JASON1: ON-GROUND GYRO-MAGNETIC ATTITUDE ESTIMATION IN CASE OF DEFINITIVE STAR TRACKERS LOSS

Johan Montel(1), Philippe Landiech(2), Odile Dunogues(3)

(1) Control and AOCS office, French National Space Center (CNES), Johan.Montel@cnes.fr
(2) Mini-satellite division, French National Space Center (CNES), Philippe.Landiech@cnes.fr
(3) *BT*-SYNTEGRA, France, odile.dunogues@syntegra.fr

1. ABSTRACT

After one year in orbit, a partial darkening of French-US mini-satellite Jason1 (PROTEUS platform) star trackers (STR) was noted. After almost three years, STRs deliver data with a degraded availability ratio (≈50%) and still allow the mission to respect the pointing specifications (better than 0.2°). An on-ground attitude estimation filter has been developed, in order to respond to the potential definitive STR unavailability. This filter is based on magnetometer, gyrometer and altimeter (nadir angle measurement) data fusion, within a dynamic extended Kalman filter. The principle behind the method is to estimate the spacecraft attitude at every visibility period and to reinitialise the on-board estimator that drifts under gyrometer errors. The filter architecture, tuning, and some preliminary results from in-flight spacecraft telemetry are presented in detail.

2. INTRODUCTION

The CNES/NASA satellite Jason1 is the first mission of the PROTEUS program. Successor of TOPEX-POSEÏDON in the field of ocean radar altimetry, Jason1 is a geocentric pointing mini-satellite. The on-board attitude estimation concept is based on a PROTEUS-generic gyro-stellar filter. After one year in orbit, a partial darkening of star trackers (STR) was noted. In order to cope with a possible definitive STR unavailability, an on-ground attitude estimation filter has been developed at *CNES Control and AOCS office*. The filter is based on magnetometer (MAG), gyrometer (GYR) and altimeter (ALTI) data fusion, within a dynamic extended Kalman filter. The general concept behind the method is to estimate the spacecraft attitude at every visibility period and to refresh the on-board attitude propagator that drifts under gyrometer errors. GYR biases will also be estimated and frequently uploaded.

This paper exposes:

- the functional concept of this back-up attitude estimation system
- the mathematical design of the filter and its tuning method
- the main results from real and simulated telemetry.

3. FILTER FUNCTIONAL CONCEPT

3.1 An on-ground multi-modes filter

This back-up filter has been installed *on-ground* because of the following reasons:

- in NOM, MAG TM acquisitions are not driven by the AOCS but by specific task monitoring. So MAG data are not directly usable on-board in this mode.
- ALTI measurements are not available for the on-board AOCS software (part of the payload data).
- The filter is rather complex: an on-board patch would be very difficult to implement.

Furthermore, the on-ground filter is adaptable to every AOCS modes:

- safe and transient modes: SHM and STAM,
- orbit operations modes: OCM2 and OCM4,
- normal mode: NOM.

3.2 Available sensors telemetry

Magnetometers (MAG)

MAGs provide a 1Hz three axis magnetic field measurement that can be compared to an earth magnetic model (IGRF), since the position of the space craft is always known with a precision much less than one kilometer. MAGs errors are:

- a white noise (std ≈ 0.1µT)
- biases, misalignments and scale factors that can be well calibrated using an on-ground filter.
- a sensibility wrt temperature that can be corrected using a model given by the manufacturer.

MAGs measurement are also disturbed by the spacecraft intern magnetic field which is due to MTB and other electrical equipment.

Gyrometers (GYR)

Two 2-axis GYRs are used in NOM, providing a 3-axis angular rate measurement. The frequency of the GYR telemetry is 1Hz. The measurement main errors are:

- the drift stability
- the angular random walk (ARW), which is a white noise on the rate measurement (≈ 10^{-4} °/s)
- the scale factor and misalignment errors, which have an important impact because of the orbital and yaw-steering movement of the satellite.

