ATTITUDE AND DIRECTION SENSOR USING GPS CARRIER PHASE DATA

Andrea Dalla Torre(1), Alessandro Caporali(1,2), Nicola Praticelli(2), Claudia Facchinetti(1)

(1)Center of Studies and Activities for Space "G.Colombo", via Venezia 15, 35131 Padova (Italy), cisas@dim.unipd.it
(2)Geology, Paleontology and Geophysics Dept.,University of Padua,via Giotto 1,35122 Padova (Italy), alessandro.caporali@unipd.it

ABSTRACT

We present a prototype attitude sensor based on Real Time GPS interferometry, suitable for navigation on the Earth surface or for platforms in Low Earth Orbit (LEO). The sensor is designed to measure both the attitude angles and the change in length of the baselines between the antennas.

Our breadboard consists of three NovAtel/CMC single frequency SmartAntennas with 5Hz sampling rate. The antennas form a baseline of 1m length with an intermediate antenna 0.2m far from one of the baseline end-sides. We have developed a real time software to log data from the receivers and to compute and interpret the phase differences between pairs of receivers.

We demonstrated that the data processing from this antenna set, at each epoch, leads to the baseline orientation angles with r.m.s. (root mean square) repeatability of 0.3° for horizontal angles and of 0.5° for vertical angles. The precision of this result is limited mainly by multipath. The effect of multipath can be mitigated using a calibration signal optimised for the structures surrounding the GPS interferometric attitude sensor, if they maintain a fixed geometry with time.

1. INTRODUCTION

In space applications there is a wide range of instruments capable of providing the orientation in space of a platform. These sensors are optical (sun or horizon sensors, star tracker), magnetic or gyroscopic. In the *Table 1*, a summary of the attitude sensors is given:

Table 1. Summary of existing attitude sensors [1]

Sensor	Precision	Initialization
Earth sensor	0.05º(GEO) - 0.1º(LEO)	No
Sun sensor	0.01º - 3º	No
Magnetometers	0.5º – 3º (< 6000 km)	No
Gyro	0.003º/h - 1º/h	Yes
Star tracker	2 arcsec (FOV ~ ±6º)	Yes

In case of ground applications the sensors mainly in use are the compass, the inclinometer or inertial platforms like gyroscopes. The compass is affected by the magnetic declination while both the compass and the inclinometer are affected by a strong inertia. The inertial platforms need an initialisation.

Using the GPS observable in an interferometric mode, a sensor with a precision of the order of some tenths of degree can be provided. Advantages of this sensor are the low cost, that it doesn't need any initialisation and that it hasn't any moving part.

The GPS Interferometric Attitude Sensor is formed by pairs of GPS receivers accordingly to the scheme described in §2.1. If these receivers are fixed at the platform, they can provide the orientation in space of this platform. Each pair of receivers forms a baseline and for each baseline, a horizontal angle a vertical angle and its length can be computed. Therefore, a single baseline provides 2 attitude angles of the platform. Not aligned baselines provide the three attitude angles.

The basic measurement performed by the GPS Attitude Sensor consists in comparing the down-converted carrier phases from a pair of receivers. Because each receiver has an independent clock, the fringe phase for each visible satellite is further differenced between satellites, to remove the drift of one receiver clock relative to the other. The baseline joining a pair of antennas defines body-fixed angles, which are estimated in real time using a two step procedure: a coarse estimation is first made with the Ambiguity Resolution Function (ARF) algorithm. A second finer estimate is made by least squares. Assuming baselines ranging from 0.2 meters to 1 meter, the r.m.s. of the repeatability at 1 Hz varies from 1° to 0.30° for the horizontal angle (e.g. azimuth or yaw), and a factor of 2 larger for the vertical angle (e.g. elevation or pitch/roll). For greater accuracy a longer baseline must be used, but it will be more difficult to select the correct integer ambiguities. To solve this problem compatibly with the requirement of real time processing, we use one intermediate antenna in a bootstrap mode: a 200 mm baseline is used to put for a first estimate of the solution; then the data from a 1000 mm baseline are used, in combination with the constraints coming from the 200 mm baseline, to obtain a refined solution. This approach yields a stable and accurate solution. Complementary use of the GLONASS and GALILEO navigation satellites has the potential to improve epoch-wise on the geometry and, hence, on the r.m.s. Figure.

