OPTIMAL FLIGHT TO NEAR-EARTH ASTEROIDS WITH USING ELECTRIC PROPULSION AND GRAVITY MANEUVERS

A. V. Chernov

Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russia, E-mail: chernov@keldysh.ru

ABSTRACT

Optimal space flight to near-Earth asteroid for deflection asteroids from the Earth and prevention their possible collision is investigated. The deflection is realized by means of impact-kinetic effect of the spacecraft on the asteroid and changing the asteroid orbit. The effectiveness of this method for preventing asteroid-Earth collision is estimated by means of optimal space flights, which are found. The flight of spacecraft (SC) is realized by means of using electric propulsion system. To increase effectiveness the optimal gravity maneuvers of spacecraft near Mars and Venus are using. Criterion of the space flight optimization is maximal deflection of the asteroid from the Earth at the moment of asteroid-Earth nearest approach. For determination of optimal trajectories the maximum Pontrjagin principle is used. It is assumed that the thrust of electric propulsion is unrestricted or corresponding to electric engine SPT-140 with solar battery as energy source. The technique of choice of first approximation for optimal trajectory determination on base of search optimal trajectories in more simple statement is used. The optimal trajectories are determined for wide ranges of the space flight times to near-Earth asteroids with different orbit elements. A comparison with a case of the space flight using a high thrust only or without gravity-assisted maneuvers is carried out.

1. INTRODUCTION

The opportunity of prevention of Earth-asteroid collision by means of impact-kinetic influence of spacecraft on the dangerous asteroid is investigated. SC is sent to the asteroid, impacts with it and, as result, the asteroid orbit is changed and the asteroid is deflected from the Earth. Efficiency of this method was investigated for various schemes of SC flight to asteroid Toutatis [1-7]. The efficiency of this method for the one-impulse SC flights is estimated in [1]. Case of the optimal two-impulse trajectories is considered in [2]. Using Lunar gravity assist for SC flight to deflect of asteroid is offered in [3]. The SC optimal flights with low thrust is studied in [4-7]. In these papers criterion of the space flights optimization is maximal deflection of the asteroid from the Earth at the moment of asteroid-Earth nearest approach. The opportunity of increase of the impact-kinetic influence efficiency by means of using low thrust and gravity assist maneuver near Mars or Venus is investigated in this paper. The main attention is given a case of the ideal low thrust. Besides optimal trajectories with the constrained low trust appropriate to electric engine SPT-140 and solar batteries as energy sources are received in vicinity of the optimal trajectories for ideal thrust.

2. THE PROBLEM STATEMENT

At first the SC with mass M_0 moves along an initial circular near-Earth parking orbit with radius R_0. The geocentric motion is realized by means of high thrust engine with the gas exhaust velocity c_1. At moment t_1 the SC is applied the velocity impulse $\Delta\boldsymbol{V}$ and the SC is inserted on the hyperbolic orbit with velocity "on infinity" $\boldsymbol{V}_{\infty 1}$:

$$\boldsymbol{V}_{\infty 1} = \boldsymbol{V}(t_1) - \boldsymbol{V}_E(t_1), \tag{1}$$

where $\boldsymbol{V}$ and $\boldsymbol{V}_E$ are velocities of the SC and the Earth respectively. The SC mass after geocentric acceleration with value of velocity $V_{\infty 1} = |\boldsymbol{V}_{\infty 1}|$ is

$$M(t_1) = M_0 \exp(-\Delta V/c_1), \tag{2}$$

$$\Delta V = |\Delta \boldsymbol{V}| = (V_{\infty 1}^{2} + 2\mu_E/R_0)^{1/2} - (\mu_E/R_0)^{1/2} \tag{3}$$