Altimeter (ALTI)
Apart from altitude and sea height measurement, the ALTI provides an estimation of the square angle between its antenna and the Nadir direction. The measurement errors are:

- a bias (≈ 0.005 deg^2),
- a white noise (std ≈ 0.02 deg^2). This white noise is constant in terms of *square angle* measurement, but variable in terms of *angle* measurement: the noise increases when the pointing gets close to the nadir direction.

ALTI measurement is available only when the satellite is flying over the sea, and when the Nadir-antenna angle is lower than ≈ 0.5°. The frequency measurement is about 1Hz.
The 1-axis angle measurement is not signed (conic angle measurement), and gives only information on the roll/pitch angles in Nadir pointing.

Other usefull TM
Since we use a *dynamic* filter, the following telemetry is also used by the filter: RWS rates and torques, MTB magnetic moment, THR torques (for OCM modes).

3.3 Operational concepts

k:k+1:k+2 plan
The general concept of the method is to estimate the spacecraft attitude at every visibility period and to refresh the on-board attitude propagator that drifts under GYR errors. GYR biases are also estimated and frequently uploaded.
The visibility periods are of about 2 hours long. After a visibility period #k, we have 2 hours to:

1. collect the satellite telemetry and ALTI measurements
2. run the estimation (Kalman) filter until the end of the visibility period #k in order to estimate the history of the satellite attitude and to identify other filter main states such as the GYR drift
3. propagate the final attitude until a date *after the beginning of the visibility period* #k+1
4. eventually produce the TC files that will update the on-board attitude.

This plan - named "k:k+1" – has an important drawback: in order to be performed at each visibility period, the propagation operation (#3) needs to know the potential TCs that have been taken into account on-board during the propagation time. Thus the plan "k:k+1" must be exluded. It is only possible to upload TC at every second visibility. That is what we call the "k:k+1:k+2" plan that is figured on Figure 1.

Context freezing
At the beginning of the visibility #k, the state vectors and covariances of the estimation filter are initialised with the values of the end of the last run, providing no discontinuity in the filtering process.

Nevertheless, in SHM mode, the first run will be initialised with a wide-angle uncertainty (up to 180°), but *the filter is robust to wide-angle errors.*

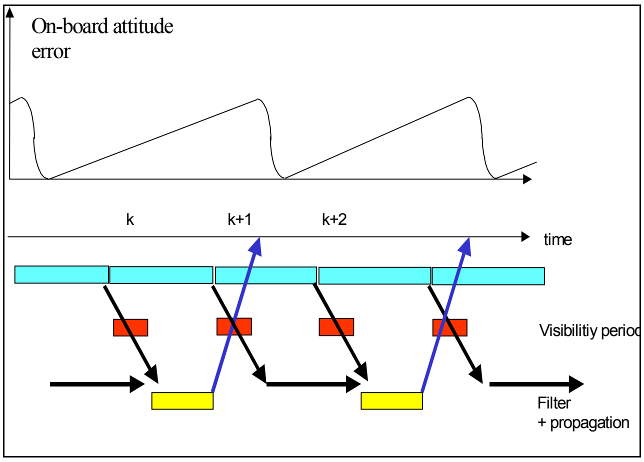

Figure 1: Attitude estimating and propagation concept.

4. FILTER DESIGN

4.1 An extended Kalman filter

The filter is an extended non-linear dynamic Kalman filter. The dynamic design is chosen because it enables filtering of GYR high frequency errors, and because the dynamic model of the spacecraft -its inertia matrix- is well identified. In order to function in degraded AOCS modes or operations, the filter is robust to wide-angle errors: the attitude state vector is the quaternion from LOF to Satellite Frame (SL).

4.2 Estimated parameters

Apart from the attitude quaternion, the other dynamic parameters are the angular rate and the external torque applied on the satellite. Estimating the external torque provides a better convergence of the filter and minimises residuals due to dynamic equations uncertainty. Some low frequency parameters have to be identified to allow good performance of the system: GYR drift and ESF, ALTI bias and misalignments, MAG biases. The evolution model of these parameters is supposed to be constant. Thus it is very simple to integrate them into the filter state vector. Let us note $\vec{X}$ the state vector of the Kalman filter:

$$\vec{X} = \begin{bmatrix} \vec{q} & \vec{\omega} & \delta\vec{g} & \vec{M}_g & \vec{T}_d & \vec{M}_a & \delta\theta^2 & \delta\vec{B} \end{bmatrix}^T$$