A possible application for a long baseline configuration (e.g. 10 m) is to provide a reference for mapping the magnetic declination, for cartographic use. The sensor has the capability to measure relatively small (>0.005 m) changes in the baseline, simultaneously with the angles. As such, it can work as a strain gauge, e.g. to monitor large deformable structures in orbit or on the ground.

Having no moving parts, the sensor can withstand the shocks of the launch and is immune from thermal and mechanical drifts, but is sensitive to the occultation of the navigation satellites produced by nearby obstacles or structures.

2. ALGORITHM DESCRIPTION

2.1 Measurements

The carrier phase measurement Φ between a GPS satellite and a receiver is modelled as follows:

$$\Phi = \frac{\rho}{\lambda} + \frac{c}{\lambda}\Delta\delta + N - \frac{d_{ion}}{\lambda} + \frac{d_{trop}}{\lambda} + \varepsilon \qquad (1)$$

where Φ is expressed in cycles at the λ wavelength. N is the ambiguity of the measure (the integer number of cycles from its emission to its reception), $\Delta\delta = \Delta t - \Delta T$ contains the receiver Δt and satellite clock offset ΔT and ε contains the residual errors, including multipath.

The data processing is based on the single differences between receivers for each satellite A tracked.

$$\Delta\Phi^A = \frac{\Delta\rho}{\lambda} + \frac{c}{\lambda}\Delta\tau + N^A - \frac{\Delta d_{ion}}{\lambda} + \frac{\Delta d_{trop}}{\lambda} + \varepsilon^A \qquad (2)$$

$\Delta\tau$ is the relative clock offset of the receivers. The first order model of the single differences for short enough baselines assumes that there is negligible horizontal ionospheric and tropospheric gradient (tropospheric effect is present only for ground applications):

$$\Delta\Phi^A = \frac{\vec{b}\cdot\hat{s}^A}{\lambda} + \frac{c\Delta\tau}{\lambda} + N^A + \varepsilon^A \qquad (3)$$

where $\vec{b}$ is the baseline vector and $\hat{s}^A$ is the line of sight unit vector to satellite A.

The Eq.3 is represented in Fig. 1 from which it can be seen that, knowing the position of the satellites ($\hat{s}^A$) and after the resolution for the ambiguity term N^A ($N^A=N_1-N_2$, where *1* and *2* refer to the two receivers), we can determine the baseline length and orientation ($\vec{b}$) in space.

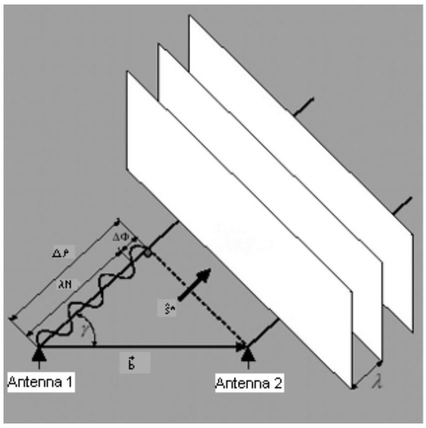

Fig. 1. phase single differences scheme. Given the navigation satellites position in sky s^A and after the solution for the ambiguity term N^A, the length and the orientation of the baseline b can be determined

Since each receiver has an independent clock, the single differences of phase must be differentiated again to remove the relative drift of the clocks. Phase observations received by pairs of receivers and transmitted by pairs of satellites are differentiated. A hub satellite (H) has to be defined, for example, as the one with higher elevation. The double difference between receivers and the A and H satellites is

$$\nabla\Delta\Phi^{AH} = \Delta\Phi^A - \Delta\Phi^H = \frac{\vec{b}\cdot(\hat{s}^A - \hat{s}^H)}{\lambda} + N^{AH} + \varepsilon^{AH} \qquad (4)$$

where N^{AH} is the difference between the ambiguities of the 2 satellites ($N^{AH}=N^A-N^H$) and ε^{AH} is the difference between their noises ($\varepsilon^{AH}=\varepsilon^A-\varepsilon^H$). The receiver clock term which was present in Eqn.3 is now absent in Eqn.4.

