Fuel-Optimum Near-Miss Avoidance Control for Clustered Satellites

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ABSTRACT

A near-miss avoidance and fuel-minimization control is constructed for clustered multiple artificial satellites with continuous thrust. A penalty function with respect to the distances between satellites is imposed on the fuel-consumption index. The function is formulated as positive if any distance is less than a defined near-miss distance and as zero otherwise. Therefore, the Euler-Lagrange equations do not require a distinction between unconstrained and constrained modes. It is enough to solve just the two-point boundary-value problem even if unspecified multiple near misses occur between satellites. The simulation procedure is to repeat shooting searches, while increasing the height of the penalty function. The initial costate values are chosen under the assumption that there is no penalty function.

1. INTRODUCTION

Advanced operations have a crucial role on various flexible missions by distributed systems with a cluster of multiple artificial satellites [1]. Clustering of two satellites has already been studied by analyzing just one relative orbit of the Clohessy-Wiltshire (hereafter called CW [2]) rotating coordinate frame [3]. However, near-miss avoidance imposes more complex constraints as a cluster is composed of more satellites. For $n \geq 3$, assume n satellites, $\mathrm{S}_i, i = 1, 2, \ldots, n$, to be flying close to each other. If near-miss avoidance strategy between all of the satellites is not fully considered, a near miss between S_1 and S_2 is probable after the near-miss avoidance between S_1 and S_3.

Potential-function guidance is considered as one solution. The respective satellites are manoeuvred as if they have virtual repulsion. Such potential fields have been formulated and imposed on the state-variable space to form an equidistant constellation around the earth [4] and a cluster center [5], respectively. The advantage is that fast computation is possible even in the case with unspecified multiple near misses by many satellites. However, finding a potential function to decrease fuel consumption is difficult, which motivates us to introduce a near-miss avoidance potential in the costate-variable space. This paper solves the fuel-optimization problem with state constraints by a penalty-function method [6].

Up to Section 3.2, this paper describes the detailed derivation and result of [8]. In Section 3.3, the problems to be solved in future work are shown.

2. FORMULATION

Consider guidance of eccentricity-inclination separation (e-i separation), a popular method of cluster formation [7]. Orbits of all satellites approaching e-i separation are restricted approximately on a fixed plane passing through the origin in the CW coordinate frame. Some near misses can be avoided orthogonally, while the in-plane positions coincide with each other if a satellite configuration is far from e-i separation. However, almost all the in-plane near misses in a process for eccentricity separation (e-separation) are projected onto three-dimensional near misses near an e-i-separation configuration. Therefore, the guidance of the planar n-satellite system to e-separation is considered around an orbiting point on a circular reference orbit.

2.1 Terminal Constraints for Cluster

Each satellite $\mathrm{S}_i, i = 1, \ldots, n$ is assumed to have continuous longitudinal thrust. Linearizing the Keplerian orbital dynamics near the reference orbit yields the CW equation [2],

$$X_i'' - 2Y_i' - 3X_i = 0,\ Y_i'' + 2X_i' = U_i(t) \qquad (1)$$

where X_i and Y_i are the radial and the longitudinal coordinates of the i-th satellite from the orbiting coordinate frame on the reference orbit, respectively, and $U_i(t)$ is the thrust acceleration. For simplification, $\boldsymbol{x}_i$ denotes and defines

$$\boldsymbol{x}_i = [x_i^{(1)}, x_i^{(2)}, x_i^{(3)}, x_i^{(4)}]^{\mathrm{T}} = \left[X_i, \frac{Y_i}{2}, X_i', \frac{Y_i'}{2}\right]^{\mathrm{T}} \qquad (2)$$

Then, the equations of motion are given by

$$\begin{aligned} \boldsymbol{x}_i' &= \boldsymbol{f}_i(\boldsymbol{x}_i, u_i), \\ \boldsymbol{f}_i &= \left[x_i^{(3)}, x_i^{(4)}, 3x_i^{(1)} + 4x_i^{(4)}, -x_i^{(3)} + u_i\right]^{\mathrm{T}} \end{aligned} \qquad (3)$$

where $u_i = U_i/2$. The unit time is adopted so that the orbital frequency is equal to 1.