The heliocentric motion is realized by means of the low thrust engine. At moment t_m SC approaches to planet (Mars or Venus) and realizes the gravity maneuvers. As result, the SC velocity in steps changes. This change is restrict:

$$\left|\boldsymbol{V}_{\infty p}^{-}\right| = \left|\boldsymbol{V}_{\infty p}^{+}\right| = V_{\infty p} \tag{4}$$

where

$$\boldsymbol{V}_{\infty p}^{-} = \boldsymbol{V}(t_m^{-}) - \boldsymbol{V}_p(t_m),\ \boldsymbol{V}_{\infty p}^{+} = \boldsymbol{V}(t_m^{+}) - \boldsymbol{V}_p(t_m), \tag{5}$$

$V(t_m^-)$, $V(t_m^+)$ are the SC velocity before and after the SC-planet approach, V_p is the planet velocity. The restriction is imposed on the pericenter radius R_π of a hyperbolic orbit in planet vicinity:

$$R_\pi \geq R^*, \tag{6}$$

where R^* is some critical distance determined by the planet radius. To fulfill $R_\pi = R^*$, the additional condition is imposed on the velocity change at moment t_m:

$$\boldsymbol{V}^+_{\infty p} \boldsymbol{V}^-_{\infty p} = \left(1 - \frac{2\mu^2}{\left(\mu + R^* V^2_{\infty p}\right)^2}\right) V^2_{\infty p}, \tag{7}$$

The spheres of planets influence is neglected and the SC heliocentric position $\boldsymbol{R}$ at moments t_1, t_m, t_2 are equal to

$$\boldsymbol{R}(t_1)=\boldsymbol{R}_E(t_1), \tag{8}$$

$$\boldsymbol{R}(t_m)=\boldsymbol{R}_p(t_m), \tag{9}$$

$$\boldsymbol{R}(t_2)=\boldsymbol{R}_A(t_2), \tag{10}$$

where $\boldsymbol{R}_E$, $\boldsymbol{R}_p$ и $\boldsymbol{R}_A$ are the Earth, planet (Venus or Mars) and asteroid heliocentric positions respectively.

At moment t_2 the SC impacts with asteriod and the asteroid is applied tu velocity impulse ΔV_A :

$$\Delta \boldsymbol{V}_A \approx (1+k)\, M(t_2)\, \boldsymbol{V}_{\infty 2}/M_A, \tag{11}$$

$$\boldsymbol{V}_{\infty 2} = \boldsymbol{V}(t_2) - \mathbf{V}_A(t_2),$$

where M is the SC mass, V_A is the asteroid velocity, coefficient k depends on the model of the SC-asteroid impact. Following [1], models of a perfectly inelastic impact (k=0) and explosive impact are considered. In explosive case on base [1], [8] the coefficient k is assumed to be 0.6 $V_{\infty 2}/V$, where critical velocity V^* is taken to be 2 km/h, $V_{\infty 2} = |\boldsymbol{V}_{\infty 2}|$. The asteroid deflection from the Earth is defined in the aim plane [9] of the Earth and defined under the formula:

$$d = |\boldsymbol{d}_0 + \Delta \boldsymbol{d}|, \tag{12}$$

where $\boldsymbol{d}_0$ is the asteroid deflection vector relative to the Earth on the aim plane without the SC influence, $\Delta\boldsymbol{d}$ is the asteroid deflection vector caused by the SC-asteroid impact relative to the non-perturbed position on this plane. The deflection vector on the aim plane equals

$$\Delta \boldsymbol{d} = C\, \Delta \boldsymbol{V}_A, \tag{13}$$

where C is the matrix of influence of the velocity impulse $\Delta\boldsymbol{V}_A$ on vectors $\Delta\boldsymbol{d}$, determined in linear statement [1].

