# EXAMINATION OF THE LIFETIME, EVOLUTION AND RE-ENTRY FEATURES FOR THE "MOLNIYA" TYPE ORBITS

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## ABSTRACT

Space vehicles of the "Molniya" series, launched into the special highly elliptical orbits with a period of ~ 12 hours, are intended for solution of telecommunication problems in stretched territories of the former USSR and nowadays − Russia, and also for providing the connectivity between Russia and other countries. The inclination of standard orbits of such a kind of satellites is near to the critical value $i \approx 63.4$ deg and their initial minimal altitude normally has a value $H_{min} \sim 500$ km. As a rule, the “Molniya" satellites of a concrete series form the groups with determined disposition of orbital planes on an ascending node longitude that allows to ensure requirements of the continuous link between any points in northern hemisphere. The first satellite of the "Molniya" series has been inserted into designed orbit in 1964. Up to now it is launched over 160 space vehicles of such a kind.

As well as all artificial satellites, space vehicles "Molniya" undergo the action of various perturbing forces influencing a change of their orbital parameters. However in case of space vehicles "Molniya" these changes of parameters have a specific character and in many things they differ from the perturbations which are taking place for the majority of standard orbits of the artificial satellites with rather small eccentricity. In particular, to strong enough variations (first of all, at the expense of the luni-solar attraction) it is subjected the orbit perigee distance $r_\pi$ that can lead to such reduction $H_{min}$ when further flight of the satellite on its orbit becomes impossible, it re-entries and falls to the Earth. Moreover, possessing the mass up to ~ 1.5 and more tons, the similar space vehicles can create the real risk situations at fall.

In paper the influence of various perturbing factors (the non-central part of a gravitational field of the Earth, an attraction from moon and the Sun, solar radiation pressure, an atmospheric drag) on changing the orbital parameters of the "Molniya" type satellites within the different time spread intervals (including the long-term ones of the order of dozens years) is researched.

The procedure and the results of corresponding calculations that allow estimating the influence of various significant perturbing factors and their joint effect on long-time evolution of the orbits and lifetime of "Molniya" type space vehicles are given.

It has been found that lifetime of space vehicle "Molniya" depends on the initial values of the longitude of ascending node $\Omega$ of the orbit, and also on the value of its argument of perigee $\omega$.

On an example of real space vehicles of the "Molniya-3" series, that recently terminated their flights, the character of the variation of the principal orbital parameters at the flight final phase, as well as the conditions and peculiarities of the de-orbiting and re-entering of such a kind of space objects are shown.

The carried out examinations allow concluding, that at the result of the orbital evolution all of the "Molniya" class space vehicles sooner or later will terminate their life on the orbit. (At the moment of representing the paper ~ 50 objects of the "Molniya" series remain in space).

The methods introduced in the paper and the obtained data can be of interest at designing the highly elliptical orbits for the artificial satellites and for investigation of the orbital motion of these objects.

## 1. INTRODUCTION

The Russian telecommunication satellites of the “Molniya” series are inserted into special high elliptical orbits, possessing certain advantages for providing communication and surveying at high