The final state is e-separation without thruster acceleration. If u_i vanishes, then

$$\begin{aligned} \boldsymbol{x}_i(t) &= A_i(t)\boldsymbol{\epsilon}_i, \\ A_i(t) &= \begin{bmatrix} 0 & -\frac{4}{3} & \cos t & \sin t \\ 1 & t & -\sin t & \cos t \\ 0 & 0 & -\sin t & \cos t \\ 0 & 1 & -\cos t & -\sin t \end{bmatrix} \end{aligned} \tag{4}$$

with in-plane orbital elements $\boldsymbol{\epsilon}_i = [\epsilon_i^{(1)}, \ldots, \epsilon_i^{(4)}]^{\mathrm{T}}$. Here, $2\epsilon_i^{(1)}/a, 2\epsilon_i^{(2)}/a$, and $(\epsilon_i^{(3)}/a, \epsilon_i^{(4)}/a)$ define a nominal longitude, a drift rate, and an eccentricity vector, respectively, where a is the semimajor axis of the reference orbit.

According to traditional formulation, the system is said to in e-separation if the following all conditions are satisfied [7].

(A) $\epsilon_i^{(1)} = \epsilon_i^{(2)} = 0$.

(B) $(\epsilon_i^{(3)}, \epsilon_i^{(4)}) \neq (\epsilon_j^{(3)}, \epsilon_j^{(4)})$ for $i \neq j$.

(C) the eccentricity $\dfrac{1}{a}\sqrt{\left(\epsilon_i^{(3)}\right)^2 + \left(\epsilon_i^{(4)}\right)^2}$ is sufficiently small.

A loose cluster is defined as the state which satisfies just (A) and (B). Considering the condition (C) is out of the scope of this paper. The condition (B) corresponds to avoidance of near-miss. In this paper, any near miss is avoided by a penalty function defined later. Hence, (B) can be excluded from the terminal constraint. Eqn. (4) with (A) yields

$$\boldsymbol{\psi}_i(\boldsymbol{x}_i(t_{\mathrm{f}}), t_{\mathrm{f}}) = \boldsymbol{0},\ \boldsymbol{\psi}_i = [x_i^{(1)} + x_i^{(4)}, x_i^{(2)} - x_i^{(3)}]^{\mathrm{T}} \tag{5}$$

which is the final constraint.

2.2 Penalty Function for Near-Miss Avoidance

In order to avoid any near miss and reduce thruster fuel consumption, the minimization of the following index is considered,

$$J(\boldsymbol{u}) = \int_0^{t_{\mathrm{f}}} L(\boldsymbol{x}, \boldsymbol{u}, t)\mathrm{d}t \tag{6}$$

where

$$L = W(\boldsymbol{x}) + \frac{1}{2}|\boldsymbol{u}|^2, \quad W = \sum_{j=1}^{n}\sum_{j>i}^{n} w(r_{ij}) \tag{7}$$

and $\boldsymbol{x} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n]^{\mathrm{T}} \in \mathcal{R}^{4n}$, $\boldsymbol{u} = [u_1, \ldots, u_n]^{\mathrm{T}} \in \mathcal{R}^n$. A penalty function for near-miss avoidance is denoted by W, where r_{ij} denotes the distance between S_i and S_j. A function w will be defined later.

Constraint of $\boldsymbol{x}_i' = \boldsymbol{f}_i$ and $\boldsymbol{\psi}_i(\boldsymbol{x}(t_{\mathrm{f}}), t_{\mathrm{f}}) = \boldsymbol{0}$ are included by considering the extremization of the following function:

$$\begin{aligned} &J^*(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}, t_{\mathrm{f}}, \boldsymbol{x}(t_{\mathrm{f}}), \boldsymbol{\nu}) = \Big[\boldsymbol{\nu}^{\mathrm{T}}\boldsymbol{\psi}\Big]^{t=t_{\mathrm{f}}} \\ &+ \int_0^{t_{\mathrm{f}}} \Big\{L(\boldsymbol{x}, \boldsymbol{u}, t) + \boldsymbol{\lambda}^{\mathrm{T}}(t)\{\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) - \boldsymbol{x}'\}\Big\}\,\mathrm{d}t \end{aligned} \tag{8}$$

by using the costate variables $\boldsymbol{\lambda}(t) \in \mathcal{R}^{4n}$ and the Lagrange multipliers $\boldsymbol{\nu} \in \mathcal{R}^{2n}$, where $\boldsymbol{f} = [\boldsymbol{f}_i, \ldots, \boldsymbol{f}_n]^{\mathrm{T}}$ and $\boldsymbol{\psi} = [\boldsymbol{\psi}_i, \ldots, \boldsymbol{\psi}_n]^{\mathrm{T}}$. The first variation is computed as