The structure of the Kalman filter has an elegant design in order to provide *good flexibility* in the choice of the estimated states. Some states that require specific guidance profiles or manoeuvres to be observable, for example GYR ESF and ALTI misalignments, can easily be "frozen" without changing the structure of the filter. *In routine mode, only attitude, angular rate, external torques and GYR biases will be estimated.*

4.3 Model evolution equations

The evolution of the state vector is based on cinematic and dynamic equations:

$$\dot{\vec{Q}} = \frac{1}{2}\vec{Q}\otimes\vec{\Omega} \qquad (1)$$

$$I_{SL}\dot{\vec{\Omega}} + \vec{\Omega}\wedge\left(I_{SL}\vec{\Omega} + \vec{H}_{RWS}\right) = \vec{T} \qquad (2) \text{ with}$$

$$\vec{\Omega} = \vec{\omega} + \vec{\omega}_{0\_SL} = \vec{\omega} + \vec{q}^* \otimes \vec{\omega}_0 \otimes \vec{q}$$

$$\vec{T} = \vec{T}_d + \vec{T}_{gg} + \vec{T}_{MAG} + \vec{T}_{MTB} + \vec{T}_{THR} + \vec{T}_{RWS}$$

and $\vec{z}_{LOF} = \vec{q}^* \otimes [0 \ \ 0 \ \ 1]^T \otimes \vec{q}$ is the direction of the 3rd vector of the LOF expressed in SL frame.

(1) and (2) can be expressed wrt the state vector:

$$\dot{\vec{q}} = \frac{1}{2}\vec{q}\otimes\vec{\omega} = F_q\left(\vec{X},\vec{U}\right)$$

$$\dot{\vec{\omega}} = -\dot{\vec{\omega}}_{0\_SL} + I_{SL}^{-1}\left[-\left(\vec{\omega}+\vec{\omega}_{0\_SL}\right)\wedge\left(I_{SL}\left(\vec{\omega}+\vec{\omega}_{0\_SL}\right)+\vec{H}_{RWS}\right)+\vec{T}\right] = F_\omega\left(\vec{X},\vec{U}\right)$$

with: $\vec{\omega}_{0\_SL} = \vec{q}^* \otimes \vec{\omega}_0 \otimes \vec{q}$

$$\dot{\vec{\omega}}_{0\_SL} = \frac{1}{2}\left(-\vec{\omega}\otimes\left(\vec{q}^*\otimes\vec{\omega}_0\otimes\vec{q}\right)+\left(\vec{q}^*\otimes\vec{\omega}_0\otimes\vec{q}\right)\otimes\vec{\omega}\right)+\vec{q}^*\otimes\dot{\vec{\omega}}_0\otimes\vec{q}$$

$$\omega_0 = \sqrt{\frac{\mu}{a^3}} \approx 0.053°/\text{s} \text{ and } \vec{\omega}_0 \approx \begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix}, \ \dot{\vec{\omega}} \approx \vec{0}$$

Apart from attitude and angular rate, the evolution of the other part of the state vector is assumed to be constant within the prediction model. So the evolution of the state vector can be expressed as:

$$\dot{\vec{X}} = F\left(\vec{X},\vec{U}\right) = \begin{bmatrix} F_q\left(\vec{X},\vec{U}\right) \\ F_\omega\left(\vec{X},\vec{U}\right) \\ \vec{0}_{21} \end{bmatrix}$$

4.4 Measurements

Measurement equations represent the relation between the measures delivered by the sensors and the state vector:

$$\vec{Y} = H\left(\vec{X},\vec{U}\right)$$

Magnetic measurement

MAGs deliver a measurement of the magnetic field in SL frame: $\vec{B}_{mes}$. In the sense of the Kalman filter, we consider the following measurements: $\vec{Y}_1 = \begin{bmatrix} \vec{b}_0 \wedge \vec{b}_{meas} \\ \vec{b}_0 . \vec{b}_{meas} \end{bmatrix}$