latitudes. The period of standard orbits of such a kind of satellites is about $12^h$ and the inclination is near to the critical value $i \approx 63.4$ deg promoting a small and slow modification of an apsides line of these orbits. The perigee altitude of a “Molniya” type orbit can vary depending on mission purposes and a spacecraft (SC) modification; however for standard orbits of SC of “Molniya-1” series the value of this parameter is close to ~ 500 km. Usually satellites of a concrete series form the constellation. The planes of satellite orbits are disposed such as to ensure requirements of full coverage of all Russia and CIS territory, and also of foreign countries. Satellites of the “Molniya” series have been launched since the mid of 1960s. At present more than 160 spacecrafts of this type were inserted into orbits. (It is necessary to notice, that afterwards other satellites, belonging to the different countries, began to be launched into orbits of the “Molniya” type subsequently).
During the flight time these orbits suffer the significant modifications at the expense of various perturbing factors. To counteract perturbations and to keep back the given properties of the orbits, the SC “Molniya” are equipped with a correcting propulsion system, and from time to time the orbital parameters are corrected. However after the end of a mission (or because of a SC emergency) the satellite turns into uncontrolled mode and nothing can stop the change of its orbital parameters due to the perturbing forces. At the expense of orbital evolution the altitude of perigee and the minimal altitude also vary, becoming in certain time so low, that the spacecraft cannot orbit the Earth and it re-entries. Thus, the conditions of the final phase of a flight and the re-entry of a “Molniya” type satellite as a rule have the other character, than the re-entry of the space objects which were flying in circular (or near-circular) orbits. The first SC of “Molniya” series re-entered in March 1967, having lifetime of ~ 1.5 years. Further similar re-entries of such SC became regular enough and now only about 50 satellites of the “Molniya” series continue their orbital flights with the various remaining lifetime.
Having the mass exceeding 1.5 tons, “Molniya” spacecrafts at uncontrollable re-entry can be considered as the real hazard objects that demands the accurate permanent orbital control of them especially at the final phase of a flight.

## 2. EVOLUTION OF THE “MOLNIA” TYPE ORBITS

Under investigation of evolution of the given class orbit we will consider as the base an orbit with the following initial parameters:

draconic period ($T$) □ 718 min,
minimum altitude ($H_{min}$) □ 550 km,
inclination ($i$) □ 62,8 deg,
argument of perigee ($\omega$) □ 280 deg.

To the indicated parameters there are a semi-major axis $a \approx 26600$ km and an eccentricity $e \approx 0.74$ osculating at the time of an ascending node.
The orbit with mentioned above parameters was most often used as the nominal one at launching the SCs of “Molniya” series. At the same time, for wider representation of orbit evolution and SC’ lifetime in these orbits, in some cases, in addition to the base orbit we will examine also the orbits slightly different from it on parameters $i$ and $\omega$.
We will carry out the analysis of the orbital parameters evolution concerned a space vehicle passive flight, i.e. we will suppose that, in case of the active SC, all corrections of its orbit have already been fulfilled, and residual impulses of its operating attitude control systems are enough small, or, that SC is not capable to fulfill any more of its goal functions, and it became a space debris.
Flying in high elliptical orbit the SC “Molniya” periodically approaches the Earth entering the atmosphere and exposes itself to its drag influence, and disposes from the Earth in the significant distances, filling strong enough luni-solar gravitational influence. Because of a design, including the shape and size of solar batteries, the SC “Molniya” has a significant reflecting area promoting solar pressure radiation.
At the analysis of the “Molniya” type orbits evolution the influence of the following major factors, perturbing spacecraft’ motion, were estimated:

- non-central part of the Earth gravity field,
- luni-solar gravitational attraction,
- solar radiation pressure,
- aerodynamic drag of the Earth's atmosphere.

The estimation of the orbital parameters evolution was fulfilled on the base of a numerical integration of the differential equations of the SC' motion for the given time periods applying one or another model of perturbing forces, or by means of using the certain analytical relations. For transformation from rectangular coordinates and velocities of SC, being particularly a result of numerical integration of the SC' motion equation, to the appropriate orbital elements and for inverse transformation were used the known formulas.

The system of the differential equations, realizing numerical model of the SC motion and taking into account the complete composition of the indicated above forces, can be presented in the rectangular inertial geocentric coordinate system (IGCS) related to the mean equinox and equator of standard epoch J2000 in the vector form given by Eq. 1:

$$\ddot{\bar{r}} = -\mu \cdot \frac{\bar{r}}{r^3} + M \cdot \overline{grad}\, U(\bar{r}') + \bar{F}_{atm}(\bar{r}, \dot{\bar{r}}) \quad + \sum_{\alpha = L,S} \bar{F}_{\alpha}(\bar{r}, \bar{r}_{\alpha}) + \bar{F}_{spr}(\bar{r}, \bar{r}_s). \tag{1}$$