$$\begin{aligned} \delta J^* = &\int_0^{t_{\mathrm{f}}} \Bigg\{\left(\frac{\partial H}{\partial \boldsymbol{x}} + \dot{\boldsymbol{\lambda}}^{\mathrm{T}}\right)\delta\boldsymbol{x} \\ &\qquad + \frac{\partial H}{\partial \boldsymbol{u}}\delta\boldsymbol{u} + (\boldsymbol{f} - \dot{\boldsymbol{x}})\delta\boldsymbol{\lambda}^{\mathrm{T}}\Bigg\}\mathrm{d}t \\ &+ \left[H + \boldsymbol{\nu}^{\mathrm{T}}\frac{\partial \boldsymbol{\psi}}{\partial t}\right]^{t=t_{\mathrm{f}}} \mathrm{d}t_{\mathrm{f}} \\ &+ \left[\boldsymbol{\nu}^{\mathrm{T}}\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{x}} - \boldsymbol{\lambda}^{\mathrm{T}}\right]^{t=t_{\mathrm{f}}} \mathrm{d}\boldsymbol{x}(t_{\mathrm{f}}) + \Big[\boldsymbol{\psi}\Big]^{t=t_{\mathrm{f}}}\delta\boldsymbol{\nu} \end{aligned} \tag{9}$$

where the Hamiltonian function is introduced as

$$H = L + \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{f} \tag{10}$$

The extremization $\delta J^* = 0$ leads to the Eular-Lagrange equations of L and the boundary condition, which gives

$$\begin{aligned} &\dot{\boldsymbol{\lambda}}^{\mathrm{T}} = -\frac{\partial H}{\partial \boldsymbol{x}}, \quad \dot{\boldsymbol{x}}^{\mathrm{T}} = \frac{\partial H}{\partial \boldsymbol{\lambda}}, \\ &\frac{\partial H}{\partial \boldsymbol{u}} = \boldsymbol{0}, \quad \boldsymbol{\lambda}^{\mathrm{T}}(t_{\mathrm{f}}) = \left[\boldsymbol{\nu}^{\mathrm{T}}\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{x}}\right]^{t=t_{\mathrm{f}}} \end{aligned} \tag{11}$$

In this paper, t_{f} is to be fixed. Note that the following condition is added if t_{f} is not constrained.

$$\left[H + \boldsymbol{\nu}^{\mathrm{T}}\frac{\partial \boldsymbol{\psi}}{\partial t}\right]^{t=t_{\mathrm{f}}} = 0 \tag{12}$$

In this paper, the Hamiltonian function is given by

$$\begin{aligned} H = \sum_{i=1}^{n}\Bigg\{&\sum_{j>i}^{n} w(r_{ij}) + \frac{1}{2}u_i^2 + \lambda_i^{(1)}x_i^{(3)} + \lambda_i^{(2)}x_i^{(4)} \\ &+ \lambda_i^{(3)}\left(3x_i^{(1)} + 4x_i^{(4)}\right) + \lambda_i^{(4)}\left(-x_i^{(3)} + u_i\right)\Bigg\}. \end{aligned} \tag{13}$$

Then, the Euler-Lagrange equations yields

$$\boldsymbol{\lambda}_i' = \begin{bmatrix} -\displaystyle\sum_{j\neq i}^{n}\frac{\partial w(r_{ij})}{\partial x_i^{(1)}} - 3\lambda_i^{(3)} \\ -\displaystyle\sum_{j\neq i}^{n}\frac{\partial w(r_{ij})}{\partial x_i^{(2)}} \\ -\lambda_i^{(1)} + \lambda_i^{(4)} \\ -\lambda_i^{(2)} - 4\lambda_i^{(3)} \end{bmatrix}, \quad u_i = -\lambda_i^{(4)} \tag{14}$$

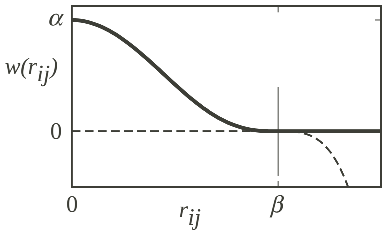

Figure 1: The penalty function for near-miss avoidance

and

$$\boldsymbol{\varphi}_i(t_\mathrm{f}) = \mathbf{0},\ \boldsymbol{\varphi}_i = \left[\lambda_i^{(1)} - \lambda_i^{(4)}, \lambda_i^{(2)} + \lambda_i^{(3)}\right]^\mathrm{T} \quad (15)$$