The scalar product is useful to treat the case where $\|\vec{q}\|. > 90°$.

$$\vec{b}_{meas} = \frac{\vec{B}_{meas}}{\|\vec{B}_{meas}\|} = \frac{\vec{B}+\delta\vec{B}}{\|\vec{B}+\delta\vec{B}\|} \approx \frac{\vec{B}+\delta\vec{B}}{\|\vec{B}\|} = \vec{b} + \frac{\delta\vec{B}}{\|\vec{B}\|} = \vec{b} + \frac{\delta\vec{B}}{\|\vec{B}_0\|}$$

(we assume here that MAG biases are small wrt magnetic field module.)

$$H_1\left(\vec{X},\vec{U}\right) = \begin{bmatrix} \vec{b}_0 \wedge \left(\vec{q}^* \otimes \vec{b}_0 \otimes \vec{q} + \dfrac{\delta\vec{B}}{\|\vec{B}_0\|}\right) \\ \vec{b}_0 . \left(\vec{q}^* \otimes \vec{b}_0 \otimes \vec{q} + \dfrac{\delta\vec{B}}{\|\vec{B}_0\|}\right) \end{bmatrix}$$

GYR measurement

GYRs deliver a measurement of the inertial angular rate of the satellite:

$$\vec{Y}_2 = \vec{\Omega}_{meas}$$

$$H_2\left(\vec{X},\vec{U}\right) = M_g\left(\vec{\omega} + \vec{q}^* \otimes \vec{\omega}_0 \otimes \vec{q}\right) + \delta\vec{g}$$

ALTI measurement

The ALTI measurement is the square angle between the antenna and Nadir directions:

$$\vec{Y}_3 = \theta_{meas}^2$$

As the measured angle is always < 0.5° (otherwise we would not be able to measure it) we can write:

$$\theta_{meas}^2 \approx 1 - 2\cos\left(\theta_{meas}\right) = 1 - 2\vec{z}_{ALTI\_SL} . \vec{z}_{NAD\_SL}$$

where:

$\vec{z}_{ALTI\_SL}$ is the ALTI antenna direction is SL frame (unit vector) and $\vec{z}_{NAD\_SL}$ is the Nadir direction expressed in SL frame (unit vector).

$$H_3 = \begin{bmatrix} \beta_x \\ \beta_y \\ \sqrt{1-\beta_x^2-\beta_y^2} \end{bmatrix} \bullet \left(\vec{q}^* \otimes \vec{z}_{NAD\_LOF} \otimes \vec{q}\right)$$

4.5 Discrete Kalman filter equations

Prediction equations

We consider a second order sampling method:

$$\vec{X}_{k+1/k} = \vec{X}_{k/k} + \frac{\delta t}{2}\left(3F\left(\vec{X}_{k/k},\vec{U}_k\right) - F\left(\vec{X}_{k-1/k-1},\vec{U}_{k-1}\right)\right)$$

$$P_{k+1/k} = F_k P_{k/k} F_k^T + Q_k$$

$$\text{where } F_k = \frac{\partial}{\partial\vec{X}}\left(F\left(\vec{X},\vec{U}\right)\right)_{|t=t_k}$$

Particular case of the quaternion prediction: we adopt a <u>Wilcox second order</u> development.

Correction equations

$$\vec{X}_{k+1/k+1} = \vec{X}_{k+1/k} + K_{k+1}\left(\vec{Y}_{k+1} - H_{k+1}\vec{X}_{k+1/k}\right)$$

$$P_{k+1/k+1} = \left(I - K_{k+1}H_{k+1}\right)P_{k+1/k}$$

$$K_{k+1} = P_{k+1/k}H_{k+1}^T\left(H_{k+1}P_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1}$$

$$\text{where } H_k = \frac{\partial}{\partial\vec{X}}\left(H\left(\vec{X},\vec{U}\right)\right)_{|t=t_k}$$

Computation of F_k and H_k
The computation of the partial derivatives of F and H wrt state vector is given in document [4].