The criterion of the SC flight optimization is the maximal asteroid deflection from the Earth for explosive model SC-asteroid impact:

$$\mathrm{d} \to \max. \tag{14}$$

3. OPTIMAL FLIGHT FOR IDEAL THRUST

The heliocentric SC motion is describe by following system:

$$\begin{cases} \dfrac{d\boldsymbol{R}}{dt} = \boldsymbol{V}, \\ \dfrac{d\boldsymbol{V}}{dt} = \boldsymbol{a} + \boldsymbol{g}, \\ \dfrac{dM}{dt} = -\dfrac{M^2 \cdot a^2}{2n}, \end{cases}, \tag{15}$$
$$t_1 < t < t_m, t_m < t < t_2$$

where $\boldsymbol{a}$ is the jet acceleration vector, $\boldsymbol{g} = -\mu_S \boldsymbol{R}/R^3$, μ_S is is the Sun gravitation parameter, n is the jet power of the low thrust engine. For this ideal thrust the vector $\boldsymbol{a}$ is not restricted and power n=*const*.

3.1 Necessary conditions for optimal trajectory

To determine optimal trajectory the maximum principle of Pontrjagin is used [10]. The differential equations system describing the optimal jet acceleration is

$$\begin{cases} \dfrac{d\boldsymbol{a}}{dt} = \boldsymbol{b}, \\ \dfrac{d\boldsymbol{b}}{dt} = \left[\dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{R}}\right] \boldsymbol{a}, \end{cases} \tag{16}$$
$$t_1 < t < t_m, t_m < t < t_2,$$

Taking into account (1-3) at moment t_1 the jet acceleration vector $\boldsymbol{a}$ is

$$\boldsymbol{a}(t_1) = \frac{n}{M(t_1)c_1} \frac{\boldsymbol{V}_{\infty 1}}{(V_{\infty 1} + 2\mu_E/R_0)^{1/2}}. \tag{17}$$

The optimal trajectories is determined with the fixed value of relative velocity $V_{\infty p}$, and then the optimal one providing the maximal criterion (14) is chosen. Under this condition the following restrictions are imposed on the vectors **a** and $\boldsymbol{b}$ at moment t_m:

$$\boldsymbol{a}(t_m^-) = \mu_1 \boldsymbol{V}_{\infty p}^- + \mu_3 \boldsymbol{V}_{\infty p}^+ , \quad (18)$$

$$\boldsymbol{a}(t_m^+) = \mu_2 \boldsymbol{V}_{\infty p}^+ + \mu_3 \boldsymbol{V}_{\infty p}^- , \quad (19)$$

$$\boldsymbol{b}(t_m^+) = \boldsymbol{b}(t_m^-) + \boldsymbol{\pi} , \quad (20)$$

where μ_1, μ_2, μ_3 are some constant coefficients, $\boldsymbol{\pi}$ is some constant vector.

If condition (7) is not essential (i.e. $R_\pi > R$), μ_3 is 0. Taking into account (14), restriction is imposed on vector $\boldsymbol{a}$ at moment t_2:

$$\boldsymbol{a}(t_2) = \frac{1}{M_A} \frac{n}{(\boldsymbol{d}, \Delta \boldsymbol{d})} (1+k) C^T \boldsymbol{d} . \quad (21)$$

Thus the problem of optimal control is reduced to boundary problem for the differential equations system (15), (16) with conditions (1-3), (8) at moment t_1, conditions (9), (18-20) at moment t_m, and (10), (21) at moment t_2.

3.2 Scheme of boundary problem solution

The boundary problem is solved by Newton modified method. The varied parameters are $\boldsymbol{V}(t_1)$, $\boldsymbol{b}(t_1)$, μ_1, μ_2, μ_3, $\boldsymbol{\pi}$, and also $\boldsymbol{e}_V = \boldsymbol{V}_{\infty p}^+ / V_{\infty p}$ ($|\boldsymbol{e}_V|=1$). Given vectors $\boldsymbol{V}(t_1)$, $\boldsymbol{b}(t_1)$ and taking into account (1-3), (17), the system of equations (15), (16) is integrated on interval $t_1<t<t_m$. After integration at moment t_m some vectors $\boldsymbol{R}(t_m^-)$, $\boldsymbol{V}(t_m^-)$, $\boldsymbol{a}(t_m^-)$, $\boldsymbol{b}(t_m^-)$ and mass $m(t_m^-)$ are obtained. Given parameters μ_1, μ_2, μ_3, $\boldsymbol{\pi}$, $\boldsymbol{e}_V$ the conditions after gravity assist are determined by formulas:

$$\boldsymbol{R}(t_m^+) = \boldsymbol{R}(t_m^-) , \quad (22)$$

$$\boldsymbol{V}(t_m^+) = \boldsymbol{e}_V V_{\infty p} , \quad (23)$$

$$\boldsymbol{a}(t_m^+) = \mu_2 \boldsymbol{V}_{\infty p}^+ + \mu_3 \boldsymbol{V}_{\infty p}^- , \quad (24)$$

$$\boldsymbol{b}(t_m^+) = \boldsymbol{b}(t_m^-) + \boldsymbol{\pi} , \quad (25)$$

$$m(t_m^+) = m(t_m^-) . \quad (26)$$

Them system of equations (15), (16) is integrated on interval $t_m<t<t_2$. After integration the conditions (4), (7), (9), (10), (18), (21) are verified. If inequality (6) is not essential (i.e. $R_\pi > R^*$), parameter μ_3 is 0 and condition (7) is not verified. The convergence of the Newton method mainly depends on the first approach. The first approach is chosen by means of following scheme. The SC trajectory is divided on two parts: from the Earth to the the planet (Mars or Venus) and from this planet to the asteroid. On each of these parts the problem of optimal control is solved.

For first part ($t_1<t<t_m$) the optimal trajectory is determined for maximal mass $M(t_m)$ criterion. This optimal trajectory is to satisfy conditions (1-3), (8), (9). This problem is solved according to method offered in [4,5], when the optimal trajectory is chosen in vicinity of some Kepler trajectory. The mass $M(t_m)$ and value of velocity $V_{\infty p}$ obtained on the first part of trajectory are chosen as initial condition for second part.

Optimal trajectory on second part ($t_m<t<t_2$) is to satisfy condition (9), (10). The optimal problem is solved for fixed initial value of velocity $V_{\infty p}$ and mass $M(t_m)$. The criterion of trajectory optimization is maximal asteroid deflection (14). The method of problem solution is described in [5]. According to this method at first the optimal control problem is solved for criterion of maximal mass $M(t_2)$, then for criterion of maximal spacecraft momentum relative to the asteroid at moment t_2, and then for criterion of maximal asteroid deflection from the Earth for perfectly elastic (k=1) impact, and at last for criterion of maximal asteroid deflection for explosive impact (14).

Obtained two optimal parts of trajectory is chosen as first approach for the stated boundary problem. Such choice of the initial approximation has provided good convergence of the modified Newton method.

4. OPTIMAL FLIGHT FOR CONSTRAINED THRUST

The optimal trajectory for constrained thrust is defined in vicinity of optimal one determined in case of ideal thrust. In this connection the SC optimal velocity $\boldsymbol{V}^{opt}(t_1)$, $\boldsymbol{V}^{opt}(t_m^-)$, $\boldsymbol{V}^{opt}(t_m^+)$ и $\boldsymbol{V}^{opt}(t_2)$ determined for ideal thrust were taken for trajectory with constrained thrust and is not varied. Thus the SC trajectory can be divided on two parts for intervals (t_1, t_m) and (t_m, t_2) with joining condition (26). On each of these intervals the SC motion is described by equation system:

$$\begin{cases} \dfrac{d\mathbf{R}}{dt} = \mathbf{V}, \\ \dfrac{d\mathbf{V}}{dt} = \mathbf{g} + \delta \dfrac{P}{M}\mathbf{e}, \\ \dfrac{dM}{dt} = -\delta \cdot q, \end{cases} \quad . \tag{27}$$

$$t_1 < t < t_m, t_m < t < t_2,$$

where P is the value of thrust vector, $\mathbf{e}$ is its nonconstrained direction, q is the mass flow rate, δ(t) is function accepting values 0 or 1. Here the constraints to P and q were taken appropriate to electric propulsion system SPT-140 [11].

