INTERPLANETARY AND LUNAR TRANSFERS USING LIBRATION POINTS

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ABSTRACT

In this paper a technique based on the invariant manifolds theory has been optimized and applied to cases of practical interest: interplanetary transfers (among inner planets) and low energy transfers to the Moon. In the former case the manifolds of Sun-Planet systems, not intersecting in the configuration space, are connected by splitting the full four-body problem into two three-body problems and linking the corresponding transit orbits through a conic arc. Low energy transfers to the Moon, instead, have been obtained by targeting, through a Lambert's three-body arc, a piece of the stable manifold associated to the *L1* point of the Earth-Moon system. The proposed *patching conic-manifolds method* exploits the two gravitational attractions of the bodies involved in the transfer to change the energy level of the spacecraft and to perform a ballistic capture and a ballistic repulsion. The effectiveness of this approach is demonstrated by a set of solutions found for Venus, Mars and Moon transfers.

1. INTRODUCTION

Preliminary design of interplanetary and lunar space trajectories is commonly done using the patching conics method and, thus, describing the motion of the spacecraft with a two-body model. For instance, the heliocentric legs, described by taking into account just the action of the Sun, are patched together with the planet-centered conics in order to define the whole interplanetary transfer. By this technique the transfer problem can be formulated analytically and this description fits well with further optimization processes.

When looking for low energy trajectories exploiting more than one gravitational attraction, a more complete model should be employed [1]. Nevertheless, the introduction of such models means the lost of the analyticity of the solution since the problem is no longer integrable. In particular, the conservation of the angular momentum vanishes and the orbital parameters can be defined just as variation of the two-body Keplerian motion. Moreover, even if the generic n-body models are governed by the Newtonian dynamics, they turn out to be chaotic systems and make the trajectory design a challenging issue.

The easiest extension of the two-body model is represented by the Circular Restricted Three-Body Problem (CR3BP). This problem involves only one integral of motion (the Jacobi constant) meaning that some "dynamical substitutes" should be introduced to assure preliminary information to the design. These objects, in the case of the CR3BP, are the libration points, the periodic and quasi-periodic orbits around them and their associated invariant manifolds.

In the last years, the works carried out by Koon, Lo, Marsden and Ross [2] led to the definition of a technique to obtain low energy transfers between two bodies. This kind of transfers can be obtained if the manifolds, associated to the periodic orbits around libration points of each three-body problem, intersect in the configuration space. Thus, interplanetary transfers among outer planets and moon-to-moon transfers around a giant planet can be computed by matching, in the configuration space, the manifolds of two different systems and performing, with a Δv maneuver, the intersection in the whole phase space. The invariant manifolds approach has also helped to understand the dynamics behind the Belbruno-Miller trajectories, or Weak Stability Boundary (WSB) lunar transfers [1]. In this case, indeed, it has been proven that the Moon's capture was due to the intersection of the manifolds associated to the periodic orbits around *L2* of the Sun-Earth and Earth-Moon systems [3].

Thus, the importance of the invariant manifolds to design space trajectories has matured in the last years, but, the main drawback of this technique is the requirement of an intersection between the manifolds in the physical space. This makes the method only suitable for transfers, as between two outer planets or two moons around a giant planet, where the physical constants and the orbital parameters allow such an intersection. Thus, a question arises: is it possible to have a low energy transfer exploiting the invariant manifolds even if an intersection does not exist?

The present paper aims to explore the possibility to combine the invariant manifolds with other techniques in order to get low energy *interplanetary* and *lunar* transfers of practical interest. Both the applications are based on the same concept: if the properties of the R3BPs do not allow a free transportation, the two bodies are linked by targeting their manifolds with an additional trajectory leg; this leg is represented by a two-body (for interplanetary transfers) or a three-body (for lunar transfers) Lambert's arc.