4.6 System observability

Attitude observability
MAGs provide a 2-axis instantaneous attitude observability: the non-observable direction is the direction of the magnetic field. In NOM, this direction varies in SL frame because of the orbital and yaw-steering motions. Two cases have to be considered:

1. yaw fix (when β<15°)
- SL Z-axis: the evolution is harmonic, with a period of ω0, and an amplitude of ≈ 32μT.
- SL Y-axis: ≈ constant = $-B_0 \cos(i)\cos(\phi) \approx -7\mu T$ for ϕ=0.
- SL X-axis: the evolution is harmonic, with a period of ω0, and an amplitude of ≈ 16μT.

2. yaw steering (when β>15°)
- SL Z-axis: harmonic evolution, with a period of ω0, and an amplitude of ≈ 32μT.
- SL Y-axis: ≈ harmonic evolution
- SL X-axis: harmonic evolution, with a period of 2ω0, and an amplitude of ≈ 10μT, and a bias of -5μT for the extreme case β = 15°.

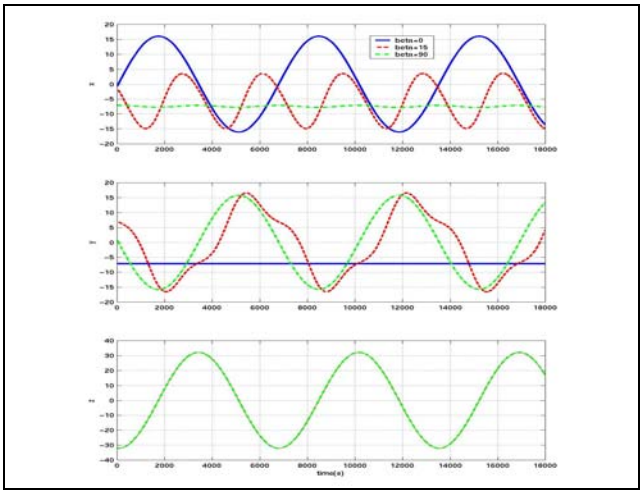

Figure 2: SL magnetic field for β=0°, 15°, 90°.

⇒ The system observability depends on the β angle. In all guidance cases, the magnetic field varies enough in SL frame to allow a good 3-axis observability over one orbit. Nevertheless, the bandwith of the system will be limited by the duration of the magnetic field movement in SL frame, i.e. $\approx\omega_0$: MAG measurement provides a low frequency 3-axis observability.

GYR drift and ESF observability
In order to distinguish GYR drift from ESF, it is necessary to have large SL angular rate variations. The standard way to estimate GYR ESF is to perform high amplitude calibration maneuvres. Because of the loss of agility due to the loss of one of the four reaction wheels, these maneuvres are no longer feasible. Thus, the foreseen method in case of definitive loss of stellar measurement is to use the orbital and yaw-steering (yaw max) movements within our Kalman filter.

ALTI bias and misalignment observability
ALTI biases and misalignment cannot be distinguished from one another in Nadir pointing mode: the altimeter provides a constant measurement that is the misalignment square angle plus its bias. Special maneuvres - called "cross maneuvres" – were developed in order to estimate these parameters.

MAG bias observability
MAG biases are observable thanks to ALTI measurement without requiring specific attitude maneuvres.

4.7 Tuning

The parameters to be tuned are: the measurement noise covariance matrix R_k (8x8 elements, positive definite), the model noise covariance matrix Q_k (28x28 elements, non negative definite) and the initial covariance matrix.

$$\vec{X}_{k+1} = \vec{X}_k + F_k\left(\vec{X}_k, \vec{U}_k\right) + \vec{\varepsilon}_X \text{ with } E\left(\vec{\varepsilon}_X . \vec{\varepsilon}_X^{\,T}\right) = Q_k$$

$$\vec{Y} = H\left(\vec{X}, \vec{U}\right) + \vec{\varepsilon}_Y \text{ with } E\left(\vec{\varepsilon}_Y . \vec{\varepsilon}_Y^{\,T}\right) = R_k$$

