensors data from 4 passes (around 10 minutes each) over Cuiabá tracking station. However, numerical results from simulated data yielded consistently better accuracy levels. The results are considered promising in view of the magnetic interference level on the SCD-1 magnetometer.

Procedure Description

The approach consists of a batch least-squares estimation of all six orbital metric elements at the initial time plus two angle corrections for the attitude of the spin axis. The observations are the components of the geomagnetic field both along the spin axis and perpendicular to it. These components are invariant to the satellite phase angle, which does not need to be estimated. Since there are too many parameters to be determined from two observations only, the algorithm requires the data to be stored over a span of few tens of minutes. Attitude of the spin axis is supposed to remain constant during this period. Also, because solar sensors are usually available on most satellites, the Sun angle is included in order to improve attitude observability.

The orbit is modeled as a Keplerian movement analytically propagated[8] during this short span. The transition matrix of the linearized dynamic is evaluated analytically too[8-9]. The geomagnetic field model[11] considers the full set of spherical harmonics with coefficients given by the IGRF-95 model. The necessary evaluation of the magnetic field gradient within the differential correction process is carried out by numerical derivation.

The algorithm initialization has to be performed based on the specific mission features. For a satellite at near circular orbit, like SCD-1, SCD-2 and SACI, a special algorithm is presented based on the comparison between expected (modeled) and observed range of the geomagnetic field intensity throughout one orbit revolution.

Batch Least-Squares Algorithm

Let r_k be the satellite position vector; $\dot{r}_k$ its velocity vector and A_k its spin axis unit vector at a given time t_k in the inertial frame. Let X_k be the orbit and attitude joint state vector and f the analytical propagation non-linear function of the system from the initial state X_0 to a final state at t_k :

$$X_k = \{r_k' \quad \dot{r}_k' \quad A_k'\}' \ , \tag{1}$$

$$X_k = f(X_0, t_k - t_0) \ . \tag{2}$$

The orbit related elements in f are given by Ref. 10, and the attitude remains constant.

Let Y_k be the measurement vector sequence tagged with t_k, corrupted by an unbiased white sequence vector ε_k, and h_k the observation non-linear function:

$$Y_k = h_k(X_k) + \varepsilon_k \ , \tag{3}$$

$$E\{\varepsilon_k\} = 0, \quad E\{\varepsilon_i \varepsilon_k'\} = \delta_{i,k} \cdot \Xi_k \ , \tag{4}$$

where $E\{\cdot\}$ is the expectance operator, $\delta_{i,k}$ is the Kröenecker delta and Ξ_k is the covariance matrix of the observation noise sequence.

Let $B(r_k, t_k)$ be the geomagnetic field vector in the inertial frame. The magnetometer observation function may be written as:

$$h_k(X_k) = \begin{Bmatrix} [B'(I - A_k A_k')B]^{1/2} \\ B'A_k \end{Bmatrix} \tag{5}$$

where I is the identity matrix on $\Re^{3x3}$.

Let S be the Sun direction unit vector in the inertial frame. Because the Sun sensor measurement is constant during the sampling period, it will be arbitrarily tagged with t_0:

$$h_0(X_0) = S'A_0 \ . \tag{6}$$

According to the Least-Squares Method the orbit and attitude determination problem may be stated as:

$$\text{Min } J(X_0) = \sum_k [Y_k - h_k(X_k)]' \Xi_k^{-1} [Y_k - h_k(X_k)] \ . \tag{7}$$