2. DYNAMICS

The equations of motion, in the second order Lagrangian form, written in the synodic dimensionless frame are [4]:

$$\begin{cases} \ddot{x} - 2\dot{y} = \Omega_x \\ \ddot{y} + 2\dot{x} = \Omega_y \end{cases} \tag{1}$$

where the subscripts denote the partial derivatives of:

$$\Omega(x, y) = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu) \tag{2}$$

with respect to the coordinates of the spacecraft (x,y). Since the synodic system (Fig. 1) has the origin in the center of mass, the two distances in the Eqn. 2 are:

$$\begin{aligned} r_1^2 &= (x+\mu)^2 + y^2 \\ r_2^2 &= (x-1+\mu)^2 + y^2 \end{aligned} \tag{3}$$

The system has a first integral of motion, called *Jacobi integral*, which is given by:

$$C(x, y, \dot{x}, \dot{y}) = 2\Omega(x, y) - \left(\dot{x}^2 + \dot{y}^2\right) \tag{4}$$

and represents a 3-dimensional manifold for the states of the problem within the 4-dimensional phase space. Once a set of initial conditions is given, the Jacobi integral defines some forbidden and allowed regions bounded by the zero velocity curves. The energy of the spacecraft and the Jacobi constant are related by:

$$C = -2E \tag{5}$$

which states that a high value of C is associated to a low energy of the spacecraft. For a low value of the energy the spacecraft is bounded to orbit around one of the two primaries. If the energy is increased the allowed regions of motion enlarge and the spacecraft is free to leave one of the primaries.

The differential system of Eqns. 1 presents five equilibria: three points (*L1*, *L2* and *L3*) are aligned with the primaries and called *collinear*; two points (*L4* and *L5*) are at the vertex of two equilateral triangles with the primaries and called *triangular* (Fig. 1). The dynamics around *L1* and *L2* is equal to the product of a *saddle* times a 4D *center*.

There are two manifolds associated to *L1* and *L2*: a stable (W^S_{Li}) and an unstable (W^U_{Li}), both 1-dimensional ($i=1,2$). The manifolds associated to the Lyapunov orbits are centered on these lines and called $W^S_{Li,p.o.}$ and $W^U_{Li,p.o.}$ ($i=1,2$). If a spacecraft is *on* a stable manifold, its trajectory winds onto the orbit while, if it is *on* the unstable one, it winds off the orbit (Fig. 2).

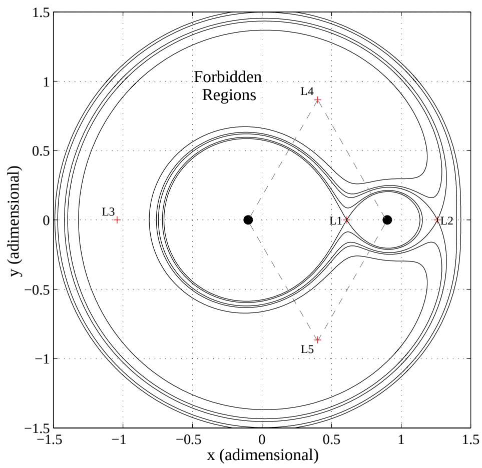

Fig. 1. Synodic system, libration points and Hill's curves for several values of C ($\mu=0.1$)

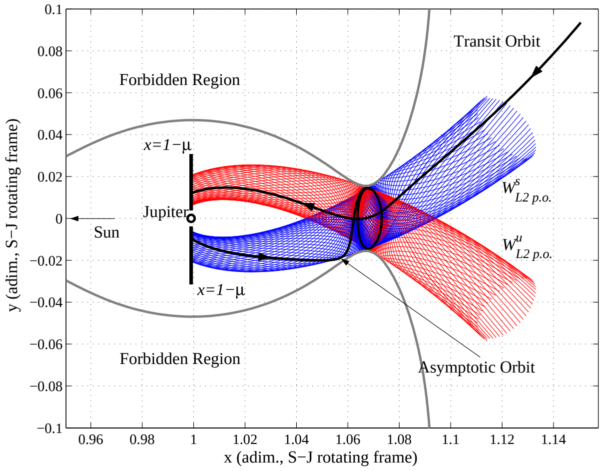

Fig. 2. Transit and asymptotic orbits in a neighborhood of the Sun-Jupiter *L2*

The method used to compute the manifolds is based on the linear approximation of the flow mapping around a periodic orbit. Thus, once the monodromy matrix M associated to a periodic orbit has been obtained, the manifolds are computed by propagating the flow along the directions associated to the Floquet multipliers of that orbit. It is important to observe that the manifolds are separatrices and they split different regimes of motion. This means that orbits starting inside the tubes, flowing under the dynamical system of Eqns. 1, will continue to remain in that tube. These trajectories are called *transit orbits* because they are the only able to go through the small allowed region and to approach the planet for a given energy value (Fig. 2).