2.2 Single baseline determination

The components of the baseline between a pair of receivers is determined by solving the double difference Eqn.4 for all the satellites in view tracked at both antennas. Additional unknowns are the *n-1* ambiguities N^{AH}, where n is the number of common navigation satellites. This solution is done at each epoch and, clearly, there are more unknowns than equations. The ambiguities are integer multiples of the wavelength (19cm) and are solved with the so-called Ambiguity Resolution Function (ARF).

The ARF is an ambiguity independent algorithm; it tests trial values of the azimuth and elevation angle of the baseline attempting to maximize the ARF function:

$$ARF(az,el) = \cos\left[\nabla\Delta\Phi_{obs}^{AH} - \nabla\Delta\Phi_{trial}^{AH}{}_{(az,el)}\right] \quad (5)$$

where $\nabla\Delta\Phi_{obs}$ are the observed double differences. The pair of test values (*az*, *el*) maximizing this function will be chosen. The theoretical maximum of the ARF function is equal to *n-1*. In the real case the value of *n-1* is never reached because of the contributions of the multipath, the path noise and the quantization error. Once a first orientation of the baseline (*az*, *el*) is estimated, this information is used to compute the pre-fit residuals and to initialize the partial derivative of the measurement model relative to the scalar baseline length *b* and the azimuth and elevation angles. Subtracting the trial values for the double difference of phase (relative to the computed *az* and *el*) from the observed double difference we will obtain an estimation of the ambiguities present on the measurement. Hence, after pre-elimination of ambiguities we are left with *n-1* equations and 3 unknowns. This process of pre-elimination of the ambiguities must be repeated at every epoch independently of the other epochs due to the possibility that cycle slips modify the values of one or more ambiguities.

The double differenced phase, corrected for the integer ambiguities, enter the normal equations which will be solved by Least Squares. The normal equations relate the residuals of the double differences of phase *y* with the vector *x* containing the azimuth *az*, the elevation *el* and the baseline length *b*:

$$y = Ax + \varepsilon \quad (6)$$

where *A* is the matrix of the partials of the Double Differences relative to the azimuth, the elevation and the length (BAE reference system), *x* is the array of the corrections to be applied to the length, the azimuth and the elevation, ε is the noise term. Assuming non-correlated observations, the use of the Least Square Solution approach leads to:

$$A^T y = A^T Ax \quad (7)$$

To be precise, double difference data are correlated and the correlation depends on the way the double differences are constructed. However we will ignore in the following this detail although the calculation does take into account this correlation.

At this stage it's possible to constrain some parameters (baseline length or orientation) using a preconditioning matrix *C*. This is a matrix with null off-diagonal elements. The higher is the value contained in the main diagonal, the higher the corresponding parameter is bound. If the A^TA+C is invertible, then an improved estimation of the length *b* and of the baseline angles *az* and *el* is obtained from:

$$x = \left[A^T A + C\right]^{-1} A^T y \quad (8)$$

For example, in the case of the attitude sensor the baseline length is bound to the a priori value while the orientation angles are unconstrained:

$$C = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

This approach has been proposed by Caporali [2] and used for the development of a GPS interferometric deformation sensor [3].

3. GPS ATTITUDE SENSOR

In order to develop a GPS interferometric attitude sensor for ground and space applications we have to define the number and the geometry of the antennas. We focus here on an attitude sensor capable of measuring 2 orientation angle (horizontal and vertical). To obtain the three attitude angles two sensors should be used.