As usual, to deal with non-linearity at the equations they have to be linearized around a given nominal value $\overline{X}_0$:

$$X_0 = \overline{X}_0 + \Psi p \ , \tag{8}$$

$$X_k = f(\overline{X}_0, t_k - t_0) + F_k \Psi p \ , \tag{9}$$

$$h_k(X_k) = h_k(f(\overline{X}_0, t_k - t_0)) + H_k F_k \Psi p \ , \tag{10}$$

where p is a reduced state correction vector which takes into account that A_0 has only two degrees of freedom since it is a unit vector,

$$p \equiv \left\{ r_0' - \bar{r}_0' \quad \dot{r}_0' - \dot{\bar{r}}_0' \quad \xi' \right\}' , \tag{11}$$

with ξ such that:

$$A_0 = \bar{A}_0 + \left[(I - \bar{A}_0 \bar{A}_0') S \;\vdots\; [S\times] \bar{A}_0 \right] \xi , \tag{12}$$

and $\bar{\bullet}$ denotes nominal value; $[V\times]$ denotes the vector product operator which is a skew-symmetric matrix with elements given by the V components $\forall V \in \Re^3$; and Ψ, F_k and H_k are defined as:

$$\Psi = \frac{\partial (X_0 - \bar{X}_0)}{\partial p} , \tag{13}$$

$$F_k = \left. \frac{\partial f(X_0, t_k - t_0)}{\partial X_0} \right|_{X_0 = \bar{X}_0} , \tag{14}$$

$$H_k = \left. \frac{\partial h_k(X_k)}{\partial X_k} \right|_{X_k = f(\bar{X}_0, t_k - t_0)} . \tag{15}$$

Ψ may be easily derived from Eqs. 1,11-13, while F_k may split as:

$$F_k = \begin{pmatrix} \frac{\partial r_k}{\partial r_0} & \frac{\partial r_k}{\partial \dot{r}_0} & 0_{3x3} \\ \frac{\partial \dot{r}_k}{\partial r_0} & \frac{\partial \dot{r}_k}{\partial \dot{r}_0} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & I \end{pmatrix} , \tag{16}$$

where the orbit derivatives are analytically given at Ref. (10), and $0_{n\,x\,m}$ represents the null matrix on $\Re^{n\,x\,m}$. As for magnetometer observations, H_k is given by:

$$H_k = \begin{bmatrix} \nabla B (I - \bar{A}_k \bar{A}_k') \bar{B}_k / \beta_k & A_k \\ 0_{3x1} & 0_{3x1} \\ -\bar{B}_k \bar{B}_k' \bar{A}_k / \beta_k & \bar{B}_k \end{bmatrix}' , \quad \forall k \neq 0 , \tag{17}$$

where the nominal field $\bar{B}_k$, its symmetric gradient matrix ∇B and the radial field component β_k are given by:

$$\bar{B}_k = B(\bar{r}_k, t_k) , \tag{18}$$

$$\nabla B = \left. \frac{\partial B(r, t_k)}{\partial r} \right|_{r = \bar{r}_k} , \tag{19}$$

$$\beta_k = [\bar{B}_k' (I - \bar{A}_k \bar{A}_k') \bar{B}_k]^{1/2} , \tag{20}$$

while for the solar sensor observation, H_k is given by:

$$H_0 = \left[0_{1x6} \quad S' \right] . \tag{21}$$

The optimal solution p^* is then given by:

$$p^* = C_p \upsilon , \tag{22}$$

where C_p is the error covariance matrix of p, and υ is the weight average of the residual:

$$C_p = \left\{ \sum_k \Psi' F_k' H_k' \Xi_k^{-1} H F_k \Psi \right\}^{-1} , \tag{23}$$

$$\upsilon = \sum_k \Psi' F_k' H_k' \Xi_k^{-1} \left[Y_k - h_k(f(\bar{X}_0, t_k - t_0)) \right] . \tag{24}$$

Once p^* is found, the state estimate is given by:

$$X_0^* = \bar{X}_0 + \Psi p^* , \tag{25}$$

which may be used as the new nominal value in a hopefully convergent iterative process. After each iteration it is recommended to normalize the attitude vector to assure it will remain a unit vector.

Algorithm Initialization

In this section one presents a procedure to initialize the iterative algorithm presented in the previous section. This initializing procedure is valid for near circular orbits only, but absolutely independent of the attitude stabilization mode.