In the present work the invariant manifold *tubes*, delimiting the appropriate dynamical ways useful to approach or depart from a planet at a low energy level, have been used to define the transit orbits. Hence, the selection of a specific energy level uniquely sets the size of the periodic orbit around the libration point involved (e.g. *L1* if the transfer is toward the interior, *L2* if it is toward the exterior); then, the unstable manifold associated to this orbit is computed in the Sun-Earth synodic and sideral systems. The latter frame is used to look for optimal intersections between this unstable manifold and the stable one, computed in the Sun-Target Planet system, and the candidate transit trajectories are chosen on the corresponding Poincaré section [5] [6].

3. INTERPLANETARY TRANSFERS

If two transit orbits are chosen, one starting from the Earth and the other approaching the arrival planet, the departure and arrival legs of the interplanetary trajectory are given. If these two orbits match in the physical space, a single Δv is required to perform the transfer [3]. When the two trajectories do not match, as occurs when looking for a transfer between two inner planets, an intermediate arc is needed to perform the transfer [5].

Thus, the whole interplanetary transfer is composed by three different arcs separated by four maneuvers: the first (Δv_S) is used to inject the spacecraft into an unstable manifold tube, toward the interior or the exterior according to the target planet (Fig. 3); the second (Δv_1) is a deep space maneuver useful to place the spacecraft on the intermediate conic arc (Fig. 4); the third (Δv_2), another deep space maneuver, places the spacecraft from the conic arc to the capture leg (Fig. 4); the fourth maneuver (Δv_E) is used to stabilize the spacecraft into a circular orbit around the arrival planet (Fig. 5).

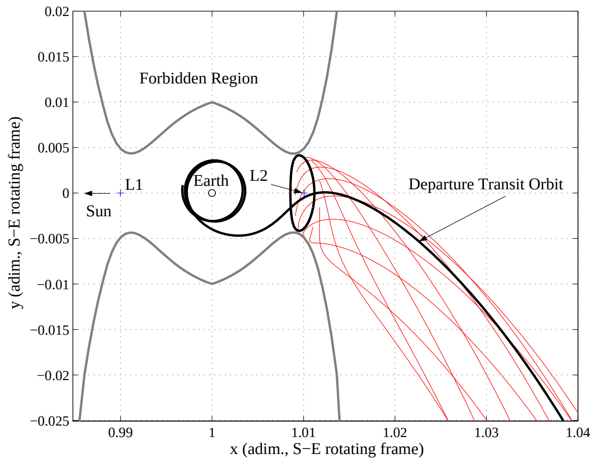

Fig. 3. First maneuver (Δv_S) used to inject the spacecraft into the unstable manifold tube

The total cost is:

$$\Delta v = \Delta v_S + \Delta v_1 + \Delta v_2 + \Delta v_E \quad (6)$$

and it is related to a transfer between two circular orbits with radiuses equal to r_S and r_E respectively around the Earth and the arrival planet. An optimization step, aiming at reducing the cost function 6, can be implemented by leaving the main parameters free to vary within certain ranges [6]. These parameters could be chosen among the sizes of the orbits around the libration points, the times concerning the three legs and the launch date. Indeed, to prove the effectiveness of these trajectories, the total cost can be compared with the one associated to the Hohmann transfer (Δv_H) linking the same departure and arrival orbits. A 3D analytical ephemeris model has been used in the design of interplanetary transfers; hence the cost function (Eqn. 6) takes also into account the Δv involved in the changes of inclination.