First the baseline length is considered. Differencing the phase data recorded by the receivers, their relative position can be determined at the order of 3-5mm. We expect a better accuracy on the estimation of the orientation angles with the increasing of the baseline length. On the other hand, an increasing on the baseline length makes more difficult to resolve for the ambiguity (see §2.1) because of the higher number of candidates, this may drive the solution to converge to wrong orientations.

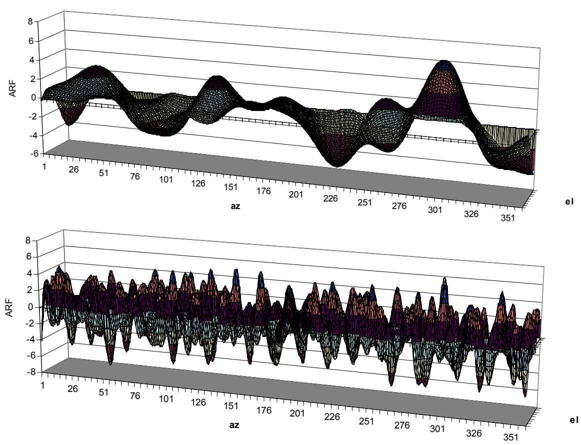

Fig. 2. ARF pattern for the 200 mm baseline (top) and the 1m baseline (bottom). While in the first case the maximum can be found unanbiguously, in the second one the solution can easily converge to wrong values

In Fig. 2 it can be seen the ARF values in the azimuth-elevation pattern for a 200mm and a 1m baselines.

While for the short baseline the maximum ARF value can be identified unambiguously, for the long one a big number of candidates appears.

A trade-off between the accuracy and the reliability is then needed. If a reliable but not very accurate solution is needed, a baseline of 200-300 mm can be used; this because the baseline length is comparable with the wavelength. If a higher accuracy is required (of the order of some tenths of a degree) a baseline of 0.8-1 m should be used, this configuration asks for an initialisation and/or a filtering approach to ensure the stability of the solution.

In our sensor these problems have been overcome using a relatively long baseline (~1m) with an intermediate antenna forming a shorter baseline with one of the two external antennas. This short baseline will provide a lower accuracy solution to be used only to "drive" the longer one to the best solution. Every solution will be independent from the previous one, therefore the sensor will not need any initialisation. Fig. 3 shows the prototype GPS interferometric attitude sensor during a kinematic test.

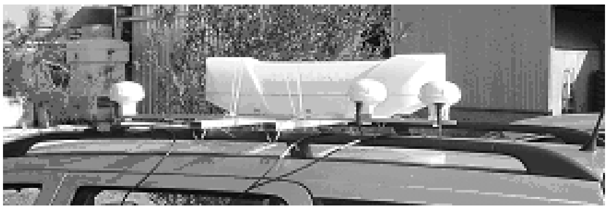

Fig. 3. prototype attitude sensor in a dynamic test together with SIMRAD heading sensor for marine applications (in the background)

We study mainly a sensor configuration of limited dimensions, of about 1 meter, but also the case of longer baselines (~10m) will be considered. This can be useful in case of big structure (e.g. boats, ISS, etc.) for which a very high accuracy (<0.1º) is needed.

Given that, in our case, the relative position between the receivers is fixed in a local reference frame, the baseline length is known with a good accuracy, so we have to solve only for the orientation angles that result to be two of the attitude angles of the platform. We can, nevertheless, obtain information also on the baseline length variation with time and then on the deformation of the platform on which the antennas are placed. Using non-parallel baselines, all the three attitude angles can be derived.

First a study on the variation of the accuracy of the orientation determination with different baseline lengths is presented. The test consists on the static measurements of the GPS observable with the variation of the baseline length. It has been considered the simpler configuration, the one with only two receivers. These tests are expected to provide an estimation of the stability of the solution and to check the dependence of the solution accuracy from the observed Navigation Satellites set (satellite numbers and satellite geometry).