There are conditions (8), (9) and

$$V(t_1) = V^{opt}(t_1). \tag{28}$$

$$V(t_m) = V^{opt}(t_m^-). \tag{29}$$

for first part of trajectory ($t_1<t<t_m$) and the conditions (9), (10) and

$$V(t_m) = V^{opt}(t_m^+). \tag{30}$$

$$V(t_2) = V^{opt}(t_2). \tag{31}$$

for second part ($t_m<t<t_2$).

To solve this optimal control problem the Pontrjagin maximum principle is also used. The necessary conditions for optimal trajectories reducing this problem to boundary value problem for system of differential equations are described in [11] and [12]. The boundary problem is solved by means of the modified Newton method. The first approach is optimal trajectory for ideal thrust. The scheme of this problem solution consists in gradual contraction of domain of admissible value of thrust and corresponds to scheme described in [6].

5. NUMERICAL RESULTS

Numerical results were obtained for SC flights to following asteroids and dates D of Earth-asteroid approaches: 4179 Toutatis, D=29/09/2004; 4660 Nereus, D= 11/12/2021; 3361 Orpheus, D=19/11/2025; 35396 (1997XF11), D= 26/10/2028; 4581 Asclepius, D= 25/03/2020; 1862 Apollo, D=08/05/2007; 5143 Heracles, D=29/11/2016; 433 Eros, D=31/01/2012; 39572 (1993DQ1), D= 12/06/2011; 887 Alinda, D=08/01/2025; 52768 (1998OR2), D=29/04/2020; 6491 (1991OA), D=3/5/2003; 5604 (1992FE), D=5604; 33342 (1998WT24), D=11/12/2018; 2340 Hathor, D=22/10/2007; 3362 Khufu, D= 15/12/2004; 5381 Sekhmet, D=17/05/2015. Masses of all asteroids M_A are assumed to equal $1.257 \cdot 10^7$, that corresponds radius 100 m and density 3g/sm^3. The SC low-Earth initial parking orbit R_0 is 6671 km, the SC initial mass M_0 is 8 t; the gas exhaust velocity of th high thrust engine c_1 is 4.5 km/s; the electric-jet power for ideal low thrust case n is 4kW.

The optimal trajectories were defined for intervals t_{23}=t_3-t_2 up to 12 year, and t_{12}=t_2-t_1 up to 5 years. At first the maximal asteroid deflection d (14) was determined on time set (t_{12}, t_{23}) for one-impulse scheme of SC flight when the velocity impulse is applied to the SC on parking orbit near Earth and then the SC motion is assumed to be passive (passive flight without gravity maneuvers).

Table 1. The characteristic of asteroids and optimal trajectories without using gravity maneuvers and low thrust

	P, yr	*e*	*i, deg*	t_{12} *yr*	t_{23}, *yr*	Δd_0 10^3 *km*	*d* 10^3 *km*
			Apollo				
Toutatis	3.98	.63	.46	1.8	11.6	60	104
Nereus	1.82	.36	1.4	.24	10.7	13	14
Orpheus	1.33	.32	2.6	.55	9.4	16	19
Asclepius	1.03	.35	4.9	.71	11.7	12	16
1997XF11	1.73	.48	4.1	.37	8.9	36	42
Apollo	1.78	.56	6.3	.30	10.5	47	53
Heracles	2.48	.77	9.1	.32	7.4	143	160
			Amor				
Eros	1.76	.22	10	.66	8.8	7	9
1993DQ1	2.91	.49	10	1.1	11.4	30	45
Alinda	3.92	.56	9.3	2.1	11.4	46	80
1998OR2	3.70	.57	5.8	1.7	10.8	46	73
1991OA	3.96	.58	5.7	3.5	10.9	1.9	4
			Aten				
1992FE	0.89	.40	4.8	.31	10.2	33	38
1998WT24	0.61	.42	7.3	.27	11.4	13	15
Hathor	0.78	.45	5.8	.30	11.1	42	47
Khufu	0.98	.47	9.9	.19	11.6	25	30
Sekhmet	0.92	.29	49	.37	9.6	54	62