3.1 Earth-Venus

Table 1 shows a set of solutions found for the transfer to Venus: the total (Δv) and the Hohmann cost (Δv_H) refers to transfers starting from circular orbits of radius r_S and ending onto circular orbits of radius r_E respectively around the Earth and around Venus. The first solution, represented in Figs. 4,5, takes *732 days* to reach Venus and costs *722 m/s* less than the corresponding Hohmann transfer; the second is *534 days* long and *381 m/s* cheaper; the third one takes *494 days* and is *806 m/s* cheaper than the Hohmann. The typical time of flight (TOF) of the Hohmann transfers is *145 days*. These solutions show that up to *16%* can be saved in total Δv for a transfer to Venus. Nevertheless, due to the asymptotic nature [2] characterizing these transfers, the associated TOF are about *300%* longer.

Table 1. Solutions for the Earth-Venus transfer

Δv (m/s)	Δv_H (m/s)	r_S (km)	r_E (km)
4248	4970	294000	220900
4565	4946	173300	300900
4051	4857	173500	294700

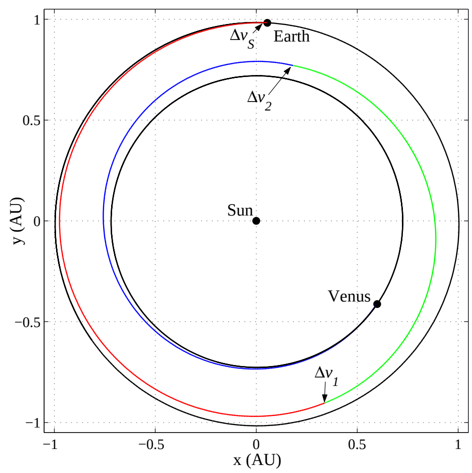

Fig. 4: Earth-Venus transfer and the four maneuvers

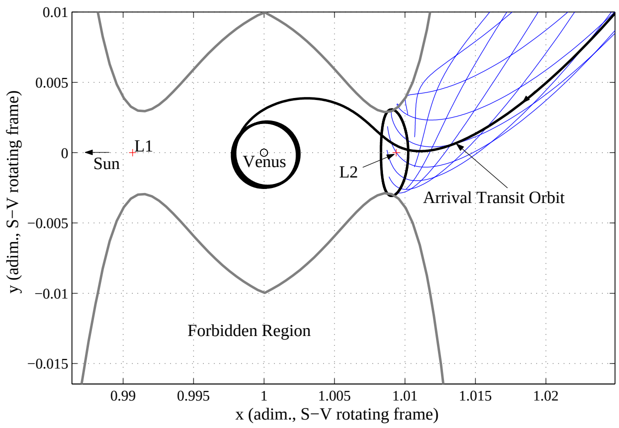

Fig. 5. Arrival leg and final circular orbit around the target planet (Venus in this case)

3.2 Earth-Mars

A typical Earth-Mars transfer is represented in Figs. 3 (escape transit orbit) and 6 (interplanetary trajectory); Table 2 shows a set of solutions found for this case. The first solution is *861 days* long and allows to save *117 m/s*; the second trajectory is *534 m/s* cheaper than the Hohmann one and *750 days* longer (the Hohmann transfer needs typically *250 days*); the third solution saves *327 m/s* and needs *919 days* to reach the Red planet. Even if the solutions have a TOF about *300%* longer, up to *12%* in total Δv can be saved with respect to a Hohmann transfer.

Table 2. Solutions for the Earth-Mars transfer

Δv (m/s)	Δv_H (m/s)	r_S (km)	r_E (km)
4457	4574	387600	484400
3755	4289	185900	142800
4607	4934	412300	241500

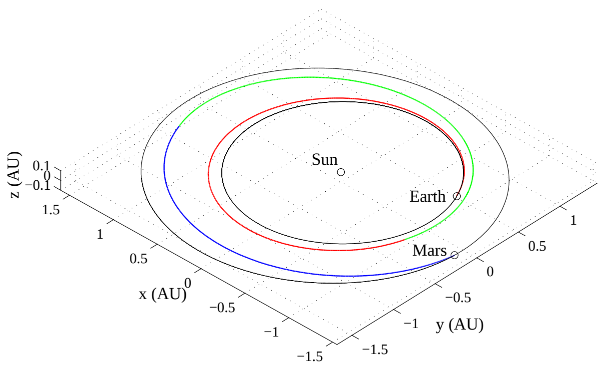

Fig. 6. Earth-Mars transfer

4. LUNAR TRANSFERS

The minimum energy required to reach the Moon, departing from the Earth, is the one corresponding to a value slightly lower than C_1, that is the Jacobi constant (Eqn. 4) corresponding to the *L1* point. With greater values, indeed, Hill's curves close and the motion is allowed only in the region around the Earth or around the Moon (Fig. 1).