The near-miss-avoidance penalty function $w(r)$ is selected with the property that $w(r)$ decreases with increasing $|r|$ for $|r| \leq \beta$ and $w(r) = 0$ for $|r| \geq \beta$. If $\lim_{r\to 0} w(r) = \infty$ is adopted for full safety from collision, the repulsive effect is too strong to search for an optimum initial $\boldsymbol{\lambda}_i$ by the shooting method. Thereby, a bounded positive $w(0)$ is chosen. Notice that the integrand of J^* has to be of class C^2 or greater. Therefore, w needs to satisfy $w(\beta) = w'(\beta) = w''(\beta) = 0$. Moreover, it is convenient if $w(r)$ is an even function. The above conditions give the following polynomial function with the lowest power of r:

$$w(r) = \begin{cases} -\alpha(r^2 - \beta^2)^3 & \text{for } |r| < \beta \\ 0 & \text{for } |r| \geq \beta \end{cases} \quad (16)$$

which is shown in Fig. 1. The repulsive region corresponds to the ellipse of which the major axis along the longitude has double the length of the minor axis in the CW space. This is suitable for real situations since both longitudinal errors of thruster acceleration and position measurement are larger than the radial errors.

The minimization of Eqn. 6 does not exactly give the fuel-minimized solution. The optimal orbits obtained by minimizing J^* is not necessarily restricted in a domain satisfying $w(r_{ij}) = 0$ even if the range of w includes zero. In this case, the solution is not optimized exactly. Even if the solution satisfies $w(r_{ij}) = 0$ for any (i, j) and $t \in [0, t_\mathrm{f}]$, the control function solved by the minimization of J^* with $n > 2$ does not necessarily minimize the total fuel consumption for all the satellites which is proportional to the total velocity change defined by

$$\mathrm{tot}|\Delta v| = \sum_{i=1}^{n} \int_0^{t_\mathrm{f}} |u_i(t)| \mathrm{d}t \quad (17)$$

The consumption difference from the truly minimum fuel will be discussed in Section 3.1 and 3.2.

2.3 Iterative Procedure for Optimization

The task is now to solve the two-point boundary problem, Eqn. 3 and Eqn. 14, with terminal constraints Eqn. 15 for given initial values $\boldsymbol{x}(0)$. Since the nonlinearity $w(r_{ij})$ is included in Eqn. 14, the shooting method is applied to this problem. Hence, an initial guess of costate values are assumed as $\boldsymbol{\mu} \in \mathcal{R}^{4n} = [\boldsymbol{\lambda}_1(0), \ldots, \boldsymbol{\lambda}_n(0)]^\mathrm{T}$. Before performing a shooting search, $\boldsymbol{\mu}$ is found with $w(r_{ij}) = 0$. In this linear case, Eqn. 14 is solved analytically as

$$\begin{aligned} \lambda_i^{(1)}(t) &= 4\mu_i^{(1)} - 3\mu_i^{(4)} + 3\mu_i^{(2)} t \\ &\quad -3(\mu_i^{(1)} - \mu_i^{(4)})\cos t - 3(\mu_i^{(2)} + \mu_i^{(3)})\sin t \\ \lambda_i^{(2)}(t) &= \mu_i^{(2)} \\ \lambda_i^{(3)}(t) &= -\mu_i^{(2)} \\ &\quad +(\mu_i^{(2)} + \mu_i^{(3)})\cos t - (\mu_i^{(1)} - \mu_i^{(4)})\sin t \\ \lambda_i^{(4)}(t) &= 4\mu_i^{(1)} - 3\mu_i^{(4)} + 3\mu_i^{(2)} t \\ &\quad -4(\mu_i^{(1)} - \mu_i^{(4)})\cos t - 4(\mu_i^{(2)} + \mu_i^{(3)})\sin t \end{aligned} \quad (18)$$

Constraint of $\boldsymbol{\lambda}_i(t_\mathrm{f})$ by Eqn. 11d gives

$$\lambda_i^{(1)}(t_\mathrm{f}) = \lambda_i^{(4)}(t_\mathrm{f}) = \nu_1, \lambda_i^{(2)}(t_\mathrm{f}) = -\lambda_i^{(3)}(t_\mathrm{f}) = \nu_2 \quad (19)$$

Eqn. 18 with Eqn. 19 gives

$$\begin{aligned} (\mu_i^{(1)} - \mu_i^{(4)})\cos t_\mathrm{f} + (\mu_i^{(2)} + \mu_i^{(3)})\sin t_\mathrm{f} &= 0 \\ (\mu_i^{(2)} + \mu_i^{(3)})\cos t_\mathrm{f} - (\mu_i^{(1)} - \mu_i^{(4)})\sin t_\mathrm{f} &= 0 \end{aligned} \quad (20)$$

