

# OPTIMUM TRAJECTORY FOR LOW-THRUST MULTIPLE TROJAN ASTEROIDS FLYBYS

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## ABSTRACT

It is known that there is an asteroid family around L4 or L5 Lagrange point of the Sun-Jupiter system, and it is called the Trojan asteroid family. As one of outer planet exploration mission, a plan to explore the Trojan asteroids using Jupiter swing-by is considered. Based on this mission, this paper discusses multiple flybys trajectory for Trojan asteroids exploration.

A trajectory optimization of Trojan asteroids exploration using multiple flybys depends largely on objects of exploration. For this reason, the stage of choosing the candidates of exploration is important, and it is indispensable to choose most qualified candidate for the mission.

As the first step, this paper proposes the flyby sequence search strategy which uses a ballistic orbit, and using these results, the multiple flybys trajectory to Trojan asteroids by impulsive velocity change is designed. Next, this paper considers an optimization of multiple slow-flybys trajectory of the spacecraft which uses low thrust propulsion. Furthermore feasibility of the sequence of main-belt asteroid flyby on the way to Trojan asteroid was studied.

## 1. INTRODUCTION

One of objects of outer planet exploration mission is asteroid. There are more than tens of thousands of asteroids whose orbits are known, between the orbits of Mars and Jupiter. And this number is increasing day by day by the observation.

These asteroids are categorized by shape, size, composition and mass. And a number of sets of asteroid called "Family" were discovered. In addition, since asteroid's size is very small, data which one asteroid has is very limited. For this reason, multiple flybys exploration is very useful for asteroid exploration.

There are crucial differences between one planet exploration and multiple asteroids exploration.

Unlike in the case of planet exploration, multiple asteroids exploration has not obvious candidates of exploration. In other words, at the stage of choosing candidates of exploration, those candidates underspecified. And a trajectory optimization of Trojan asteroids exploration using multiple flybys depends largely on objects of exploration. For this reason, the stage of choosing the candidates of exploration is important, and it is indispensable to choose most qualified candidate for the mission.

So, in this study, first, the flyby sequence search strategy which uses a ballistic orbit is proposed, and using these results, the multiple flybys trajectory to Trojan asteroids by impulsive velocity change is designed. Next, this paper considers an optimization of multiple slow-flybys trajectory of the spacecraft which uses low thrust propulsion. Furthermore feasibility of the sequence of main-belt asteroid flyby on the way to Trojan asteroid was studied.

## 2. TROJAN ASTEROID

There are thousand of asteroids around L4 or L5 Lagrange point of the Sun-Jupiter system. And those asteroids are called the Trojan asteroid. Trojan asteroid family is one of recent discovery of the asteroids group. It is in the area that a spacecraft has not explored yet.

There are several mission design candidates to visit the Trojan asteroids. The most straightforward way is the direct transfer from the earth. However, the accelerating of the spacecraft to reduce the relative velocity at the asteroids encounter results in requiring higher delta-V. Another way, more practical one, is taking advantage of the Jupiter gravity assist to increase the speed and fly adjacent to the Jupiter. This way can achieve the slow flyby with less fuel compared to the former method.

This type of mission can be divided into three stages. The first is the energy accumulation stage to the Electric Delta-V Earth Gravity Assist (EDVEGA) [4]. The Next

stage is the transfer to Jupiter from the Earth, and the spacecraft perform Jupiter swing-by. The Last stage is the transfer to Trojan asteroids from Jupiter. This paper discusses the third segment of this mission sequence.

Figure 1 shows the distribution of Trojan asteroids.

