LOW-THRUST OPTIMAL TRAJECTORIES FOR RENDEZVOUS WITH NEAR EARTH ASTEROIDS

Régis BERTRAND and Jacques FOLIARD

Centre National d'Etudes Spatiales (CNES),18 avenue Edouard Belin, F31401 Toulouse Cedex 9, France,
E-mail:Regis.Bertrand@cnes.fr,Jacques.Foliard@cnes.fr

ABSTRACT

The interest of the scientific community for small bodies has been growing for the last decades. A direct consequence is the number of missions or projects envisaged by national space agencies considering asteroids and comets as potential targets. We can quote for example NEAR to asteroid Eros and ROSETTA to comet 67P/Churyumov-Gerasimenko. A crucial point for the mission design is the cost, in terms of propellant requirement. Some interesting bodies, indeed, may not be reached by means of classical chemical propulsion because of the propellant mass having to be loaded on board. In this framework, the use of electric propulsion allows to consider either targets needing a lot of energy to be reached or multiple targets in a one way mission. Moreover, the computation of low-thrust optimal trajectories is a very complex task. The solution of the associated optimal control problems is done by combined means of the maximum principle, non-classical smoothing techniques and decomposition-coordination methods. After the general formulation and definition of the problems, we will present different optimal trajectories to various targets (such as Orpheus, Anteros, 1999FG3…) including the Earth escape phase and the heliocentric cruise. Complex options with flyby of intermediate asteroids or with gravity assist of the Earth are also proposed.

1. INTRODUCTION

The number of missions or projects (present or past) to asteroids translates the interest of the international scientific community for these primitive bodies. Presently, the main space missions or projects are: NASA's missions NEAR to asteroid Eros and DEEP SPACE 1 to asteroid Braille, NASA's project DAWN to asteroids Vesta and Ceres, ESA's project SIMONE to six Near Earth Asteroids (NEA) and MUSES-C a mission of the Japanese Space Agency to asteroid Itokawa. Four of these five missions (except NEAR) consider electric propulsion as the prime propulsion system in order to benefit of the high specific impulse of the low-thrust engines. This is a way to reduce efficiently the propellant consumption, and a direct consequence is an increase of the payload mass.

The goal of this paper is to show the opportunity to achieve low-cost missions to multiple NEAs using electric propulsion. The basic idea is to define the spacecraft as an auxiliary passenger of an Ariane 5 GTO launch and use the low-thrust engines in order to achieve the interplanetary transfer. The problem of the trajectory optimization then arises. This is a quite involved task which implies the use of advanced numerical methods.

After the presentation in section 2 of the low-thrust interplanetary trajectories optimization problems treated in this paper (formulation, target asteroids…), we will briefly define the numerical methods applied (section 3). Then, numerous results will lead to a complex interplanetary trajectory including the flyby of multiple NEAs (section 4).

2. PROBLEM STATEMENT

2.1 Low-thrust Interplanetary Trajectories

In the framework of preliminary mission analysis, an interplanetary trajectory is generally split into different parts by using the patched conic approximation [1]. These parts are: the escape phase around the Earth, the heliocentric cruise and the insertion phase around the target body. The particularity of low-thrust trajectories is that the acceleration due to the engine thrust is very low. So the burning phases must be continuous and during these ones the trajectory is slowly and continuously modified. The optimization of low-thrust trajectories consists mainly in finding the number of burning and coasting arcs and the thrust direction during the burning periods. This kind of problems falls into the optimal control problem category. Other scalar parameters may also have to be determined like, for example, the departure and arrival dates. If an intermediate gravity assist maneuver is added in the mission scenario, the date and the altitude of the perigee during the planetocentric phase are also optimization variables. So, each part of the trajectory is optimized separately and linked to each other to build the entire optimal trajectory. In this paper, we will focus exclusively on the escape and heliocentric phases. The cost of the insertion around the target bodies may be neglected indeed in the ΔV mission assessment (this will be explained in details in section 4.5).