Classes of asteroids, periods P, eccentricities e, inclinations i of with respect to the ecliptic plane of asteroids orbits, optimal times t_{12}, t_{23} for one-impulse flight of SC to these asteroids and maximal deflections Δd_0 are shown in Table 1. For these optimal flights times (t_{12}, t_{23}) the optimal trajectories of SC flights using low thrust (without gravity maneuvers) were determined and corresponding the asteroid deflections Δd are shown in Table 1. Then the passive flights of SC with gravity maneuvers near Mars or Venus were determined on time set (t_{12},t_{23}). The maximal asteroid deflections Δd_{go} and times of SC flight $t_{1m}= t_m-t_1$, $t_{2m} = t_2-t_m$, t_{23} for this case are shown in Table 2 (for gravity maneuvers near Mars) and Table 3 (near Venus). For these optimal times (t_{1m}, t_{m2}, t_{23}) the optimal trajectories of SC flight using gravity assist and low thrust were determined. The asteroid deflections Δd_g for these trajectories are also shown in Tables 2, 3.

Table 2. The characteristics of optimal trajectories with gravity maneuvers near Mars

	t_{1m} , *yr*	t_{m2} , *yr*	t_{23} , *yr*	Δd_{g0}, 10^3 *km*	Δd_g, 10^3 *km*
Toutatis	0.59	0.30	11.8	36	50
Nereus	1.77	0.66	11.1	15	29
Orpheus	1.45	0.63	9.4	12	22
Asclepius	0.81	0.62	11.7	11	17
1997XF11	0.37	1.59	10.7	30	54
Apollo	0.83	0.58	10.7	25	40
Heracles	1.21	0.32	9.7	133	210
Eros	0.44	1.15	10.6	10	16
1993DQ1	0.66	0.29	11.4	39	53
Alinda	0.80	0.82	11.7	19	31
1998OR2	0.17	1.26	11.0	16	25
1991OA	3.67	0.93	11.9	1.7	4.9
1992FE	0.55	0.49	9.0	9	13
1998WT24	0.65	0.63	12.0	3	5
Hathor	0.70	0.58	11.7	9	14
Khufu	0.40	1.14	8.52	11	18
Sekhmet	0.81	0.44	11.4	14	21

Table 3. The characteristics of optimal trajectories with gravity maneuvers near Venus

	t_{1m} , *yr*	t_{m2} , *yr*	t_{23} , *yr*	Δd_{g0}, 10^3 *km*	Δd_g, 10^3 *km*
Toutatis	0.50	0.88	11.8	65	98
Nereus	0.23	0.68	8.8	17	23
Orpheus	0.38	0.37	9.1	19	24
Asclepius	0.40	0,19	10.5	18	22
1997XF11	0.40	0.41	10.5	46	58
Apollo	0.25	0.71	8.6	41	54
Heracles	0.46	0.51	9.7	142	179
Eros	0.64	0.81	8.7	6	8
1993DQ1	0.88	0.79	11.5	21	34
Alinda	0.46	1.07	11.6	57	87
1998OR2	1.30	1.10	10.9	23	42
1991OA	1.35	1.68	11.1	1	2
1992FE	1.51	1.91	11.0	34	77
1998WT24	0.35	0.34	11.9	18	23
Hathor	0.16	0.23	11.2	29	33
Khufu	0.41	0.75	10.6	24	36
Sekhmet	0.59	1.02	9.6	38	61