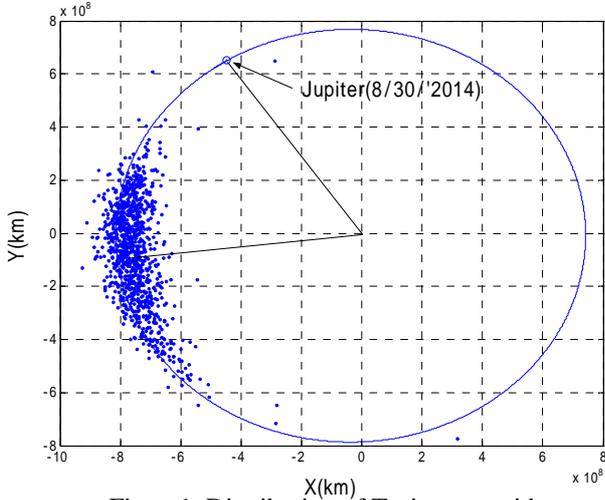


Figure1. Distribution of Trojan asteroids

### 3. THE ASSUMPTION FOR THIS MISSION

#### 3.1 Initial Condition

This paper discusses the transfer to Trojan asteroids from Jupiter. Initial condition of this problem is assumed as follows,

- In the previous study of the Jupiter exploration, a launch opportunity in 2010 was applied, which was one of the best conditions to transfer from Earth to Jupiter. The spacecraft arrives at Jupiter about 2014. So this paper assumed that the date for Jupiter flyby is August 30, 2014.

- The relative velocity of the spacecraft with respect to Jupiter is assumed to be 7.0 km/s, for the acceleration by the Jupiter gravity assist. And this relative velocity direction can be changed by the Jupiter swing-by condition.

#### 3.2 Flyby Threshold

We assume that the spacecraft uses chemical propulsion. And in case that fuel mass is estimated at ten percent of a total spacecraft mass, usable delta-V is about 0.3km/s. Based on this reason, flyby threshold can be written as follows,

$$R_i = 0.3 [km/s] \times |x[s] - x_i[s]| \quad (1)$$

where  $x[s]$  is Flight time from Jupiter swing-by to target asteroid, and  $x_i[s]$  is Flight time from Jupiter swing-by to  $i$ th asteroid.

When  $r_i < R_i$ ,  $i$ th asteroid is selected as flyby candidates. Where  $r_i$  is the closest distance of the spacecraft with respect to  $i$ th asteroid.

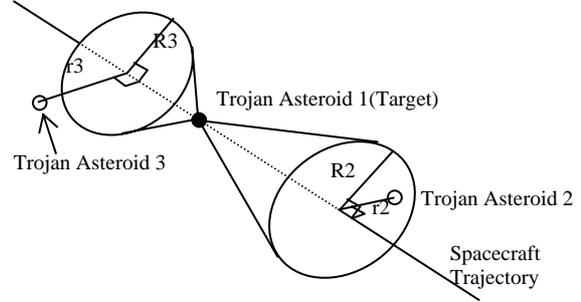


Figure2. Flyby Threshold

### 4. EXPLORATION OF THE TROJAN ASTEROIDS

System model is considered two-body problem. This model considers only the Sun gravity, since the sphere of influence of the Jupiter is small enough compared with the scale of the trajectory considered here.

In this section, as the first step, multiple flyby sequence using impulsive maneuvers is discussed. Next, the trajectory for Trojan asteroids exploration using low-thrust propulsion is optimized.

#### 4.1 Multiple Flyby Sequence using Impulsive Maneuvers

In this section, multiple flyby sequence using impulsive maneuvers is searched and evaluated.

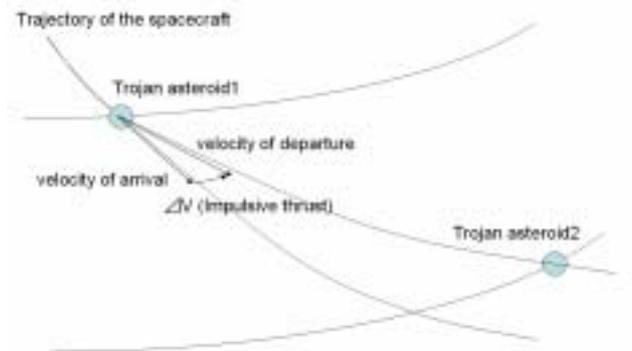


Figure3. Impulsive thrusts transfer

#### • Flyby sequence search strategy

This strategy is divided into two phase, as follows,

### Flyby candidates selection phase

1. The trajectory is searched to fly from Jupiter to each Trojan asteroid.
2. Candidates asteroids are selected from Trojan asteroid family within flyby threshold area.