Here $\bar{r}, \dot{\bar{r}}, \ddot{\bar{r}}$ − position vector, velocity vector and acceleration vector of a SC in a mentioned inertial system of coordinates; $\mu$ − a gravitation constant of the Earth; $U(\bar{r}')$ − a non-central part of the Earth gravity field represented in decomposition to a series by spherical functions (harmonics), $\bar{r}' = M^T \bar{r}$ − a SC position vector in geocentric Earth-fixed rotating coordinate system ($M$ − a matrix of transformation from the rotating to the inertial system of coordinates); $\bar{F}_{atm}$ − an acceleration called by atmospheric drag; $\bar{F}_{\alpha} = \bar{F}_L, \bar{F}_S$ − an acceleration due to a gravitational attraction of the "third" body (the Moon or the Sun); $\bar{F}_{spr}$ − an acceleration caused by the solar radiation pressure; $\bar{r}_L, \bar{r}_S$ − position vectors of the Moon and the Sun in IGCS.

The usage of Eq. 1 for the Earth satellite motion model gives a possibility simply enough to take into account the various perturbing forces and to obtain the appropriate calculation data, allowing estimate influence of these perturbing factors on the evolution of the satellite orbit parameters both separately and jointly. Thus, the obtaining of the basic calculation data – the position and velocity vectors of the SC at the given epoch in each case was carried out by means of the numerical integration of appropriate variants of Eq. 1 with the help of the effective numerical method [1, 2].

As the orbital elements used at the analysis of an orbit evolution, were considered classical Keplerian elements $a, e, i, \omega, \Omega$, and also perigee distance $r_{\pi} = a\,(1 - e)$ and the minimum altitude of SC over the Earth surface − $H_{min}$. The last two parameters are especially important from the point of view of an estimation of the remaining SC's lifetime.

## 2.1. <u>The influence of non-central part of the Earth gravity field on "Molniya" type orbit evolution</u>

The influence of non-central part of the Earth gravity field is the main perturbing factor practically for all orbits of artificial Earth satellites. Due to this factor the orbital parameters of an artificial satellite have a short-period, a long-period and secular variations.

The short-period variations representing changing in orbital parameters within one revolution take place for all elements. In Figs. 1-3 the short-period variations of a semi-major axis, an eccentricity and a perigee distance of the "Molniya" type orbit due to the influence of a non-central part of the Earth gravity field on two neighboring revolution are shown. (In this case the harmonics up to the order and degree (16×16) were taken into account). The represented results correspond to the basic orbit and to the orbit different from it on the parameter $\omega$ (which in the second variant had the value

$\omega$ = 300°). From these figures, in particular, follows that in spite of the maximum value of the parameter $a$ variations on one revolution reaches ~150 km, due to a synchronous changing of parameter $e$ the variation of perigee distance $r_\pi$, determined by dependence $\delta r_\pi = \delta a(1-e) - a\,\delta e$, in both cases do not exceed 3-5 km.