From these Tables it shows that use gravity assist near Mars for asteroids Nereus, Eros, 1993DQ1 and near Venus for Toutatis, Nereus, Orpheus, Asclepius, 1997XF11, Alinda, 1992FE, 1998WT24 reduces to increase (up to 50%) asteroid deflection in comparison with passive flight. Use low thrust increases deflection (up to 120%) additionally. Search of optimal SC trajectories with gravity assist in the given domain of times didn't bring increase of asteroid deflection in comparison of the SC trajectories without gravity assist for flight to asteroids Apollo, Heracles, 1998OR2, 1991OA, Hathor, Khufu, Hathor, Sekhmet.

For case of constrained thrust numerical analysis showed that asteroid deflection decreases on 5-30% in comparison with case of ideal one.

6. CONCLUSION

The scheme for determination of optimal space flight with using gravity maneuver (near Mars or Venus) and low thrust engine for SC impact-kinetic influence on dangerous asteroid was offered. The effectiveness of impact-kinetic influence is investigeted for wide ranges of the space flight times to near-Earth asteroids with different orbit elements. Numerical results showed that use of gravity assist and low thrust can essentially increase asteroid deflection from the Earth in comparison with passive flight.

7. REFERENCES

1. V.V. Ivashkin, A.V. Zaytsev. Analysis of possible changes of the orbit of an asteroid approaching the Earth by impact of a apacecraft, *Cosmic Research V. 37, No.4,* 382 - 392. 1999

2. V.V. Ivashkin, A.V. Chernov, Ph.I. Baum. An analysis of spacecraft optimal two-impulse flights to near-Earth asteroid. *Preprint, Keldysh Inst. Appl. Mathem. RAS, N34,* 1999

3. V.V. Ivashkin, Ph.I. Baum. Using Lunar gravity assist for space flight to near-Earth asteroid, *Preprint, Keldysh Inst. Appl. Mathem. RAS, N67*, 2000

4. V.V. Ivashkin, A.V. Chernov. Determination of optimal trajectories for space mission to near-Earth asteroid with using of low-thrust engine, *Preprint, Keldysh Inst. Appl. Mathem. RAS, N19,* 1997

5. V.V. Ivashkin, A.V. Chernov. Optimal trajectories of the spacecraft mission to near-Earth asteroid with using low thrust", *Cosmic Research, V. 38, No. 2,* 141 – 152, 2000

6. A.V. Chernov, An analysis of spacecraft optimal flights to near-Earth asteroid with stepped selectric-jet thrust, *Preprint, Keldysh Inst. Appl. Mathem. RAS, No. 86,* 2001

7. A.V. Chernov, The optimal trajectories to the near-Earth asteroid with using low thrust, *54th International Astronautical Congress, IAC-03-U.1.09. Bremen, Germany,* 2003.

8. K.P Stanyukovich, Unsteady motions of continuous medium, *Moscow: Nauka*, 1971

9. Fundamentals of space flight theory, Eds: G. S. Narimanov, M. K. Tikhonravov, *Moscow: Mashinostroenie*, 1971

10. L.S. Pontrjagin, V.G. Boltjansky, R.V. Gamkrelidze, E.F. Mishenko, "Mathematical Theory of optimal processes", *Moscow: Phismatgis,* 1961

11. G.S. Zaslavskiy, V.G. Zharov, A.V. Chernov, Optimal transfer from Earth satellite orbit to Mars satellite orbit with using electric propulsion system at cruising phase, *The Proceeding of the 17th International Symposium of Space Flight Dynamics. Keldysh Institute of Applied Mathematics, Space Informatics Analytical Systems, Vol. 1, 305-311, Moscow, Russia,* 2003.

12. E.L. Akim, G.S. Zaslavskiy, V.G. Zharov , A.V. Chernov, Interplanetary flight control with electric engine in view of thrust errors. *Present proceeding.*