Let b_k be the total geomagnetic field intensity at a given time t_k ; $\dot{b}_k$ its time derivative and $\Im_{i,j,*}$ a set of four independent scalar observations:

$$b_k = \left|B(r_k, t_k)\right| \ , \tag{26}$$

$$\dot{b}_k = B(r_k, t_k)' \dot{B}(r_k, t_k) / b_k \ , \tag{27}$$

$$\Im_{i,j,*} \equiv \left\{b_i, \dot{b}_i, b_j, \dot{b}_j\right\} , \quad i \neq j \ , \tag{28}$$

supposing for a while that $\dot{b}_k$ could be observed in some way. Since circular orbits have four degrees of freedom only, namely the semi-major axis a, the inclination i, the longitude of ascending node Ω and the sum true anomaly plus argument of perigee $M + \omega_\pi$, it should be possible in principle to evaluate those four elements from $\Im_{i,j,*}$. For the sake of non-ambiguity, the redundant set $\Im_{i,j,k}$ could be considered instead, which includes one more scalar observation than $\Im_{i,j,*}$,

$$\Im_{i,j,k} \equiv \left\{b_i, \dot{b}_i, b_j, \dot{b}_j, b_k\right\} . \tag{29}$$

The considered set of scalar observations have the advantage of being invariant to three-axis attitude changes, but have an hindrance due to the fact that time derivatives of the magnetic field are not an usual output of magnetometers. So, a small adjustment is necessary.

Let $b_{\min}$ and $b_{\max}$ be respectively a local minimum and a local maximum of the sequence b_k. Now, neglecting the observation noise and the effect of a finite sampling rate, one has:

$$\dot{b}_i = 0, \quad \forall i : \quad b_i \in \{b_{\min}, b_{\max}\} \ . \tag{30}$$

So, one propose to evaluate the orbit from a modified set $\Im_{*,*,k}$ containing only three magnetic measurements, actually plus two time related measurements:

$$\Im_{*,*,k} \equiv \left\{b_{\min}, b_{\max}, b_k, i, j : b_i = b_{\min}, b_j = b_{\max}\right\} . \tag{31}$$

Since the field rotates together with the Earth, $\dot{B}$ is given by:

$$\dot{B} = [\omega_\oplus \times] B + \nabla B \left\{\dot{r} - [\omega_\oplus \times] r\right\} , \tag{32}$$

where $\omega_\oplus$ is the Earth's angular velocity vector.

Therefore, whenever $\dot{b} = 0$ it holds:

$$g' \dot{r} = g' [\omega_\oplus \times] r \ , \tag{33}$$

with g being the gradient vector of b :

$$g = \nabla B \cdot \frac{B}{\left|B\right|} \ . \tag{34}$$

Then the velocity vector can be evaluated as follows. Let (x, u, v) be an orthogonal base defined for all r non co-linear with g :

$$x = \frac{r}{|r|} \ , \tag{35}$$

$$u = \frac{(I - xx')g}{\left|(I - xx')g\right|} \ , \tag{36}$$

$$v = [x \times] u \ . \tag{37}$$

For a circular orbit, $\dot{r}$ lies in the u - v plane which in view of Eq. 33 yields:

$$\dot{r} = \sqrt{\frac{\mu_\oplus}{|r|}} y \ , \tag{38}$$

where y is a component of the orthogonal base (x, y, z) of the orbital frame:

$$y = [z \times] x \ , \tag{39}$$

$$z = \lambda u \pm \sqrt{1 - \lambda^2} v \ , \tag{40}$$

$$\lambda = \frac{g' [\omega_\oplus \times] r}{\sqrt{\frac{\mu_\oplus}{|r|}} g' u} \ , \tag{41}$$

The signal ambiguity being easily removed by considering that the third component of vector z must be positive for direct orbits and negative for retrograde ones.