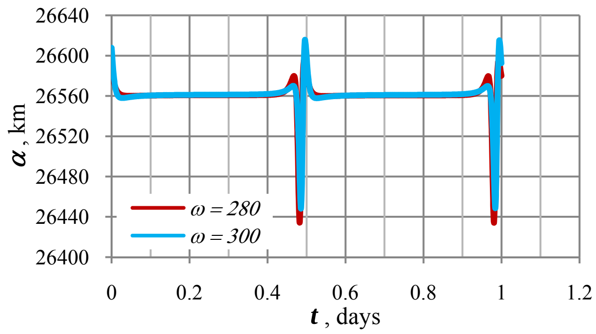



Fig. 1. “Molniya” type orbit’s semi-major axis variation on two revolutions

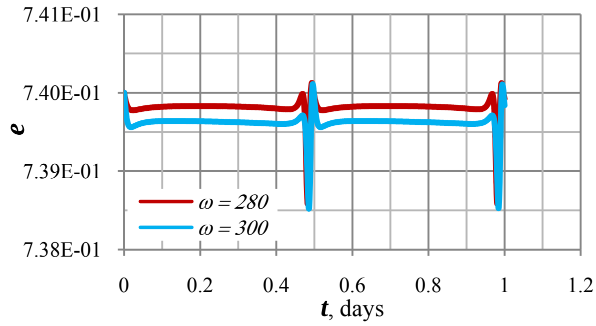



Fig. 2. “Molniya” type orbit’s eccentricity variation on two revolutions

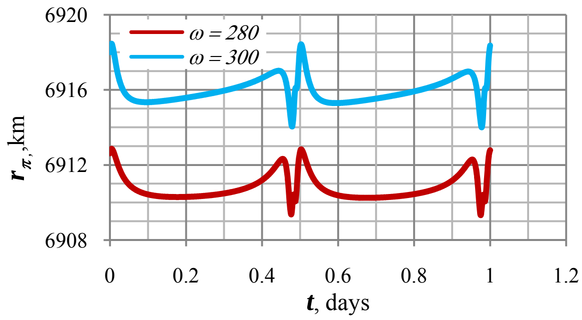



Fig. 3. “Molniya” type orbit’s perigee distance variation on two revolutions.

Long-period perturbations (having a considerably greater oscillation period in comparison with a SC’ orbit one) appear in all elements, except for a semi-major axis. However, the value of variations of orbital parameters (due to the Earth gravity field) is rather small. The character of the long-period variations caused by the influence of the Earth gravity field of order and degree (16×16) in the example of parameter $r_\pi$ is shown in Fig. 4. Here the results of calculations for the basic orbit and for the orbits different from it or only on inclination ($i$ = 65°), or only on perigee longitude ($\omega$=300°) are presented. As it is seen from the figure, for the basic orbit the value of $r_\pi$ decreased on ~ 50 km within the period of 15 years. In the case of orbit with $\omega$ = 300° the degradation of $r_\pi$ for the same time has made ~ 35 km, and for an orbit with $i$ = 65° the fifteen-year history of the perigee changing has led to a $r_\pi$ rising on ~ 10 km.

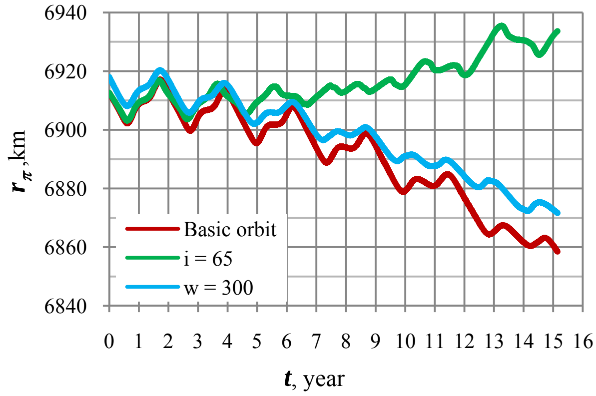



Fig. 4. Variations of perigee distance of "Molniya" type orbit's over fifteen years due to the Earth gravity field.

Secular drifts of orbital parameters become much more important from the point of view of a long-term evolution. Among the parameters interesting for us these drifts are presented at the longitude of ascending node $\Omega$ and in argument of perigee $\omega$. Secular drifts of the indicated parameters are stipulated by the influence of even zonal harmonics of geopotential, first of all – by the influence of the second zonal harmonic. Secular drifts of these parameters $\delta\Omega_{sec}$, $\delta\omega_{sec}$ due to influence of the second zonal harmonic can be presented in the form: $\delta_{sec}\Omega = \dot{\Omega}_2\left(t-t_0\right)$, $\delta_{sec}\omega = \dot{\omega}_2\left(t-t_0\right)$, where within the first order accuracy the coefficients $\dot{\Omega}_2$, $\dot{\omega}_2$ are determined by Eq. 2 as it follows from [3]:

$$\dot{\Omega}_2 = -\frac{3}{2}\cdot J_2 \cdot n \cdot \left(\frac{a_e}{a}\right)^2 \cdot \frac{\cos i}{\left(1-e^2\right)^2}, \qquad \dot{\omega}_2 = \frac{3}{4}\cdot J_2 \cdot n \cdot \left(\frac{a_e}{a}\right)^2 \cdot \frac{5\cdot\cos^2 i - 1}{\left(1-e^2\right)^2} \tag{2}$$

Here $n$ − mean motion of SC, $a_e$ − equatorial radius of the Earth.
The values of secular drift (over a year) for parameters $\Omega$ and $\omega$ of the "Molniya" type orbits at different values of inclination $i$ are given in Tab. 1.