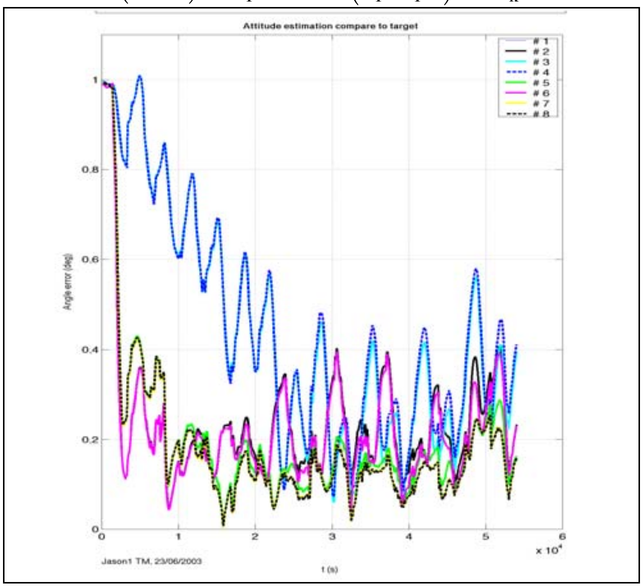

Figure 3: Attitude errors wrt several tunings

The tuning logic is:
- to consider only matrix diagonal elements
- to set the measurement noise equal to the sensor predicted noises (as seen on ground and during the in-orbit check out)
- to set the model noise in order to have the best filtering results (performance, robustness)
- to do a sensibility study wrt each parameters to find the optimal tuning (optimal attitude and GYR drifts estimations)

The system has been tuned on real Jason1 telemetry in order to take into account actual sensors' errors. The attitude of the satellite is given by the SST measurements that show that the spacecraft attitude is very close to the target attitude (better than 0.05°). Then, we are able to estimate the performance of the filter. Next figures show the temporal evolution of attitude errors based on actual telemetry.

5. MAIN RESULTS

Results based on simulated telemetry

Results based on simulated telemetry are very good, since the simulated sensor errors are the same as the errors that are taken into account in the Kalman filter (gaussian white noise).

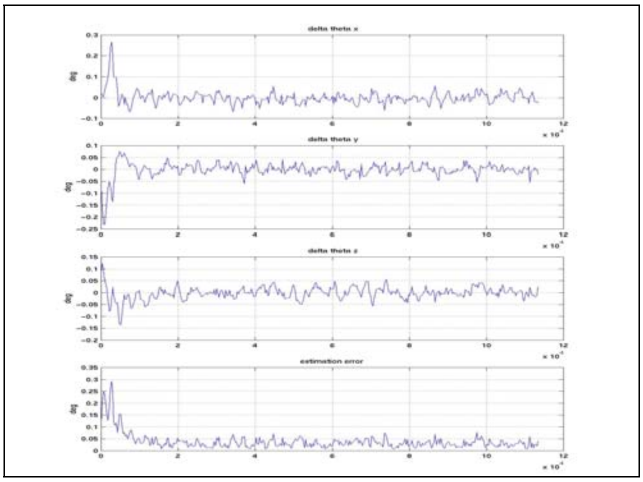

Figure 4: estimated attitude errors. Yaw max.

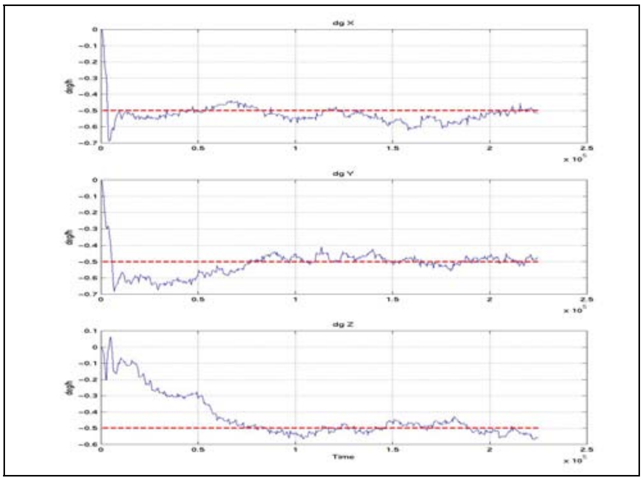

Figure 5: estimated GYR drifts. Yaw max.