### Evaluation and optimization phase

3. We apply Lambert's law to above results, and the trajectory to pass through the two asteroids is searched.
4. The trajectory of minimization of fuel consumption is chosen.
5. The preferable results of two asteroids flybys are applied to Lambert's law. The trajectory to pass through the three asteroids is searched and optimized.

### • Results - Impulsive thrusts transfer

First, the results of two Trojan asteroids flybys trajectories are categorized by total delta-V and the case of more than 0.3 km/s are eliminated for candidates.

Table1. Results of the first selection

Total V	The number of Trajectory
~ 0.03 km/s	16
~ 0.05 km/s	32
~ 0.3 km/s	212

Next, the results of three Trojan asteroids flybys trajectories are categorized by total delta-V and the case of more than 1.0 km/s are eliminated for candidates.

Table2. Results of the second selection

Total V	The number of Trajectory
~ 0.5 km/s	3
~ 1.0 km/s	17

We assume that the spacecraft uses chemical propulsion. In case that the requested delta-V is 0.3km/s, fuel mass needs about ten percent of a total spacecraft mass. Therefore, we insist the flyby exploration of two or three Trojan asteroids is feasible.

## 4.2 Low thrust transfer problem

In this section, the orbital transfers from Jupiter to Trojan asteroids are adjusted by low-thrust propulsion systems. To find the steering law the optimum control

problem is considered. The problem can be formulated as a multi-Point Boundary Value Problem which is difficult to solve analytically. So the steering law is optimized by using DCNLP (Direct Collocation with Nonlinear Programming).

### • Formulation

#### -The equation of motion-

This paper uses the equation of motion of the two-body problem. The equation considers only the Sun gravity. The equation of motion of the spacecraft is written as follows,

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{a} \quad (2)$$

where  $\ddot{\mathbf{r}}$  is the inertial acceleration and  $\mathbf{r}$  is the position vector.  $\mu$  is the gravitational parameter of the sun and  $\mathbf{a}$  is external forces.

#### -Constraints-

Suppose the spacecraft is equipped with electric propulsion, an ion engine, so that the thrust is very small. This thrust is assumed by

$$0 \leq |\mathbf{a}| \leq 4 \times 10^{-5} (m/s) \quad (3)$$

#### -Criterion Function-

The minimum fuel consumption problem is treated, so the criterion is written as follow,

$$J = \int_{t_0}^{t_f} |\mathbf{a}| dt \quad (4)$$

#### -Boundary Condition-

According to the section 2, as the spacecraft leaves the Jupiter at the initial time, the initial conditions can be written as follows,

$$\begin{pmatrix} \mathbf{r} - \mathbf{r}_j \\ \mathbf{v} - (\mathbf{v}_j + \mathbf{v}_0) \end{pmatrix} = 0 \quad (5)$$

where  $\mathbf{r}$  and  $\mathbf{v}$  are the position and inertial velocity of the spacecraft.  $\mathbf{r}_j$  and  $\mathbf{v}_j$  are the position and inertial velocity of the Jupiter. And  $\mathbf{v}_0$  is the relative velocity of the spacecraft with respect to Jupiter.

And as we consider the condition of the spacecraft when the spacecraft passes through the asteroid, the other boundary conditions are written as follows,

$$(\mathbf{r} - \mathbf{r}_i) = 0 \quad (6)$$

where  $\mathbf{r}_i$  is the position of the Trojan asteroid. And  $i$  is number of the Trojan asteroid.

**• Results - Low thrust transfer**

In this numerical simulation, we applied DCNLP to typical results of the multiple flyby sequence on the section 3.1. Numerical results of this problem are shown in Figure 4-6 and Table 3. The total delta-V of this to achieve trajectory is 0.4688 km/s.