Table 1. Rates of secular drift for angular elements of the "Molniya" type orbits

| Inclination, $i$ | 62° | 62.8° | 63.4° | 65° |
|---|---|---|---|---|
| $\dot{\Omega}_2$, deg/year | - 56.36 | - 54.87 | - 53.75 | - 50.73 |
| $\dot{\omega}_2$, deg/year | 6.12 | 2.68 | 0.15 | - 6.42 |

As it follows from the table, the rates of secular drift in longitude of ascending node $\Omega$ for the considered orbits due to the Earth oblateness make over 50 degrees per a year. The rates of secular drift in argument of perigee $\omega$ are much lower. At inclinations more than critical value ($i = 63.4°$) the parameter $\omega$ tends to constant reduction, and at $i$ <63.4° the $\omega$ variation tendency is the inverse.

## 2.2 Luni-solar perturbations of the "Molniya" type orbits

The gravitational attraction of an artificial satellite by the Moon and the Sun perturbs the parameters of its orbit. Luni-solar perturbations can be divided into three groups: short-term, long-term and secular.
Short-term variations of orbital elements induced by these perturbations are small enough and are not interesting for our investigations. Secular and long-term perturbations due to the Moon and the

Sun essentially depend on a semi-major axis and an eccentricity of orbit: they become greater if the values $a$ and $e$ increase. That is why these perturbations substantially appear in high elliptical orbits, in particular in the "Molniya" type orbits.

### ***2.2.1 Secular perturbations of the orbital elements***

Secular variations of the parameters of the considered orbits due to the luni-solar perturbations have a complicated enough character, and depend not only on a semi-major axis and an eccentricity, but also on other orbital elements: $i$, $\Omega$ and $\omega$.

For an estimation of secular variations of orbital parameters first of all we will examine such variations for one revolution of SC. The estimation of perturbations due to the "third body" we will carry out in a geocentric rectangular coordinate system $\widetilde{X}\widetilde{Y}\widetilde{Z}$, its plane $\widetilde{X}\widetilde{Y}$ coincides with the orbital plane of perturbing body (the Moon or the Sun). As it follow from [4] in this coordinate system the variations of orbital elements $\{\widetilde{q}_k\}$ for one revolution can be determined (as a first approximation) by the Eq. 3:

$$
\begin{aligned}
&\delta\,\widetilde{a} \cong 0, \\
&\delta\,\widetilde{e} = \frac{1}{2}\cdot A\cdot e\cdot\sqrt{1-e^2}\cdot sin^2\widetilde{i}\cdot sin2\widetilde{\omega}, \\
&\delta\,\widetilde{i} = -\frac{1}{4}\cdot A\cdot\frac{e^2}{\sqrt{1-e^2}}\cdot sin\,2\widetilde{i}\cdot sin2\widetilde{\omega}, \\
&\delta\,\widetilde{\omega} = A\cdot\frac{1}{\sqrt{1-e^2}}\cdot\left[\left(e^2-sin^2\widetilde{i}\right)\cdot sin^2\widetilde{\omega}+\frac{2}{5}\cdot\left(1-e^2\right)\right], \\
&\delta\,\widetilde{\Omega} = -A\cdot\frac{1}{\sqrt{1-e^2}}\cdot cos\widetilde{i}\cdot\left[e^2\cdot sin^2\widetilde{\omega}+\frac{1}{5}\cdot\left(1-e^2\right)\right].
\end{aligned}
\tag{3}
$$

Here $\widetilde{i}$ − inclination of the SC orbit to the orbital plane of perturbing body; $\widetilde{\omega}$ − angular distance of perigee of the SC orbit, relative to its ascending node in $\widetilde{X}\widetilde{Y}$ plane; $\widetilde{\Omega}$ − the angular distance (counted out in $\widetilde{X}\widetilde{Y}$ plane) between a direction in a cross point of $\widetilde{X}\widetilde{Y}$ plane and the Earth equator and a direction in an ascending node of the SC orbit on this plane (see Fig. 5).