Simultaneous equality restricts the initial values to

$$\mu_i^{(3)} = -\mu_i^{(2)}, \quad \mu_i^{(4)} = \mu_i^{(1)} \quad (21)$$

Substituting Eqn. 21 for Eqn. 18 yields

$$\begin{aligned} \lambda_i^{(1)}(t) &= \lambda_i^{(4)}(t) = \mu_i^{(1)} + 3\mu_i^{(2)} t \\ \lambda_i^{(2)}(t) &= -\lambda_i^{(3)}(t) = \mu_i^{(2)} \end{aligned} \quad (22)$$

The optimum control function is given by Eqn. 11c,

$$u_i(t) = -\lambda_i^{(4)}(t) = -\mu_i^{(1)} - 3\mu_i^{(2)} t \quad (23)$$

Then, $\boldsymbol{\mu}_i$ will be formulated by the initial orbital element set $\boldsymbol{\epsilon}_i(0)$ instead of $\boldsymbol{x}_i(0)$. From Eqn. 3 and Eqn. 23, $\boldsymbol{x}_i(t)$ is computed explicitly as

$$\boldsymbol{x}_i(t) = A_i(t)\boldsymbol{\epsilon}(0)_i + B_i(t)\begin{bmatrix} \mu_i^{(1)} \\ \mu_i^{(2)} \end{bmatrix} \quad B_i(t) = \begin{bmatrix} 4(-t + \sin t) & 6(2 - t^2 - 2\cos t) \\ \frac{1}{2}(-8 + 3t^2 + 8\cos t) & \frac{3}{2}(-8t + t^3 + 8\sin t) \\ 4(-1 + \cos t) & 12(-t + \sin t) \\ 3t - 4\sin t & \frac{3}{2}(-8 + 3t^2 + 8\cos t) \end{bmatrix} \quad (24)$$

Substitute Eqn. 24 for $[\boldsymbol{\psi}_i]^{t=t_\mathrm{f}} = \mathbf{0}$. Then, the initial values of control acceleration are given by

$$\begin{aligned} \mu_i^{(1)} &= -2(3\epsilon_i^{(1)} + 2\epsilon_i^{(2)} t_\mathrm{f})/(3t_\mathrm{f}^2) \\ \mu_i^{(2)} &= 2(2\epsilon_i^{(1)} + \epsilon_i^{(2)} t_\mathrm{f})/(3t_\mathrm{f}^3) \end{aligned} \quad (25)$$

which are independent of $(\epsilon_i^{(3)}, \epsilon_i^{(4)})$. The optimum thrust is hence given by Eqn. 22 and Eqn. 25 if no near miss happens for $t \in [0, t_\mathrm{f}]$, which is proportional to t for any initial value $\boldsymbol{x}(0)$.

Here, the optimization of t_f is considered. Compute the cost function $J(\boldsymbol{u})$ directly.

$$J = \frac{4}{9t_\mathrm{f}^3}\left\{\left(\epsilon_i^{(2)}\right)^2 t_\mathrm{f}^2 + 3\epsilon_i^{(1)}\epsilon_i^{(2)} t_\mathrm{f} + 3\left(\epsilon_i^{(1)}\right)^2\right\} \quad (26)$$

Then,

$$\frac{\mathrm{d}J}{\mathrm{d}t_\mathrm{f}} = -\frac{4}{9t_\mathrm{f}^4}(\epsilon_i^{(2)} t_\mathrm{f} + 3\epsilon_i^{(1)})^2 \leq 0 \quad (27)$$

The result is that J does not increase with the increasing t_f. Therefore, t_f is fixed in this paper.

If any near miss happens at the initial step, the following is a procedure for a sequential shooting search for an optimum $\boldsymbol{\mu}$. Let $\delta\alpha$ and $\delta\beta_\kappa$ for $\kappa = 0, 1, \ldots$ be small positive constants, and β_C be a critical distance considered as a near miss. A sequence of shots $\boldsymbol{\mu}$ denotes $\boldsymbol{\mu}^m$, where m means a time of shooting iteration. Let $C(\alpha, \beta)$ denote that $\boldsymbol{\mu}^m$ converges by a shooting search with given α and β. The negation of the expression is denoted by $\neg$. The expression $x \leftarrow y$ is defined as "substitute y for x."