Static tests give the statistic properties of our sensor. They show the stability and the repeatability of the solution. As has been already underlined, for each epoch we obtain a solution completely independent from the previous one, so the dynamic data acquisition can be considered as a concatenation of single epoch static acquisitions.

The static acquisition has been made with an approximately horizontal baseline. This is done for configurations with baselines ranging from 200mm to ~10m. Observations have been performed in a static mode for time span of about 8 hours.

Also dynamic tests in which the orientation of the baseline was varied with time have been performed. The test has been performed placing the baseline over the roof of a car. The test has been operated in order to check if the dynamic was affecting the system in some way and to look for some critical orientation. With our final configuration, the one with two baselines of different lengths, we didn't find any dependence of the solution from the dynamic of the system: even with rotation speeds of the order of 50 deg/sec the sensor proved to be robust. We tested our sensor together with the SIMRAD heading sensor for marine applications (2 GPS forming a 700mm baseline + gyro system). The obtained results were in good agreement with the advantage for our sensor to be capable of higher accuracy thanks to a longer baseline and that it doesn't need any initialization time.

Since the orientation measurements show to be strongly affected by multipath, we are inspecting a procedure to reduce multipath effect studying the correlation between errors in the orientation angles determination and the position of the HUB satellite (see Eqn.4) relatively to the body-fixed reference system.

3.1 Results

Tests have been done on the attitude sensor formed by the GPS interferometric attitude software and pairs of commercial GPS receivers.

From the tests performed with different baseline lengths, a decreasing of the standard deviation of the solutions with the increase of the baseline length can be observed. The standard deviation decreases evidently both in azimuth and in elevation, as it can be seen in *Table 2*.

Table 2. standard deviation of the azimuth and elevation solutions depending on the baseline lengths

Baseline length	Azimuth Std [deg]	Elevation Std [deg]
200 mm	1.29	2.31
300 mm	0.71	1.41
600 mm	0.40	1.05
800 mm	0.35	0.60
1 m	0.30	0.52
3.16 m	0.13	0.23
5.02 m	0.1	0.1
7.03 m	0.03	0.05
9.67 m	0.03	0.04

It is also important to consider the variations in the solution accuracies with the variation of the observed satellite set. Tests show that a high number of navigation satellites used in order to perform the solution, improves mainly the elevation solution accuracy.

As already underlined, these configurations, with long and short baseline, present some advantages and some drawbacks. For the short baseline case, of the order of the GPS wavelength, there is a quite fast ambiguity resolution. As observed in Fig. 2, the ARF function has a main peak on the azimuth-elevation space that can be identified unambiguously. On the other hand with a longer baseline we can have a higher accuracy in the solutions, but the ARF pattern present many peaks where the solution can converge.

The approach we adopted is to use the solution provided by a short baseline to limit the search for the ARF maximum of the long baseline. The limited search area in the azimuth-elevation space to be used for baseline longer than 200mm depends on the distance of the main ARF peak to the closer secondary peak. As seen in Fig. 2, this distance decreases with the increasing of the baseline length. In the *Table 3* these peak-to-peak distances are given for the different configurations.

Table 3. ARF peak to peak average distance in degree as a function of the baseline length

Length [m]	0.6	0.8	1	3.16	5.02	7.03	9.67
Peak-to-peak [˚]	40	20	10	4	2.5	1	0.7

In Fig. 4 a static test performed with the configuration shown in Fig. 3 (200mm + 1m) is described. The 3σ of the 200mm baseline solution (*Table 2*) are below the peak to peak average distance for the 1m baseline (*Table 3*). The azimuth and the elevation solutions are given for an occupation time of about 12 hours.