In Eq. 3 the value $A$, designated further as $A_\alpha$ to indicate its dependence on the concrete perturbing body, is determined by the following formula:

$$
A_\alpha = \frac{15}{2}\cdot\pi\cdot\frac{\mu_\alpha}{\mu_0}\cdot\left(\frac{a}{a_\alpha}\right)^3\cdot\sqrt{1-e_\alpha^2}\,.
$$

In this formula index $\alpha$, accepting value $L$ or $S$, specifies the perturbation (lunar or solar) in the relevant parameters, $\mu_0$ − gravitation constants of the Earth. In the assumption, that a semi-major axis of the "Molniya" type orbit $a \approx 26600$ km, $A_L$ and $A_S$ will have the following average values:

$$
A_L = 0{,}95\cdot 10^{-4}, \qquad A_S = 0{,}44\cdot 10^{-4}\,. \tag{4}
$$

Thereby the value of perturbations of elements $\{\delta\,\widetilde{q}_k\}$ due to lunar influence will exceed more than 2 times the similar perturbations induced by the solar attraction.

Angular elements $\widetilde{i}$, $\widetilde{\omega}$, $\widetilde{\Omega}$ in Eq. 3 depend on angular elements of SC orbit in the basic equatorial system of coordinates (IGCS), and also on inclination angle $I$ of a perturbing body orbit to the Earth equator. When the Sun is a perturbing body the angle $I$ will coincide with the obliquity of the ecliptic to the Earth equator $\varepsilon$. Dependence between angular orbital elements referred to IGCS and elements $\tilde{i}, \tilde{\omega}, \tilde{\Omega}$ referred to the coordinate system $\tilde{X}\tilde{Y}\tilde{Z}$ is determined by the following relations:

$$
\begin{aligned}
&\cos \tilde{i} = \cos I \cdot \cos i + \sin I \cdot \sin i \cdot \cos \Omega', \\
&\sin \tilde{\Omega} \cdot \sin \tilde{i} = \sin \Omega' \cdot \sin i, \\
&\cos \tilde{\Omega} = \cos \Omega' \cdot \cos d - \sin \Omega' \cdot \cos i \cdot \sin d \\
&\tilde{\omega} = \omega - d, \\
&\sin d \cdot \sin \tilde{i} = \sin I \cdot \sin \Omega',
\end{aligned} \tag{5}
$$

Similarly, there are formulas for transformation from elements $\tilde{i}, \tilde{\omega}, \tilde{\Omega}$ to elements $i, \omega, \Omega'$.
The element $\Omega'$ in Eq. 5 represents an arc of the Earth equator counted out from a cross point of orbital plane of a perturbing body with equator to an ascending node of SC orbit on equator. Obviously, that in a case, when the plane $\tilde{X}\tilde{Y}$ is an ecliptic plane (a perturbing body − the Sun) $\Omega' = \Omega$. In the case when the Moon is a perturbing body, will take place: $\Omega' = \Omega - \Omega_L$, where $\Omega_L$ − a longitude of ascending node of the Moon orbit on the Earth equator.