2.2 Optimal Control Problems

The formulation of the optimal control problem associated with each part of the interplanetary trajectory may be expressed as follows:

$$\begin{cases} \underset{U}{\mathrm{Min}} \quad g(t_0, t_1, X(t_1)) \\ \dot{X}(t) = f(X(t), U(t), t) \\ X(t_0) = X_0 \\ y_i(t_i, X(t_i)) = 0 \\ y_1(t_1, X(t_1)) = 0 \end{cases} \tag{1}$$

where X is the state vector, U the control variable, g the objective function, f the function of the dynamics equations, X_0 the initial conditions, y_i the function of intermediate constraints (such as flyby or swing-by or maneuver) and y_1 the function of final constraints. t_0, t_i and t_1 are respectively the departure, intermediate and encounter dates of the considered phase. The components of the state vector $[x\ y\ z\ v_x\ v_y\ v_z\ m]^T$ are: the position, velocity (in a convenient frame) and mass of the probe. The control vector is defined by the engine state (on or off) and the thrust vector. Finally, depending on the mission phase (escape or heliocentric cruise), the objective function may represent the duration of the phase, i.e. $g(t_0,t_1,X(t_1)) = t_1 - t_0$, or the propellant consumption, i.e. $g(t_0,t_1,X(t_1)) = m_0 - m(t_1)$, where m_0 is the mass of the spacecraft at the beginning of the mission phase. Applying the patched conic approximation, the dynamics equations are directly derived from the two body problem [1] between the spacecraft and the central body (Earth or the sun) of the phase. These equations are not detailed in this paper but they can be found in [1].

2.3 Main Targets

Three different NEAs have been chosen as mission targets for this preliminary analysis: 1996FG3, Orpheus and Anteros. These asteroids have been already selected by scientific teams in the framework of agency projects because of their type, size or accessibility (in terms of ΔV requirement). In this way, 1996FG3 is one of the targets of the ESA's project SIMONE [2] whereas Orpheus and Anteros were respectively initial and backup targets of the NASA's mission NEAR [3].

Asteroid 1996FG3 was discovered in 1996 by R.H. Mc. Naught at the observatory of Siding Spring (Australia). Its equatorial radius is estimated between 0.35 km and 0.75 km. The next closest pass near Earth will occur in May 2009 and the distance between the two bodies will be approximately 0.156 AU. Orpheus (or 1982 HR) was discovered in 1982 by C. Torres at the observatory of Santiago-Cerro El Roble (Chili). Orpheus is smaller than 1996FG3 because its equatorial radius is estimated between 0.25 km and 0.55 km. In January 2006 the distance between Orpheus and the Earth will reach a minimum value of 0.1597 AU. Finally, Anteros (or 1973 EC) is the biggest asteroid considered in this paper: its equatorial radius is close to 0.9 km. It was discovered by J. Gibson in March 1973 at the observatory of El Leoncito (Argentina). The next Earth's close approach is planned in May 2005 with a minimum distance of 0.132 AU. The main orbital elements (perihelion, eccentricity, inclination with respect to the ecliptic plan and the orbital period) of these three asteroids are summarized in table 1.

Table 1: Main orbital elements of the three NEAs

	1996FG3	**Orpheus**	**Anteros**
perihelion	0.69 AU	0.82 AU	1.058 AU
eccentricity	0.35	0.32	0.26
inclination	1.99 deg.	2.68 deg.	8.70 deg.
orbital period	394 days	485 days	624 days

Obviously, asteroid Anteros is very difficult to reach (especially by classical probes using chemical propulsion) because its orbital inclination with respect to the ecliptic plan is quite high (8.70 deg.). The correction of this orbital parameter will require a large ΔV. Orbital parameters of Asteroids 1996FG3 and Orpheus are close to each other in terms of eccentricity and inclination. So, the ΔV required for these two asteroids should be similar.