The algorithm can now be described as follows. First a set of candidate position vectors $\{r_i^m\}$ is constructed by scanning a net covering the whole space within an altitude range from 500Km to 1,500Km, selecting those for which $\left|B(r_i^m, t_i)\right| \cong b_{\min}$. Then, for every candidate position vector r_i^m the velocity vector $\dot{r}_i^m$ and the normal to orbital plane z_i^m are determined. The candidate solution is then propagated from t_i to both t_j and t_k, so generating r_j^m and r_k^m. If $\left|B(r_j^m, t_j)\right| \cong b_j$ and $\left|B(r_k^m, t_k)\right| \cong b_k$, then the normal to orbital plane z_j^m is analogously evaluated at r_j^m and compared with z_i^m to check if $\arccos(z_i^m \cdot z_j^m) \cong 0$. The solution for r_i corresponds to the candidate r_i^m with maximum weight average residual of all the referred amounts.

Once an orbit approximation is found at t_i, it may be propagated to a reference time t_0 and attitude may be estimated by any well-known method for 1-axis attitude determination[12,13]. This completes the algorithm to obtain the initial approximation $\overline{X}_0$.

Numerical Results

Both least squares and initializing algorithms have been implemented and tested using simulation and real data. The simulation conditions correspond to Brazilian satellites SCD-1&2 and SACI. Their main characteristics are listed in Table 1. SCD-1 and SCD-2 are very similar and SACI has not been launched yet. For this reason, real data were taken from SCD-1 only.

Table 1: Simulation Scenario

Satellite:	SCD-1&2	SACI
Altitude:	750 Km	770 Km
Inclination:	25^o	98^o
Magnetometer Accuracy:	1 mG (1)	< 1 mG (2)
Sun Sensor Accuracy:	0.5^o	0.5^o
Sampling Interval:	16 s (3)	16 s (3)

(1) After pre-processing[14]. Rough data have 3 mG of uncertainty.
(2) Payload magnetometer placed far from satellite magnetic interference sources. Includes model error.
(3) After ground pre-processing[14]. Rough data have sampling interval of 0.5 s.

Table 2 summarizes the simulation results of batch least squares algorithm. The simulation uses the same dynamical model considered in the least-squares algorithm: Keplerian orbit and constant spin axis attitude. Measurements were corrupted by unbiased gaussian random errors. The purpose of such naive simulation test conditions is only to give an idea of the expected error magnitudes in ideal cases for different inclinations.

The benefit of a bigger sampling period is clear, but at expenses of more iterations to achieve convergence. As for the inclination, the error distribution for the satellite with high inclination presented standard deviations much bigger than the obtained for the low inclination satellite with a short sampling period, but the median was not so affected. In all tests the convergence was achieved in less than 50 iterations from an initial error per axis of 100 Km in position, 100m/s in velocity and 1^o in attitude.

Table 2: Batch Least-Squares Algorithm – Summary of Estimation Errors from 20 Simulations

Satellite	Sampling Period [min]	Position Error [Km]		Velocity Error [m/s]		Attitude Error [arc-min.]		Number of Iterations	
		Median	σ	Median	σ	Median	σ	Median	σ
SCD-1	20	66	59	61	60	35	25	9	3
	30	23	49	22	42	9	21	12	3
	45	17	20	19	19	11	12	15	11
SACI	20	62	165	82	184	24	86	12	8
	30	22	21	32	38	14	16	12	4
	45	14	8	9	10	7	6	21	13

Concerning the initializing algorithm, the results were very sensitive to the weights arbitrarily set to the residuals at different simulation conditions. The position errors varied from few hundreds of Kilometers, which is quite acceptable for the purpose, to embarrassing several thousands of Kilometers. The position error median was about 1000 Km and the velocity error median about 700 m/s. The above mentioned results correspond to weights of $(5mG)^{-1}$ to the magnetic field intensity residuals and $(1^o)^{-1}$ to the misalignment between angular momentum vectors. Finding a way to assign proper values to these weights will possibly require an exhaustive simulation effort that could not be accomplished by the authors till now. Despite this admitted necessity of further adjustments, in the scope of a preliminary analysis, the several cases where the chosen weights worked fine represent an evidence of the concept validity.