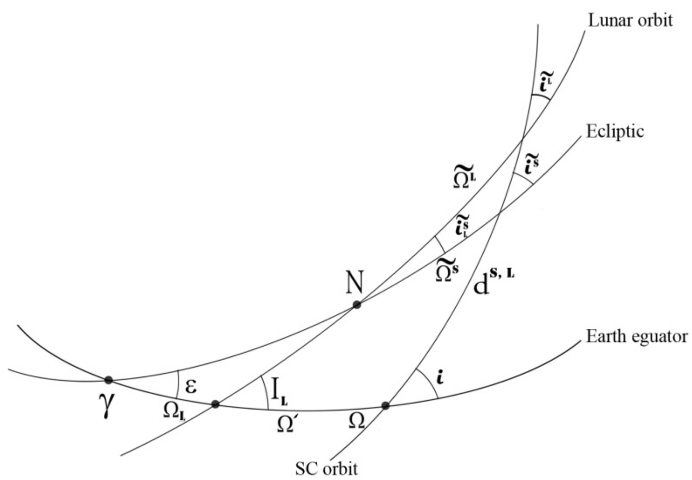


Fig. 5. Geometry of orbital motion of the SC "Molniya" and the "third" body

On the basis of the known formulas for transformation of angular elements dependences for determination of secular variations for one revolution for elements $i, \omega, \Omega$ in the basic coordinates system IGCS can be obtained. Namely, for $\delta i, \delta\omega, \delta\Omega$ it takes place Eq. 6:

$$
\begin{aligned}
&\delta i = \cos d \cdot \delta\tilde{i} - \sin\Omega' \cdot \sin I \cdot \delta\tilde{\Omega}, \\
&\delta\omega = \delta\tilde{\omega} + \cos\tilde{i} \cdot \delta\tilde{\Omega} - \cos i \cdot \delta\Omega, \\
&\delta\Omega = \frac{1}{\sin i} \cdot \left(\sin d \cdot \delta\tilde{i} + \cos d \cdot \sin\tilde{i} \cdot \delta\tilde{\Omega}\right)
\end{aligned} \tag{6}
$$

At obtaining these formulas it was supposed, that the inclination angle $I$ of a perturbing body orbit to the Earth equator remains constant within one revolution of SC and even during more significant periods of time. Such assumption is quite comprehensible within the accepted accuracy. For calculation of perturbations from the Sun $I = \varepsilon$, however, by examining the perturbations from the Moon it is necessary to consider, that the value of inclination $I$ will vary within the limits from

$I$ = 18.2° to $I$ = 28.6° with a period ~ 18.6 years due to precession of the Moon orbit (that is revealed in the moving of the ascending node $N$ of this orbit on the ecliptic).
It is necessary to note, that a semi-major axis and eccentricity variations are invariant concerning various rectangular coordinate systems. Therefore in the case of IGCS it will take place: $\delta a = \delta\tilde{a} \approx 0,\ \delta e = \delta\tilde{e}$ .
As $\delta a = \delta\tilde{a} \approx 0$ , for an estimation of variation of perigee distance $\delta\,\tilde{r}_\pi$ the following dependence will be valid:

$$\delta\,\tilde{r}_\pi = \delta\, r_\pi = -a\cdot\delta\tilde{e} = -\,a\cdot\delta e. \tag{7}$$

According to Eq. 3 the variations of elements $\{\tilde{q}_k\}$ after one revolution generally it is possible to present in the form:

$$\delta\tilde{q}_k = A\cdot f_{\tilde{q}_k}\ \ \left(\tilde{q}_k = \tilde{e},\tilde{i},\tilde{\omega},\tilde{\Omega}\right), \tag{8}$$

where functions $f_{\tilde{q}_k} = f_{\tilde{q}_k}\left(e,\tilde{i},\tilde{\omega}\right)$ explicitly depend on $\Omega$ if to take into account Eq. 5.
Using Eq. 8 and Eq. 6 it is possible to establish, that in the basic geoequatorial coordinate system the variation of the elements $e, i, \omega, \Omega$ for one revolution can be presented in the form similar to Eq. 8:

$$\delta q_k = A\cdot f_{q_k},\ \left(q_k = e, i, \omega, \Omega\right) \text{ and } f_{q_k} = f_{q_k}\left(e, i, \omega, \Omega, I\right),$$

We will denote $\delta' q_k$ the adjusted value of $\delta q_k$, so: $\delta' q_k = \dfrac{\delta q_k}{A}\cdot 10^{-4}$.
The variations of functions $\delta' q_k$ depending on the parameter $\Omega'$ at the different inclination angles $I$ $\left(I_1 = 18.2^\circ, I_2 = \varepsilon = 23.4^\circ, I_3 = 28.6^\circ\right)$ for the case of basic "Molniya" type orbit are illustrated in Figs. 6-9. From the shown graphics follows, that depending on the current value $\Omega'$ (and also an inclination angle of perturbing body' orbital plane to the Earth equator) orbital elements will increase, or decrease from a revolution to a revolution.