3. SOLUTION METHODS

The optimal control problems such as (1) are solved by means of the Pontryagin's maximum principle [4]. This indirect method gives necessary optimality conditions but also yields the solution of a complex Multi-Point Boundary Value Problem (MPBVP), see for example [5] for further details. Various numerical methods exist to solve MPBVPs [6], but a classical single shooting approach has been chosen to solve problems described in this paper. This approach consists in finding zeros of an algebraic and differential function called shooting function. The complexity of this search is mainly due to the structure of the optimal control and the presence of an intermediate constraint, see (1).

The bang-bang structure of the optimal control (engine on or off) causes numerical issues in the evaluation of the shooting function. To overcome this concern, we introduce a smoothing technique [7]. It consists in adding a perturbing term, depending on a positive parameter e, into the objective function in order to obtain a regular and continuous optimal control. Then the original bang-bang solution is found after a continuation approach generating successive smoothed solutions and in which the perturbing term progressively disappears, i.e. e tends to zero. In the case of heliocentric trajectories including an intermediate maneuver (flyby or swing-by), the shooting function is very sensitive and unstable. A way to reduce this sensitivity is to split the problem into two more stable single problems defined respectively before and after the intermediate maneuver (decomposition approach). Then a coordination technique, based on a fixed-point algorithm, leads to an optimal link between the two paths just before and just after the date of the intermediate maneuver. This decomposition-coordination method [8] is very efficient and allows to obtain complex interplanetary trajectories with flyby or swing-by maneuvers.

The combined use of smoothing approach and decomposition-coordination method represents a powerful tool for low-thrust interplanetary trajectory optimization. This is the reason why these algorithms have been implemented in the CNES preliminary mission analysis tool called ETOPH.

4. INTERPLANETARY TRAJECTORIES

In this section, we will first present the spacecraft model, then the escape trajectory and heliocentric direct and complex transfers. Finally, a summary of all the proposed missions concludes this section.

4.1 Spacecraft Model

Since the spacecraft is viewed as the maximum auxiliary payload of a heavy Ariane 5 launcher, the launch mass is assumed to be 600 kg. The spacecraft is equipped of one hall effect thruster. Its characteristics at 1 AU are: a thrust magnitude of 0.23 N, an electrical power of 5 kWe and a specific impulse of 2700 s. The maximal propellant mass loaded on board is approximately equal to 200 kg and the scientific payload mass is close to 20 kg. This rough model is largely sufficient for the needs of a preliminary mission analysis. A global “system” study should be achieved in order to improve the spacecraft model, but this is not the goal of this paper.

4.2 Escape Trajectory

The escape of the Earth’s attraction is exclusively performed by the use of the low-thrust engine. In this way, the trajectory is continuously and slowly modified because the ratio between the acceleration due to the thrust and the gravitational acceleration is very low. Thereby, the optimization criterion, which has to be minimized, chosen for this part of the mission represents the escape duration in order to obtain realistic escape scenarios. The escape is considered achieved as soon as the relative orbit energy is nullified. Indeed, the spacecraft starts this phase on an ellipsoidal Geostationary Transfer Orbit (GTO), with a negative relative energy, and achieves the escape on an asymptotic parabolic orbit. Then, the relative velocity v_{∞} with respect to the Earth is equal to 0. The GTO is characterized by a perigee of 567 km and an apogee of 35786 km. In this paper, the problem of the correction of the orbital inclination during the escape phase is not considered.

The optimal escape trajectory is represented in Fig. 1. The duration is equal to 103 days, which leads to 74 revolutions around the Earth. The time spent by the probe into the radiation belts is approximately equal to 28 days. Depending on internal characteristics of the vehicle, this last duration may be seen as a constraint for an operational point of view. The probe achieves the escape phase with a total mass of 522 kg, the relevant consumption (78 kg of propellant) represents then a velocity increment ΔV of 3.67 km/s.