Thus, assuming $C \leq C_1$, transfers to the Moon can occur through the small neck opened at *L1*. But, even if these transfers may occur theoretically, designing a trajectory crossing this region is very difficult in a chaotic regime like the R3BP. Based on the technique of *targeting*, some studies [7] [8] have shown that such trajectories require a long time and are extremely sensitive, so they seem to be unfeasible.

To overcome these difficulties, the invariant manifold theory is again considered. This is a clear example of the power of the manifolds since they provide for *additional structure* within the restricted problem frame.

Fig. 7 shows a piece of the interior stable manifold associated to *L1* (W^S_{L1}) and the corresponding exterior unstable manifold (W^U_{L1}). As can be seen, even if the transit region is very thin, these two trajectories represent a *transit orbit* between the forbidden region. So, if a spacecraft is on the W^S_{L1}, the system, by itself, brings it from a region close to the Earth to the region close to the Moon by simply exploiting the intrinsic dynamics of the three-body problem.

This kind of capture is different to both the "Belbruno-Miller trajectories" and to the "Koon et al patched manifolds" since there the Moon approach occurs from the exterior (i.e. from *L2*). The two invariant manifolds considered here allow a Moon transfer from the interior and with the smallest energy possible! Strictly speaking, this approach exploits the Moon resonances to “pump up” the apogee of the spacecraft’s orbit until it is definitively captured by the Moon.

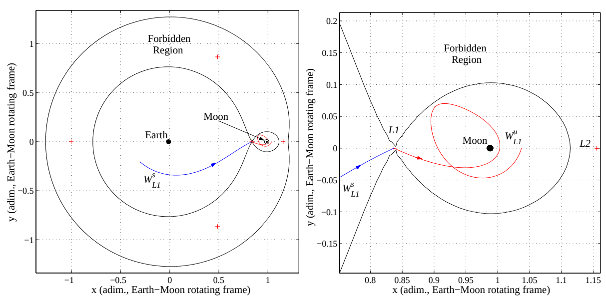

Fig. 7. The stable and unstable manifolds of the point *L1* in the Earth-Moon system (left). The two are transit trajectories through the small neck opened at *L1* (right).

Thus, the matter is to put a spacecraft on the interior stable manifold associated to *L1* and wait for the natural evolution of the system. Unfortunately, this manifold does not reach the Earth. Integrating backward W^S_{L1} for several Moon's periods, one can observe that this orbit performs several loops but it is never close to the Earth [9]. Moreover, the minimum Earth distance seems to be constant and almost equal to 0.35 Earth-Moon unit distances.

4.1 Approach and Results

Starting from circular orbits around the Earth, a numerical procedure, called Lambert's three-body problem [9], has been developed to compute some trajectory arcs that target a point on W^S_{L1}. An example, found in literature [10], solves this problem in regularized coordinates; here the Lambert's three body problem (i.e. a two-point boundary value problem) has been solved in physical coordinates using a shooting method that avoids Earth and Moon impacts. Thus, given two points, the orbit linking them in a prescribed time of flight is computed under the R3BP dynamics (Eqns. 1).

A first maneuver (Δv_1) is used to place the spacecraft into a translunar trajectory starting from a low Earth orbit; a second maneuver (Δv_2) injects the spacecraft on the capture trajectory represented by W^S_{L1}.

The total cost of the transfer is:

$$\Delta v = \Delta v_1 + \Delta v_2 \tag{7}$$

This formulation, by itself, leads to a final unstable orbit around the Moon with mean altitude equal to *21600 km*. This final orbit, shown in Fig. 8, can be further stabilized with additional maneuvers; nevertheless, in order to compare the found solutions with the classical Hohmann transfer, this mean orbit around the Moon has been considered.