As for the tests with real data, Figs. 1-3 shows the residual in both radial and axial components of the observed magnetic field at representative steps of the iterative algorithm. The sample period cover orbit #5268 to #5271, which held in February 9th, 1994.

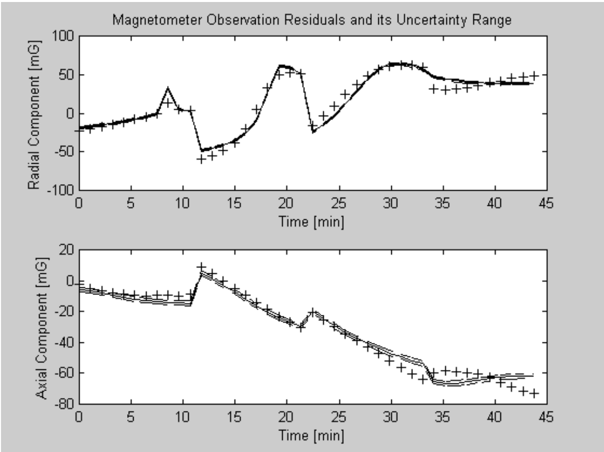

Figure 1: Residuals of SCD-1 at first iteration; Orbit # 5268-5271, real data

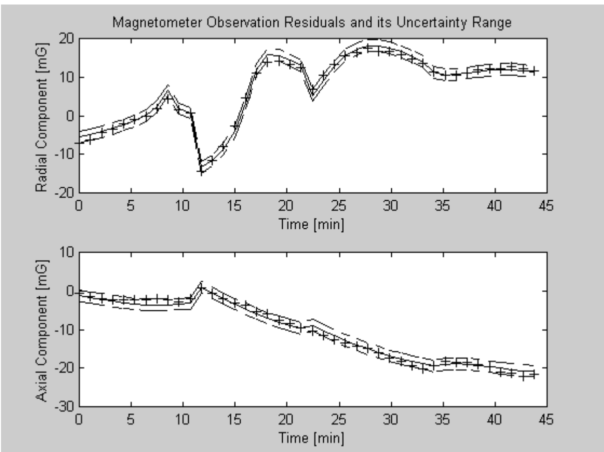

Figure 2: Residuals of SCD-1 at 30th iteration; Orbit # 5268-5271, real data

Because SCD-1 data is available on ground with the reported sampling rate only during passes over Cuiaba ground station, there are large gaps on the data sample since they cover four orbit revolutions. Those gaps were purged from Figs. 1-4 for the sake of clarity, causing the time discontinuities clearly seen on Fig. 1. Each pass takes about 10 min.

Since the residual presented a highly time correlated pattern after convergence, it was not necessary to process the data in full sapling rate. So, only one data per 64 s was processed leading to practically the same results obtained when processing the full sample (compare Figs. 3 and 4).