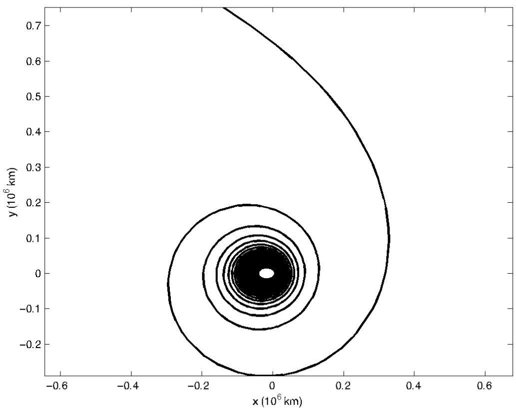

Figure 1: Escape trajectory around the Earth

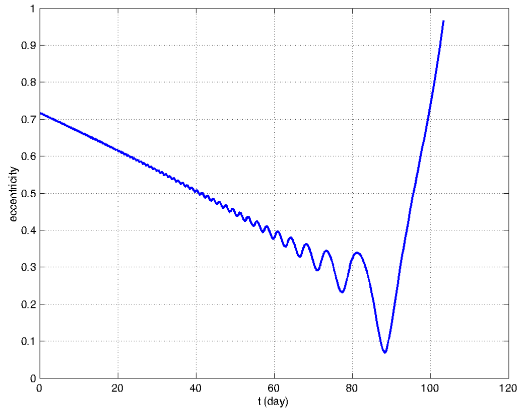

Figure 2: Orbit eccentricity during the escape phase

The escape strategy consists in switching on the engine during the overall transfer. This strategy implies a decrease of the orbital eccentricity until obtaining a quasi-circular orbit, see Fig. 2. Then, during the last revolutions the control law leads quickly to the desired parabolic orbit. This strategy depends strongly on the eccentricity of the initial orbit. Moreover, the ΔV requirement depends also on the initial orbit, it could be reduced by choosing more energetic initial orbits, i.e. orbits with higher semi-major axis.

4.3 Direct Heliocentric Trajectories

The next part of the entire trajectory is the heliocentric path. This section is then focussed on direct heliocentric trajectories. This means that no intermediate body is visited between the departure and arrival dates. Only the Earth and the target body are taken into account in the trajectory optimization. The probe is assumed to start the cruise with the same position and velocity as the Earth since the escape phase allows to transfer the probe on a parabolic relative trajectory. The rendezvous with the target asteroid is achieved as soon as the probe has reached the same heliocentric conditions as those of the asteroids. For this type of trajectory the target is successively set to Orpheus and Anteros. The initial and final dates are given by the optimization process. Thereby, the departure date (for this heliocentric phase) is taken in June 2013 for Orpheus and in November 2012 for Anteros. The characteristics (duration, final mass and velocity increment) of these heliocentric trajectories are summarized in table 2.

Table 2: Characteristics of direct trajectories

	duration	**final mass**	DV
Orpheus	294 days	422 kg	5.63 km/s
Anteros	629 days	400 kg	7.05 km/s

The encounter with the asteroids occurs in June 2014 after a cruise of 294 days for Orpheus and a cruise of 629 days for Anteros. The optimal trajectories are represented respectively in Fig. 3 for Orpheus and Fig. 4 for Anteros.