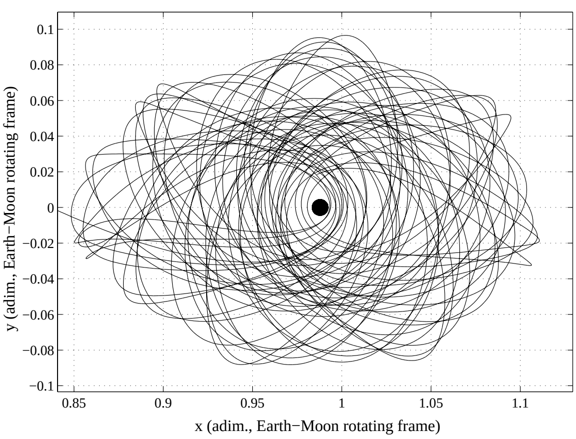

Fig. 8. Final unstable orbit around the Moon

Table 3 shows a set of solutions found for the transfers to the Moon. Two sample departure orbits have been considered: a *200 km* Low Earth Orbit (LEO) and a *200 x 35840 km* Geostationary Transfer Orbit (GTO). The first solution is remarkable since allows to reach the Moon with a cost equal to *3081 m/s* and with a time of flight of *49 days*. The corresponding solution obtained departing from a GTO has a cost equal to 914 m/s and the same time of transfer. Note that the Hohmann transfer from the LEO costs *3344 m/s* and is *6.5 days* long. If the Hohmann solution starts from the GTO the cost reduces down to *1177 m/s* with the same time.

Even if the time of flight increases, these results show that a low energy transfer to the Moon can save up to 22% in total Δv departing from a GTO and up to 8% departing from a LEO. Furthermore, a low Moon orbit could be obtained with an additional maneuver performed at L1 where the two gravitational attractions balance and small changes in the velocity vector produce large deviations in the final trajectory [7]. This concept applies also when free fall trajectories to the Moon are required.

Table 3. Solutions for the Earth-Moon transfer

Δv (m/s)		Δt (days)
LEO	GTO	-
3081	914	49
3085	918	119
3091	924	47

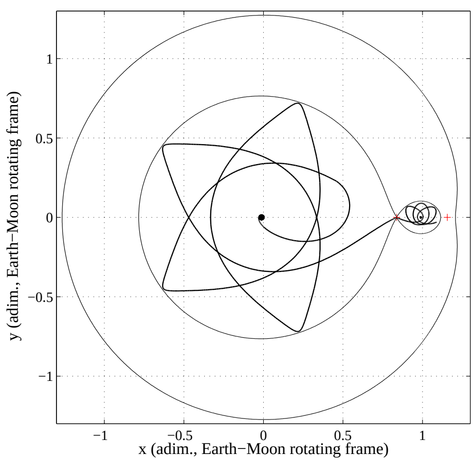

Fig. 9. A typical Earth-Moon transfer obtained with the developed approach.

5. CONCLUSIONS

The paper describes an approach that allows to extend the technique of the manifolds to systems where the gravity constants and the relative distances of the bodies do not allow any intersection in the configuration space. In particular, low energy interplanetary transfers and Earth-Moon transfers have been considered to prove the effectiveness of such highly non-linear trajectories. In particular, the invariant manifolds reveal to be good "dynamical substitutes" when the dynamical problem loose its analyticity and more information are required to design the path.

Since the trajectories defined in the R3BP frame need a certain time to exploit the two gravitational attractions, and so to reduce the cost of the transfer, the time of flight associated to these transfers increases; but, up to *16%* can be saved in total cost for a transfer to Venus while up to *22%* for a transfer to the Moon. This leads to employ these trajectories for cargo missions where the payload mass must be maximized without any particular constraint on the transfer time.

The possibility to combine the invariant manifolds technique together with the use of low-thrust systems represents an advanced topic for the trajectory design and an interesting application for the future space missions; therefore the authors are currently investigating such an opportunity.

6. ACKNOWLEDGMENTS

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