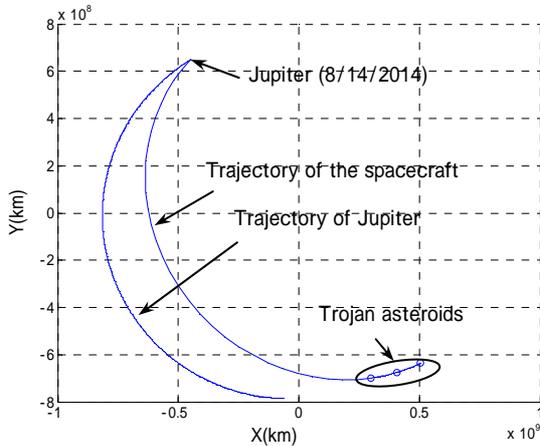


Figure4. Flyby trajectory

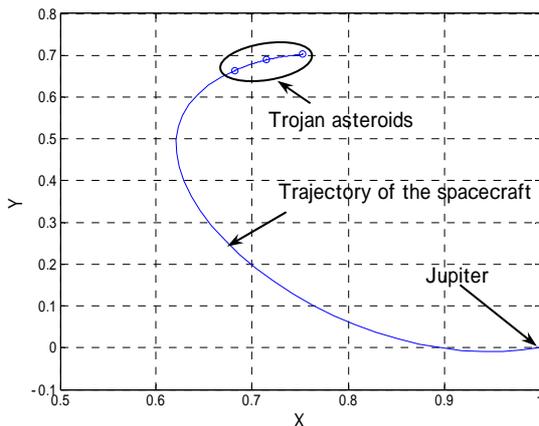


Figure5. Flyby trajectory in the Sun-Jupiter System

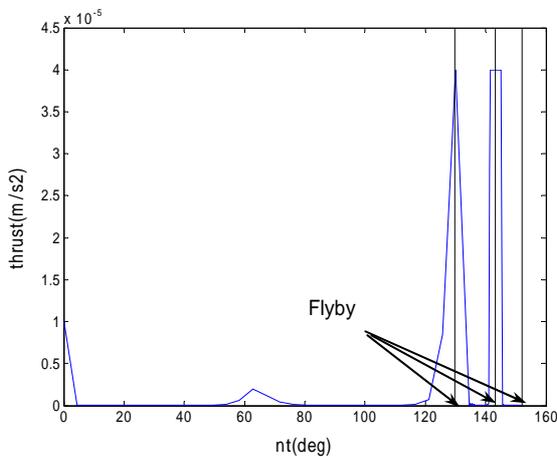


Figure6. Control inputs

In this simulation, each flyby time is divided into 30 segments. And the time of each boundary condition is free. This result has three peaks, and they are almost bang-bang control inputs.

By thrusts-level increasing, this result comes to the impulsive transfer.

Table3. Summary of flyby trajectory

	1	2	3
Designation(and name)	1998YW6 (Irus)	5151T-2 (Tydeus)	1999YR12
H(Absolute visual magnitude)	11.8	11.8	14.1
Flight time from Jupiter swing-by[day]	1619.9	1723.2	1825.6
relative velocity [km/s]	6.6463	5.4957	6.1665
$\theta_1$ [deg]	100.87	101.45	101.71

For this result, we insist the flyby exploration using established low thrust propulsion is feasible.

$\theta_1$  in Table3 is the observational parameter which is defined as follows,

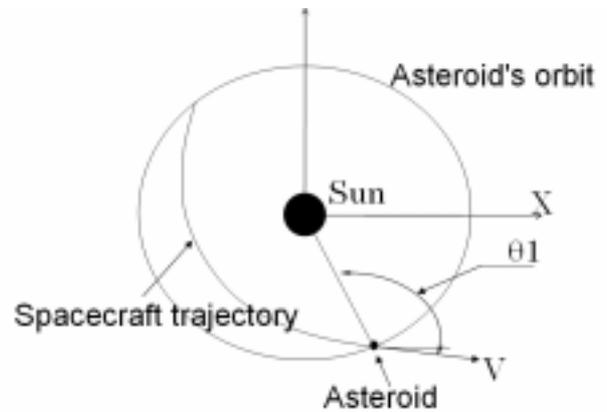


Figure7. Setting of the observational parameter

V is inertial velocity of the spacecraft and  $\theta_1$  is the supplementary angle between r and v. When this parameter is  $\theta$ , the spacecraft have a good view of an asteroid.