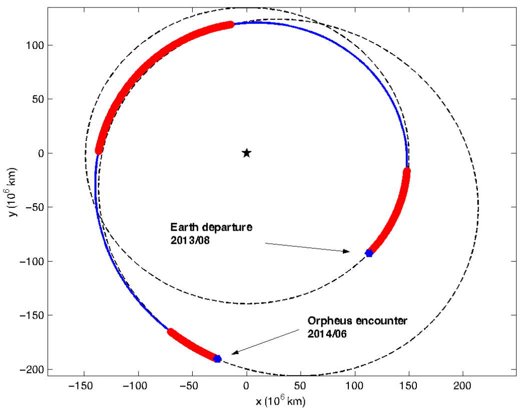

Figure 3: Earth to Orpheus cruise

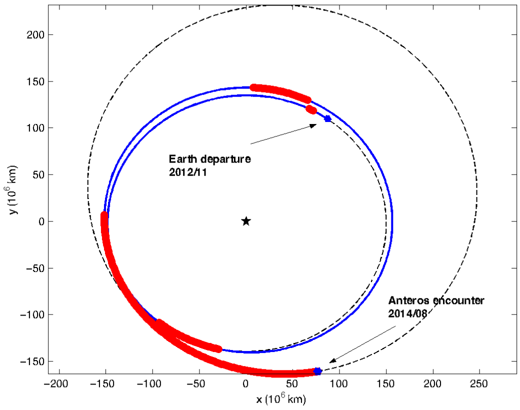

Figure 4: Earth to Anteros cruise

The strategy employed to transfer the probe from the Earth to Orpheus requires three burning periods (36 days + 62 days + 23 days), see the bold part of the trajectory in Fig. 3, and two ballistic arcs. The transfer is very short and needs less than one revolution around the sun. In the case of the mission to Anteros, the optimal control law is defined by four burning arcs (2 days + 28 days + 24 days + 126 days) and three ballistic periods. The propellant mass required to complete the transfer to Orpheus, respectively to Anteros, represents 19.1 %, respectively 23.4 %, of the spacecraft mass at the beginning of the heliocentric phase. The propellant ratio is then quite high for the trajectory to Anteros. A way to decrease the consumption is to introduce an Earth Gravity Assist (EGA) maneuver in the cruise scenario.

4.4 Complex Heliocentric Trajectories

In this section, we propose the study of two complex heliocentric trajectories. The first one is an EGA Earth to Anteros trajectory in order to show the benefit of the EGA maneuver. The second example is focussed on an Earth to Orpheus trajectory including an intermediate fly-by of asteroid 1996FG3.

Earth-EGA-Anteros trajectory

All the dates of the present scenario, date of departure, arrival and gravity assist are left free and determined by the optimization procedure. The altitude of the perigee during the gravity assist maneuver is also an optimization variable. The solution of this complex problem is obtained by the combined use of the smoothing techniques (in order to overcome issues due to the bang-bang control) and the decomposition-coordination method (in order to take into account the EGA intermediate maneuver in the relevant optimal control problem), see section 3. The optimal trajectory is proposed in Fig. 5. The departure date (for this heliocentric phase) occurs in July 2012, the EGA is planned the 4th of June 2013, and the encounter with

Anteros is achieved in July 2014 after a cruise of 756 days. The control strategy is based on five different burning periods (9 days + 8 days + 23 days + 4 days + 118 days). The EGA maneuver occurs during the third coasting arc. This means that all the gravity assist conditions are mainly obtained by means of the third thrust phase. The altitude of the perigee on the EGA hyperbolic trajectory is set to 26127 km by the optimization procedure. The incoming and outgoing velocity norms are both equal to 2.4 km/s since the EGA maneuver is not powered. The probe completes the heliocentric transfer with a total mass of 409 kg which corresponds to a velocity increment ΔV of 6.46 km/s. The propellant mass required for this heliocentric phase represents then 21.6 % of the departure mass.