The iteration procedure
$\alpha \leftarrow 0$, $\kappa \leftarrow 0$, $\beta \leftarrow \beta_\mathrm{C} + \delta\beta_0$, and $\gamma \leftarrow \beta$.
Iterate to perform the following five conditional commands after the shooting search;

(P1) if $C(\alpha, \gamma)$, then $\alpha \leftarrow \alpha + \delta\alpha$;

(P2) if $\neg C(\alpha, \gamma)$, then $\kappa \leftarrow \kappa + 1$ and $\gamma \leftarrow \gamma + \delta\beta_\kappa$;

(P3) if $\gamma \neq \beta$ and $C(\alpha, \beta)$, then $\gamma \leftarrow \beta$;

(P4) if $C(\alpha, \beta)$ and $\min r \geq \beta_\mathrm{C}$, then the search for optimum $\boldsymbol{\mu}$ is terminated successfully;

(P5) if $\neg C(\alpha, \varepsilon\beta)$ for arbitrary $\varepsilon > 0$, then the search for optimum $\boldsymbol{\mu}$ ends in failure.

Here, $\min r$ denotes a minimum instantaneous distance given by

$$\min r = \min\Big\{\min\{r_{ij}(t); \forall(i,j)\}; t \in [0, t_\mathrm{f}]\Big\} \quad (28)$$

Therefore, $\min r > \beta_\mathrm{C}$ in the terminating condition (P4) shows that any near miss is avoided. When each shooting search ends in (P1) or (P3), the last $\boldsymbol{\mu}$ is regarded as an initial guess $\boldsymbol{\mu}^0$ for the next shooting search. When each shooting search ends in (P2), $\boldsymbol{\mu}$ that satisfies the convergence condition latest is regarded as $\boldsymbol{\mu}^0$ for the next search.

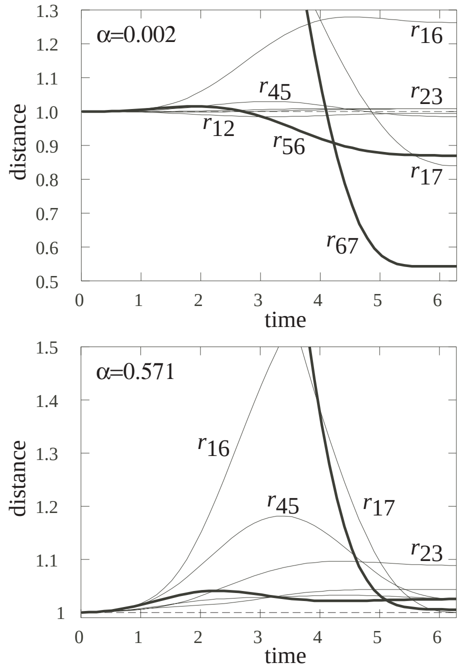

Figure 2: Distances between the respective satellites with $\beta = 1.04$.

A shooting search corresponds to the Newton-Raphson method. Consider $\Lambda = [\boldsymbol{\psi}_1(t_\mathrm{f}), \ldots, \boldsymbol{\psi}_n(t_\mathrm{f}), \boldsymbol{\varphi}_1(t_\mathrm{f}), \ldots, \boldsymbol{\varphi}_n(t_\mathrm{f})]^\mathrm{T} \in \mathcal{R}^{4n}$ as a function of $\boldsymbol{\mu} \in \mathcal{R}^{4n}$. The $4n$-dimensional tangential plane of Λ at $\boldsymbol{\mu}^m$ intersects the $4n$-dimensional $\boldsymbol{\mu}$ plane in the $8n$-dimensional space. This intersection point is regarded as $\boldsymbol{\mu}^{m+1}$.

In principle, a shooting method of the boundary-value problem is repeated with fixing β and increasing α, following the conditional command (P1). There is another way to expand w such that α is fixed and β is increased from zero. Changing β, however, means varying repulsive-interaction regions in the phase space, which is different from an increase of α corresponding to just strengthening the interaction. If α is not small, even a small change of β results in divergence of $\boldsymbol{\mu}^m$ because of the radical change of interaction dynamics. However, changing β can be the second best way while α is close to zero. In particular, this is considered to be effective if $\boldsymbol{\mu}^m$ neither converge nor diverge. The conditional commands (P2) and (P3) are hence included.

3. NUMERICAL EXPERIMENT

3.1 Initial Value

The fuel-optimum injection into clustering satellites is now evaluated. Let the near-miss distance β_C be the unit distance, 1, and the initial values be

$$\boldsymbol{\epsilon}_i(0) = \begin{cases} [0, 0, \sin\phi_i, \cos\phi_i]^{\mathrm{T}}, i = 1, \ldots, 6 \\ [-t_{\mathrm{f}}\pi/4, \pi/4, 0, 0]^{\mathrm{T}}, \ i = 7 \end{cases} \quad (29)$$

where $\phi_i = \pi i/3$. It corresponds to e-separation for six satellites and eastward drift of the seventh satellite. Let t_{f} be 2π, i.e., one period of the reference orbit. If no thrust is performed, the satellites move along