**5. EXPLORATION WITH THE MAIN-BELT ASTEROIDS**

As is shown in Figure 8, the trajectory of Trojan asteroids exploration passes through the area between 3.5 AU and 5 AU, where main-belt asteroids exist (Asteroid-Belt). And so feasibility of the sequence of main-belt asteroid flyby on the way to Trojan asteroid is studied.

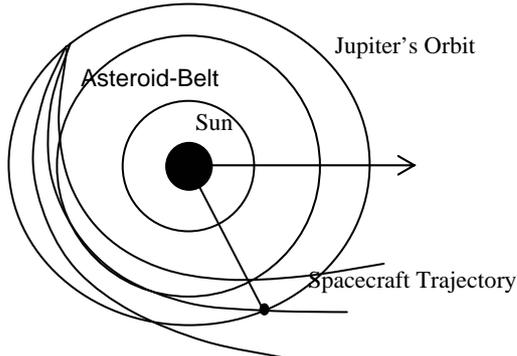


Figure8. Asteroid Belt

### 5.1 Selection of Main-belt Asteroids

Presently, there are tens of thousands of asteroids whose orbits are known, among the main-belt asteroids. We considered relation between the period of each main-belt asteroid orbit and trajectory design of Trojan asteroids exploration, and choose the main-belt asteroids. Selected asteroids are shown in Figure 9.

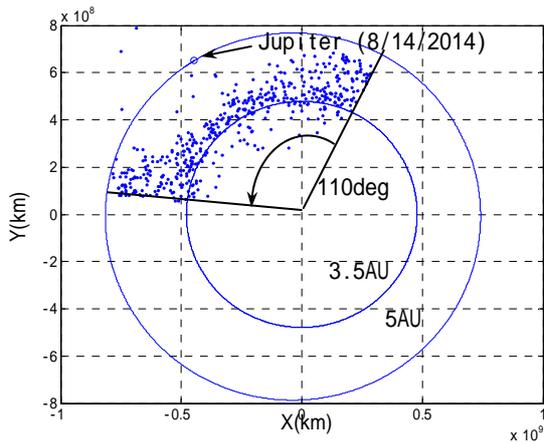


Figure9. Distribution of searched main-belt asteroids

### 5.2 Multiple Flyby Sequence using Impulsive Maneuvers

In this section, multiple flyby sequence using impulsive maneuvers is searched and evaluated.

#### • Flyby sequence search strategy

The Numerical method is the same as the section 4.1. This method applies to the trajectories that pass through two Trojan asteroids and consume fuel within total delta-V 0.3km/s. In this section, we solved the trajectory that pass through one main-belt and two Trojan asteroids.

#### • Results - Impulsive thrusts transfer

Numerical results of this problem are shown. The results of one Main-belt asteroid and two Trojan

asteroids flybys trajectories are categorized by total delta-V and the case of more than 0.5 km/s are eliminated for candidates.

Table4. Results of the selection

Total V	The number of Trajectory
~ 0.3 km/s	16
0.3 ~ 0.5 km/s	25

We assume that the spacecraft also uses chemical propulsion, and insist the flyby exploration of one main-belt asteroid and two Trojan asteroids is feasible.

### 5.3 Low thrust transfer trajectory optimization

In this section, the orbital transfers from Jupiter to the main-belt asteroids and Trojan asteroids are adjusted by low-thrust propulsion systems. This problem is solved by similar methods of the section 4.2.

#### • Formulation - Low thrust transfer

The assumption of the flyby to main-belt asteroid is written as follow,

$$(\mathbf{r} - \mathbf{r}_a) = 0 \quad (7)$$

where  $r_a$  is the position of the main-belt asteroid. And  $a$  is number of the main-belt asteroid. The formulation of this problem is the same as section 3.3 with the exception above.

#### • Results – Low thrust transfer

In this numerical simulation, we applied DCNLP to typical results of the multiple flyby sequence on the section 5.2. Numerical results of this problem are shown in Figure 10-12 and Table 5. The total delta-V of this to achieve trajectory is 0.1541 km/s.