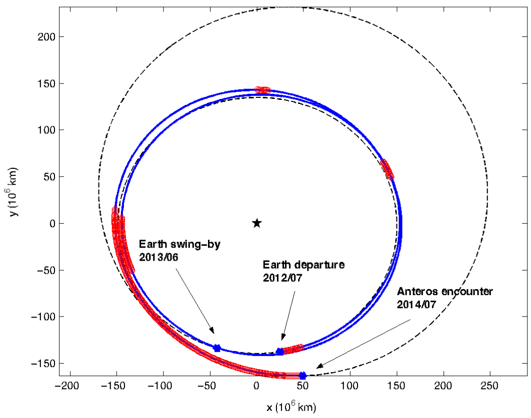

Figure 5: Earth to Anteros EGA cruise

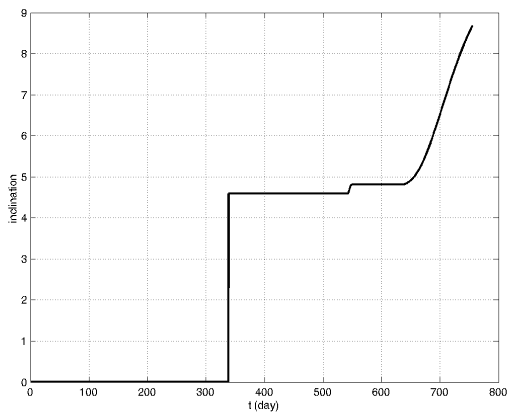

Figure 6: Inclination during the Earth to Anteros EGA cruise

In comparison with the direct trajectory to Anteros proposed in section 4.3, one can notice that the EGA maneuver allows a gain in terms of ΔV of 0.59 km/s. To explain this propellant preservation, we have to analyze the history of the orbital parameters during the overall transfer. In Fig. 6, the inclination with respect to the ecliptic plan is represented. The EGA maneuver leads to 53 % of the global correction of the orbital inclination. This mainly explains the gain between the direct trajectory and the EGA one.

Earth-1996FG3-Orpheus trajectory

The last mission proposed in this paper concerns an Earth to Orpheus trajectory including an intermediate fly-by of asteroid 1996FG3. As mentioned in the previous section, all the dates (departure, encounter and fly-by) of the scenario are left free and determined by the optimization procedure. Furthermore, the same numerical methods are used to obtain the optimal trajectory. This last one is proposed in Fig. 7. The heliocentric departure date is taken in December 2012, the fly-by of asteroid 1996FG3 occurs the 14th of January 2014, and the transfer is completed in June 2015. Thus, the duration of the interplanetary cruise is equal to 916 days. The spacecraft flies by 1996FG3 with a relative velocity of 14.58 km/s.

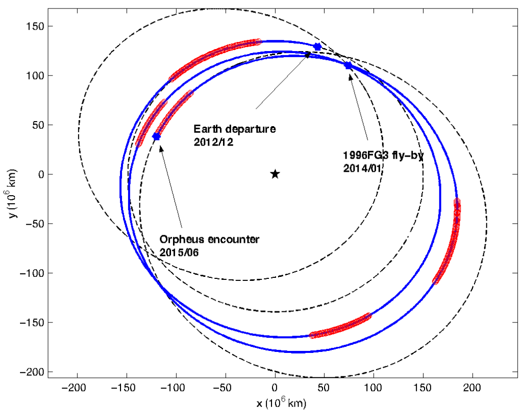

Figure 7: Earth-1996FG3-Orpheus cruise

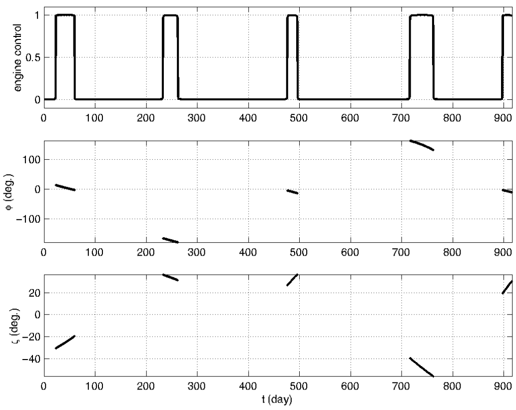

Figure 8: Optimal engine control and thrust angles

The optimal control law is defined by five burning periods (37 days + 28 days + 19 days + 46 days + 18 days). The flyby of the intermediate asteroid happens during a coasting arc just after the two first thrust phases. The associated control law and control angles (collinear (f) and orthogonal (z) to the relative orbital frame) are proposed in Fig. 8. The ΔV requirement for this cruise is 5.69 km/s which yields a final spacecraft mass of 421 kg.

In comparison with the direct trajectory presented in section 4.2, only an additional ΔV of 63 m/s allows the flyby of 1996FG3. This makes immediately the initial direct mission to Orpheus less attractive. Even if only the flyby of 1996FG3 is desired, other asteroids may be visited passively, i.e. without the use of specific maneuvers. The passive conditions of flyby are defined as follows: a maximum distance of 5.10e+6 km and a maximum relative velocity modulus of 15 km/s. Under these constraints two additional asteroids are found: asteroid 2002CA26 and asteroid 2003FJ8. The flyby conditions of each asteroid are proposed in table 3.