$$\begin{bmatrix} x_i^{(1)}(t) \\ x_i^{(2)}(t) \end{bmatrix} = \begin{cases} \begin{bmatrix} \sin(\phi_i - t) \\ \cos(\phi_i - t) \end{bmatrix}, i \in [1, 6] \\ \begin{bmatrix} -\pi/3 \\ \pi(t - 2\pi)/4 \end{bmatrix}, i = 7 \end{cases} \quad (30)$$

which puts all the satellites in the following dangerous situations. The minimum distance of an initial cluster is equal to the near-miss distance β_{C}. Moreover, S_7 approaches S_4 and S_5. This is unrealistic since a cluster should be operated to leave a safety margin. These are, however, considered adequate for testing the procedure. The first step, i.e., the fuel-optimum guidance without a penalty function, causes a collision between S_6 and S_7 since the satellites follow

$$\begin{bmatrix} x_i^{(1)}(t_{\mathrm{f}}) \\ x_i^{(2)}(t_{\mathrm{f}}) \end{bmatrix} = \begin{cases} [\sin(\phi_i), \cos(\phi_i)]^{\mathrm{T}}, i \in [1, 6] \\ [0, 1]^{\mathrm{T}}, i = 7 \end{cases} \quad (31)$$

The penalty-function guidance is then necessary.

The sequence $\boldsymbol{\mu}^m$ is considered convergent if every component difference between the sequence is less than 10^{-5}. If $\boldsymbol{\mu}^m$ does not converge during 100 iterations, it is considered non-convergent. The increments of β at the conditional command (P2) are fixed as $\delta\beta_1 = 0.1\beta$, $\delta\beta_2 = 0.02\beta$, and $\delta\beta_\kappa = 0$ for $\kappa = 3, 4, \ldots$, respectively. Let $\delta\beta_0$ be given later. Each search is computed by a fourth-order Runge-Kutta integration with a 0.002π time step (one thousandth of a period), providing $\delta\alpha = 10^{-4}$ for $\alpha \leq 0.01$, and 10^{-3} otherwise.

3.2 Optimized Orbit

Fig. 2 shows the result with $\delta\beta_0 = 0.04\beta_{\mathrm{C}}$. All the time-varying distances between satellites are graphed at $\alpha = 0.002$ and the final command (P4), respectively. The parameter α is increased to 0.571. Two bold curves show r_{56} and r_{67} in each figure. The horizontal broken line represents β_{C}. Although $r_{12}, r_{45}, r_{56}, r_{67}$, and r_{17} are judged near misses at $\alpha = 0.002$, all the satellites avoid any near miss in the final stage. The complete near-miss avoidance orbits of $(x_i^{(1)}, x_i^{(2)}), i = 1, \ldots, 7$ with the same $\delta\beta_0$ are drawn in Fig. 3. The open and the filled circles represent the initial and the final points, respectively.

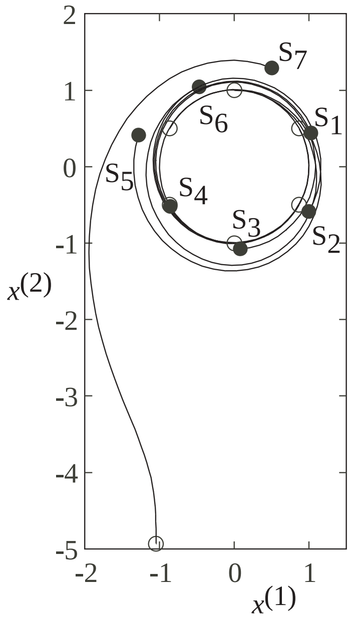

Figure 3: Orbits of the satellites obtained by the penalty function with $\beta = 1.04$ and $\delta\beta_1 = +0.1\beta$.

The final state that S_7 is injected between S_6 and S_1 is obtained. For $t \geq t_{\mathrm{f}}$, every satellite revolves once per orbital period around the origin with each final radius.