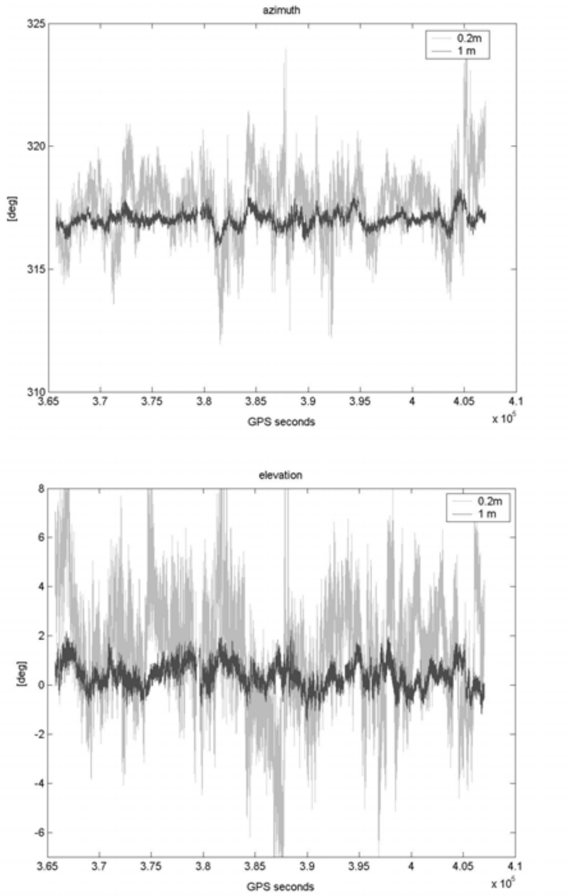

Fig. 4: azimuth (top) and elevation (bottom) determination in static conditions for the 0.2m (light) and 1m (dark) baselines. Data have been acquired for a time interval of about 12 hours with a sampling rate of 5Hz. The r.m.s. of the azimuth solutions are of 1.29˚ and 0.30˚ for the 0.2m and 1m baseline respectively and 2.31˚ and 0.52˚ for the elevation solution.

What can be observed in Fig. 4 for the two baseline configurations is an oscillatory behaviour of the azimuth and elevation. This effect decreases for longer baselines. This effect is present also in the Double Differences of the phase data and is mainly given by multipath.

The multipath is an effect given by the fact that the signals arrive to the receiver via more than one path. The multipath depends on the GPS system geometry and on the position of structures around the GPS receivers. We assumed that, if the structures surrounding the receivers are supposed to be in a static configuration, the multipath effect on a satellite's signal depends only on the GPS satellite position in the body-fixed coordinate system.

This showed to be true for static ground-based measurements. For static measurements, multipath-related features on the results repeat every sideral day, period after which the GPS satellites constellation repeats the same geometry on the sky. A multipath calibration signal may be generated from the observations of previous days separating multipath from other terms with a low-pass filer. Using this approach a calibration on the baseline computations have been operated with a strong reduction of the standard deviation of the solutions (between 40 and 50%).

The next step is to develop a multipath mitigation approach in a LEO satellite case where the receivers-satellites geometry does not present the same repetition law.

Since all the double differences are obtained with reference to a HUB satellite (eqn.4) we may think to calibrate the multipath effect on the HUB satellite signal under the assumption that signals from the same direction in the satellite-fixed reference system will be affected by multipath equally.

A first test have been operated dividing the portion of sky seen by the receivers in a grid of 2˚x2˚ areas. For each pixel of the grid, corresponding to the position of the HUB satellite (regardless of the satellite ID), we averaged the estimation of the azimuth and elevation angles of the baseline. We obtained a map of the residuals (positive or negative) of the attitude angles determination with reference to the average values both for azimuth and elevation. We used this calibration signal to correct the azimuth and elevation computations shown in figure Fig. 4. We observed a r.m.s. reduction from 0.30˚ to 0.19˚ for the azimuth results and from 0.52˚ to 0.31˚ for elevation. The main oscillation terms are removed. This preliminary test have been conducted in static conditions since we were not able to move all the reflectors together with the sensor (like in a LEO satellite case). Further investigations will be carried out to obtain a better representation of the multipath related oscillations .

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