The attitude estimation performance is around 0.05° and the long term GYR drift estimation performance is around 0.05°/h. The estimation of GYR ESC when the amplitude of the yaw-steering movement is maximum is around 500ppm (5.10^{-4}). Figure 4 and 5 present the evolutions of the attitude errors and estimated GYR drifts.

Results based on actual telemetry

Actual MAG measurements are not as good as expected. The level of the measurement remainders (wrt estimated attitude) is around 0.5µT (3σ), and is due to several factors: disturbance of the measured magnetic field due to temperature influence on MAG or on-board Foucault courrents, precision of the IGRF (earth magnetic field model), MAG biases and scale factors calibration performance…

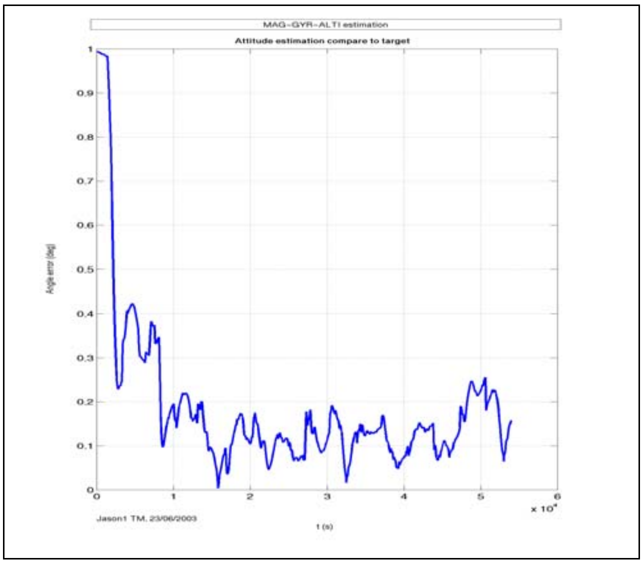

Figure 6: attitude estimation error

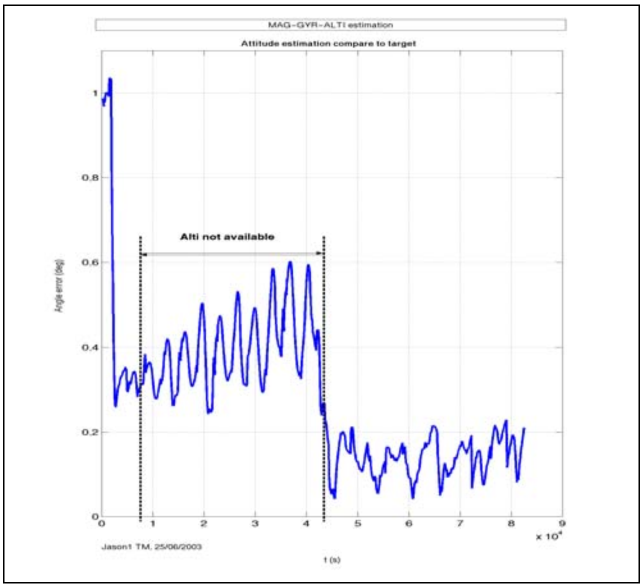

Figure 7: influence of ALTI availability

The performance of *a posteriori attitude estimation* is obtained with comparison to the on-board GYR-SST attitude estimation. *The performance is around 0.2° (3σ).* Figure 6 presents the attitude error evolution during 20 hours period, with an initial error of 1°. The duration of the convergence phase is about one orbit. Furthermore, the performance is dependant on the ALTI measurement availability. If the ALTI is not available during a long phase, the performance is reduced, as shown in Figure 7. The performance of the attitude estimation does not allow us to estimate GYR ESC. The quality of the estimation of GYR drifts is difficult to predict. Nevertheless, the actual Jason1 GYR drift variations and uncertainties are rather low (<0.05°/h in almost one year). In the case of a k:k+1:k+2 plan - i.e. ≈7 hours of attitude propagation - the expected performance of the global system "estimation–propagation" is *0.3° (typical case) - 0.55° (worst case).* This performance enables us to prolong the Jason1 mission in case of definitive loss of stellar measurements.