Table 3: Conditions of passive flybys

	date	distance	flyby velocity
2002CA26	2013/02/05	2.3e+06 km	14.34 km/s
2003FJ8	2013/03/26	4.1e+06 km	12.55 km/s

Finally, the proposed trajectory to Orpheus is very interesting from a scientific point of view because four different NEAs (2002CA26, 2003FJ8, 1996FG3 and Orpheus) may be visited with a low ΔV requirement. In this way, this scenario may be considered as a reference example for ambitious NEA mission projects.

4.5 Summary

In conclusion of these numerical results, we propose a summary of all the presented missions. For all the missions the ΔV required for the escape phase represents roughly 37 % of the mission ΔV. This value is very important and a basic way to reduce it is to consider a dedicated launch. The upper bound of the propellant mass loaded on board is equal to 200 kg. This value may be split into 78 kg (which may be preserved in the case of a dedicated launch) for the escape phase and 122 kg for the heliocentric cruise. The ΔV requirement for the insertion phase around the target asteroids may be neglected for the ΔV assessment. Indeed, the expected values are very low because of the weak gravitational acceleration of the target body.

The duration of the proposed missions remains realistic, i.e. less then 3 years, and the ratio between the propellant mass and the launch mass is equal to 29.8 % for the ambitious 1996FG3-Orpheus mission (see section 4.4). In comparison, for the NEAR mission this ratio climbs to 40 % with in addition the need for a dedicated launch.

5. CONCLUSIONS

In this paper the optimization of trajectories to various NEAs has been solved through the use of advanced numerical methods, based on smoothing techniques and decomposition-coordination approach. The presented results showed the opportunity to achieve ambitious low-cost missions to multiple asteroids, such as the mission to Orpheus with the flyby of 1996FG3. The use of low-thrust engines allows to design the entire trajectory from the escape phase to the encounter with the target asteroid. This allows the probe to be launched as a GTO auxiliary payload on Ariane 5 and thereby the cost of a dedicated launch is saved.

Yet, in order to improve this preliminary mission analysis and make the results more realistic, a global “system” study should be done with more complex dynamical models (derived from N-body problems for example) and a complete spacecraft model.

6. REFERENCES

1. Prussing J.E. and Conway B.A, *Orbital mechanics*, Oxford University Press, 1993.

2. Massari M. and Bernelii-Zazzera F., Options for Optimal Trajectory Design of a Mission to NEOs using Low-Thrust Propulsion, *14th AAS/AIAA Space Flight Mechanics Conference*, AAS 04-137, 2004.

3. Farquhar R.W., Dunham D.W. and Mc Adams J.V., NEAR Mission Overview and Trajectory Design, *AAS/AIAA Astrodynamics Conference*, 1995.

4. Pontryagin L.S., Boltyansky V.G., Gamkrelidze R.V. and Mishchenko E.F., *The mathematical theory of optimal processes*, Pergamon Press LTD, 1964.

5. Bryson A.E. and Ho Y.C., Applied optimal control, Hemisphere Publishing corporation, 1975.

6. Betts J.T., *Survey of Numerical Methods for Trajectory Optimization*, Journal of Guidance Control and Dynamics, vol. 21, No. 2, 193-207, 1998.

7. Bertrand R. and Epenoy R., *New Smoothing Techniques for Solving Bang-Bang Optimal Control Problems – Numerical Results and Statistical Interpretation*, Optimal Control Applications and Methods, vol. 23, 171-197, 2002.

8. Bertrand R. and Epenoy R., *Decomposition Techniques for Computing Optimal Low-Thrust Interplanetary Trajectories with Path Constraints*, 2nd International Symposium on Low-Thrust Trajectories, Toulouse, France, June 2002.