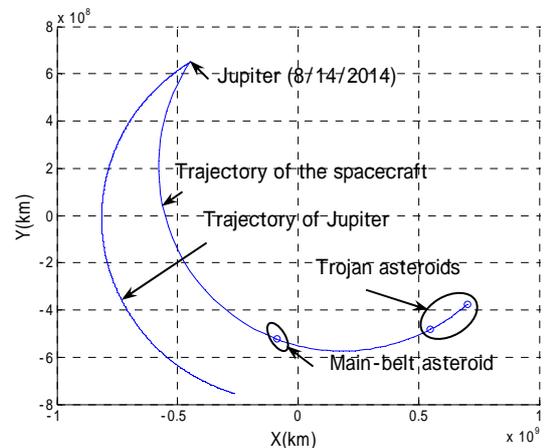


Figure10. Flyby trajectory

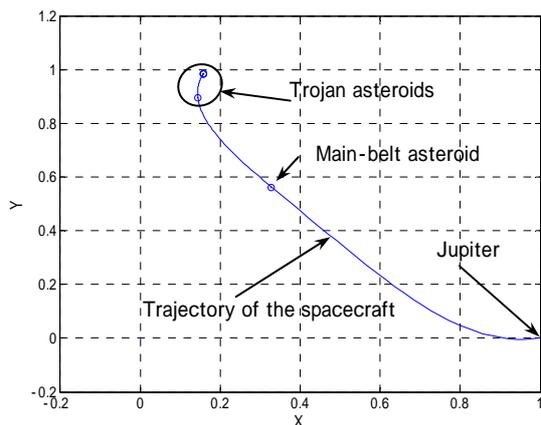


Figure11. Flyby trajectory in the Sun-Jupiter System

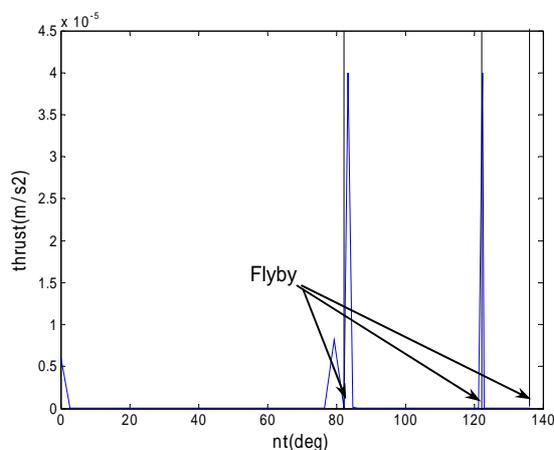


Figure12. Control inputs

In this simulation, each flyby time is divided into 30 segments. And the time of each boundary condition is free. This result has three peaks, and they are almost bang-bang control inputs.

By thrusts-level increasing, this result comes to the impulsive transfer.

Table5. Summary of flyby trajectory

	1	2	3
Designation(and name)	2003FK11 0	1996TS49 Augeias	1996TN49
H(Absolute visual magnitude)	14.6	10.8	11.9
Flight time from Jupiter swing-by[day]	989.2	1475.3	1642.7
relative velocity [km/s]	6.4238	6.8874	7.9014
$\theta_1$ [deg]	98.249	111.00	112.20

For this result, we insist the flyby exploration using established low thrust propulsion is feasible.

## 6. CONCLUSION AND FUTURE WORKS

First, the search strategy was constructed for multiple flyby exploration using impulsive transfer, and we concluded the multiple flyby exploration of two or three Trojan asteroids is feasible. Next, using those results, the flyby Trajectory of Trojan asteroids exploration using low-thrust propulsion was optimized. Furthermore feasibility of the sequence of main-belt asteroid flyby on the way to Trojan asteroid was studied and we concluded the multiple flyby exploration of one asteroid and two Trojan asteroids is feasible.

In the future, more practical trajectory that includes several observational constraints, such as relative velocity of the spacecraft with respect to each Trojan asteroid, would be considered. And we will perform in-depth analysis of the trajectory design using constraints of various low-thrust propulsion systems.

## 7. REFERENCES

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