6. CONCLUSION

This on-ground coarse attitude estimation concept based on MAG, GYR and ALTI measurements has been developed to deal with a potential definitive loss of both STR of Jason1. The predicted performance of the global system – 0.3° (typical case) to 0.55° (worst case) - allows us to prolong the Jason1 mission in case of definitive loss of stellar measurements.

7. NOMENCLATURE

MAG	Magnetometers
GYR	Gyrometers
ALTI	Altimeter
STR	Star Tracker
MTB	Magneto-Torquer Bars
AOCS	Attitude and Orbit Control System
SHM	Safe Hold Mode
STAM	Stars Aquisition Mode
NOM	NOrmal Mode
OCM	Orbit Control Mode
TM	Telemetry
LOF	Local Orbital Frame
SL	Satellite Frame
ESF	Equivalent Scale Factor (of the gyrometers)
std	Standard Deviation

$\vec{q}$	Spacecraft attitude quaternion from LOF to SL.
$\vec{\omega}$	Angular rate of the spacecraft wrt ROL, expressed in SL frame.
$\delta\vec{g}$	Gyrometers bias, expressed in SL frame.
M_g	ESF matrix. Size 3x3.
$\vec{T}_d$	External residual disturbance torques on the spacecraft, expressed in SL frame.
$\vec{T}_{gg}$	Gravity gradient torque, expressed in SL frame.
$\vec{M}_{SL}$	Spacecraft internal magnetic disturbance momentum, expressed in SL frame.
$\vec{M}_{MTB}$	MTB magnetic momentum expressed in SL frame.
$\vec{T}_{THR}$	THR torque, expressed in SL frame.
$\vec{T}_{RWS}$	RWS torque *applied on the satellite*, expressed in SL frame.
$\vec{H}_{THR}$	RWS cinetic momentum, expressed in SL frame.
I_{SL}	Spacecraft inertia matrix
$\vec{M}_a$	Misalignment bias between the magnetometer reference frame and the altimeter axis: $\vec{M}_a = [\beta_x \quad \beta_y]^T$.
$\delta\theta^2$	Altimeter bias (rad^2)
$\delta\vec{B}$	Magnetometer bias, expressed in SL frame (T)
$\vec{B}$	Magnetic field expressed in SL frame.
$\vec{b}$	$\vec{B}/\|\vec{B}\|$
$\vec{B}_{meas}$	Measured magnetic field expressed in SL frame.
$\vec{b}_{meas}$	$\vec{B}_{meas}/\|\vec{B}_{meas}\|$
$\vec{B}_0$	Predicted magnetic field expressed in LOF, stemmed from IGRF model.
$\vec{b}_0$	$\vec{B}_0/\|\vec{B}_0\|$
$\vec{\Omega}$	Inertial spacecraft angular rate, expressed in SL frame.
$\vec{\Omega}_{meas}$	Measured inertial spacecraft angular rate, expressed in SL frame.
$\vec{\omega}_0$	Orbital inertial angular rate, expressed in LOF frame.
θ^2_{meas}	Altimeter measurement (square Nadir-ALTI angle, rad^2)
$\vec{Q}$	Spacecraft attitude quaternion from J2000 inertial frame to SL.
$\vec{z}_{NAD\_LOF}$	Nadir direction expressed in LOF. (unit vector)

8. REFERENCES

[1] J. Hashmall, K.S. Liu, M. Rokni Computer Sciences Corporation, *Accurate spacecraft attitudes from magnetometer data,* GSFC MS congress 1995, Toulouse.

[2] Ph. Landiech, CNES, *Extensive use of magnetometers and magnetotorquers for small satellites attitude estimation and control*, AAS 1995 Congress, Keystone.

[3] M. Psiaki, F. Martel, Ithaco , *Three axis attitude determination via Kalman Filtering of Magnetometer data,* Journal of Guidance Vol 13 n°3

[4] J. Montel, *Jason1: On-ground gyro-magnetic attitude estimation in case of definitive star tracker loss: annexe equations.*