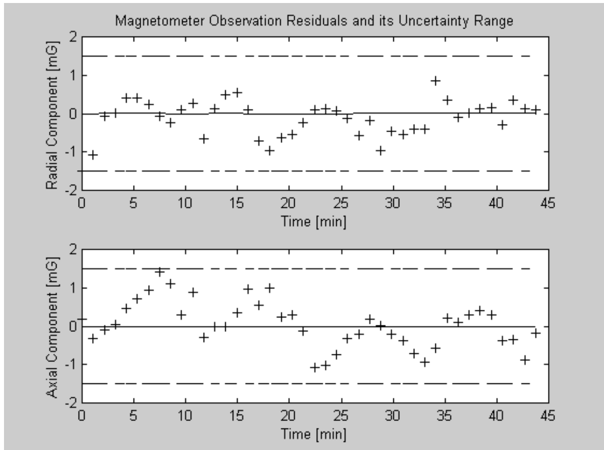

Figure 3: Residuals of SCD-1 after convergence Orbit # 5268-5271, real data

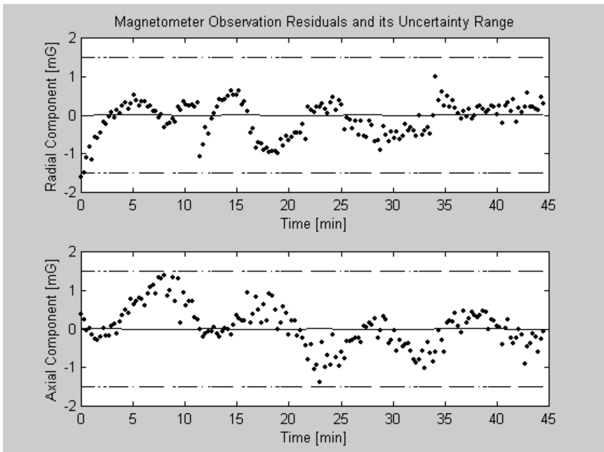

Figure 4: Residuals of SCD-1 after convergence Orbit # 5268-5271, real data, full sampling rate

In all cases convergence was achieved in less then (45) 15 iterations from an initial error per axis of (300) 100 Km in position, (300) 100m/s in velocity and (3^o) 1^o

in attitude. The relatively low convergence rate is due to a conservative factor applied to the least-squares correction to avoid divergence.

The data-preprocessing algorithm of SCD-1 removes bias efficiently from radial component of the observed field but not from its axial component. This bias is not relevant to the particular INPE's attitude determination software for SCD-1, though in the present application it would cause a position error magnitude quite bigger than the obtained consistently by simulation. Due to this problem, an empirically estimated bias of -13 mG had to be removed from the axial component of the observed field in order to lower the bias effect. The low frequency, which is still evident on the residuals, may be due to several reasons: the uncertainty on the IGRF model coefficients; the non-modeled orbit perturbations (J2, drag, etc); and an indication that the bias problem was not completely solved, for instance.

The real data results are summarized in Table 3, where the "errors" were evaluated comparing the obtained solution against orbit and attitude determination data files for the epoch, from INPE's Control Center. Their orbit determination estimates are based on range measurements from Cuiaba ground tracking antenna while the attitude is determined on ground from the Sun sensor and magnetometer observations, given the position vector.

The benefit of accumulating data from multiple passes is evident from the results, especially for position and velocity errors.

Table 3: Errors on SCD-1 Orbit and Attitude Determination Using Attitude Observations Only

Orbit No.	Position, Km	Velocity, m/s	Attitude
5268	45	267	27''
5269	287	214	13'
5270	134	132	45'
5271	198	125	3^o
5268-69	74	83	20'
5269-70	79	106	17'
5270-71	69	67	55'
5268-70	101	89	33'
5269-71	84	106	23'
5266-71	93	82	20'

Conclusions

A proposed batch least squares algorithm for orbit and attitude determination of LEO spin stabilized satellites, from attitude observations only, has been implemented and tested successfully. An error magnitude in the order of tens of kilometers has been found from digital simulation tests covering both low orbit inclination and high orbit inclination cases. Such results are compatible with previously reported results for three-axis stabilized satellites.

An initializing algorithm has also been presented and tested. A better idea of its overall performance still requires further testes and analysis. However, the obtained preliminary results show that it is also possible to initialize the least-squares algorithm based only on attitude observations. This holds for LEO satellites in near circular orbits, regardless it is one or three-axis stabilized.

Ground processing of in-flight data from Brazilian satellite SCD-1 presented errors around 100Km in position. In view of an apparently remaining bias at SCD-1 magnetometer measurements even after data pre-processing, especially on the axial component, these results are considered promising. The concept has been clearly proved and the approach should be regarded as an attractive alternative envisaging to lower costs of future missions. The present study represents an initial step in this direction. The authors are currently engaged in applying an extended Kalman filter to the same problem as a next step towards an autonomous on board orbit and attitude determination system based on heritage hardware of spin stabilized satellites.

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