The shooting searches are simulated for various $\delta\beta_0$. The consumed fuel and the α at the terminating condition (P4) or (P5) are evaluated in Table 1, where

$$\max|u| = \max\Big\{\max\big\{|u_i(t)|; \forall i\big\}; t \in [0, t_{\mathrm{f}}]\Big\} \quad (32)$$

For $\delta\beta_0 \leq 0.02$, $\boldsymbol{\mu}^m$ diverges before $\min r \geq \beta_{\mathrm{C}}$. Although the interruption condition (P5) suggests trying searches for any ε, the simulation is abandoned without changing ε since α is already large. For $\delta\beta_0 \geq 0.04$, near-miss avoidance succeeds. A final α decreases and $\mathrm{tot}|\Delta v|$ increases with increasing $\delta\beta_0$. Assuming that each shooting search requires a constant duration, a final α is nearly proportional to the computation time. Thereby, a quick search is possible if additional fuel is used. Accordingly, $\delta\beta_0 \sim 0.1$ and $\delta\beta_0 \sim 0.2$ lead to $\mathrm{tot}|\Delta v| \sim 1.1$ with $\alpha \sim 0.1$ and $\mathrm{tot}|\Delta v| \sim 1.2$ with $\alpha \sim 0.01$, respectively. Table 1 implies that $\mathrm{tot}|\Delta v|$ by the exact optimum solution is expected as about 0.9. According to our defined α increment step, a 10% increase in β causes a 20% increase in fuel with about 200 time shooting searches, and a 20% increase in β brings a 30% increase in fuel with about 100 searches. Therefore, guidance with moderate increases in fuel and reasonable shooting duration searches are constructed.

Feasibility of the maximum instantaneous thrust

Table 1: Fuel consumed for various β ($\delta\beta_1 = +0.1\beta$)

$\delta\beta_0$	$\min r$	tot$\lvert\Delta v\rvert$	$\max\lvert u\rvert$	final α	$J(\boldsymbol{u})$
0.01	0.99719	0.947	0.1381	8.600	—
0.02	0.99924	0.966	0.1382	2.700	—
0.04	1	0.998	0.1376	0.571	0.0343
0.08	1	1.05	0.1353	0.111	0.0378
0.16	1	1.15	0.1305	0.021	0.0452
0.32	1	1.21	0.1389	0.0033	0.0589

Table 2: Fuel consumed for various β ($\delta\beta_1 = -0.1\beta$)

$\delta\beta_0$	$\min r$	tot$\lvert\Delta v\rvert$	$\max\lvert u\rvert$	final α	$J(\boldsymbol{u})$
0.04	1	0.970	0.1827	0.7539	0.0365
0.08	1	1.01	0.1863	0.134	0.0406
0.16	1	1.08	0.1889	0.025	0.0493
0.32	1	1.32	0.1955	0.0066	0.0066

$(2\max|u|) \sim 0.28$ shown in Table 1 is confirmed by fixing scales. Consider cluster formation in geosynchronous orbit. A unit time is a period divided by 2π and then given by 13713.4[s]. A unit distance, i.e., the near-miss distance, is assumed as 1000[m]. A unit acceleration is then interpreted as 5.31753×10^{-6}[m/s^2]. The maximum instantaneous thrust approximates 1.5[mN] for 1000[kg] mass.

3.3 Problem for Future Work

The primary problem of the proposed optimization procedure is that a search for global minimum is impossible if any local minimum exists and initial-guess values are iterated towards the local minimum. Fig. 4 shows the orbit with the same $\delta\beta_0$ as the one in Fig. 3 but with $\delta\beta_1 = -0.1\beta$ instead of $+0.1\beta$. The final configuration by $\delta\beta_1 = -0.1\beta$ is that S_7 is injected between S_5 and S_6, which is different from the result by $\delta\beta_1 = +0.1\beta$. Simulated annealing or genetic algorithm is indeed a solution to be applied to the iterative search instead of the shooting method. Before that, however, the relation between the penalty-function formula and the existence of local minimum should be investigated in detail.

The secondary problem is that tot$|\Delta v|$ by $\delta\beta_1 = +0.1$ is slightly larger than the one by $\delta\beta_1 = -0.1$ (see Table 2) although the former case is optimized, concerning minimization of $J(\boldsymbol{u})$. It is necessary to consider more realistic formulation of costs.

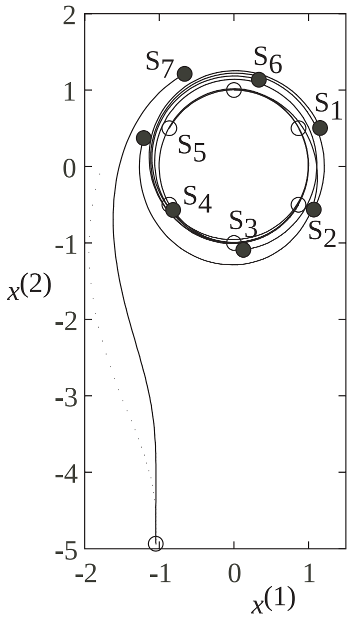

Figure 4: Orbits of the satellites obtained by the penalty function with $\beta = 1.04$ and $\delta\beta_1 = -0.1\beta$.

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