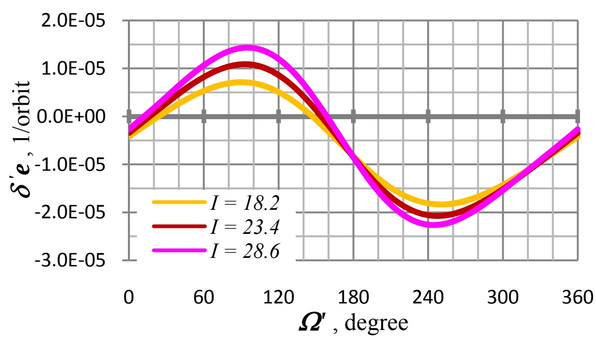



Fig. 6. Dependence of $\delta' e$ on $\Omega'$ and on I within one revolution

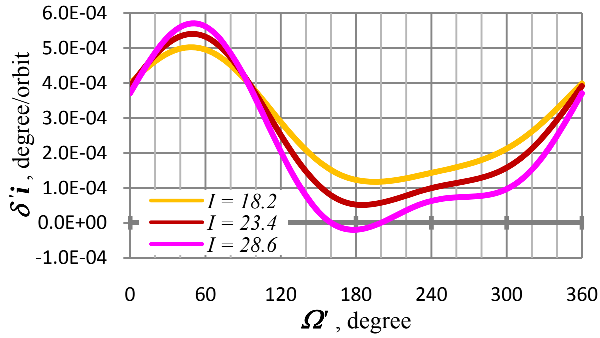



Fig. 7. Dependence of $\delta' i$ on $\Omega'$ and on I within one revolution

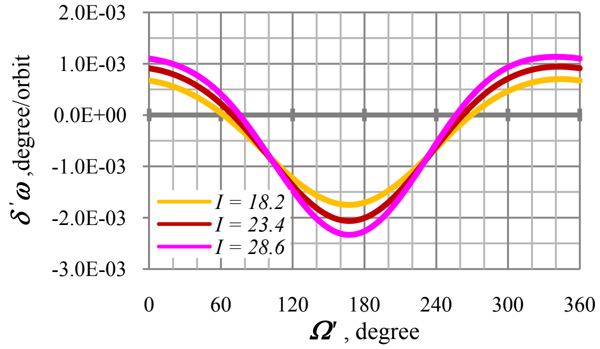



Fig. 8. Dependence of $\delta'\omega$ on $\Omega'$ and on I within one revolution

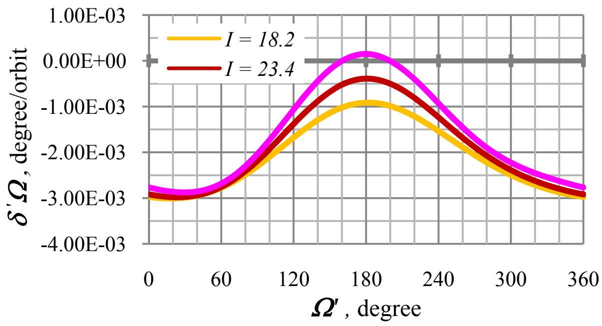



Fig. 9. Dependence of $\delta'\Omega$ on $\Omega'$ and on I within one revolution

As a result of the analysis of presented graphics, taking into account that secular drifts of examined elements are proportional to $A_L$ or $A_S$, determined in Eq. 4, and for $\delta r_\pi$ it is right Eq. 7, it is possible to conclude, that due to the gravitational attraction of the Moon and the Sun the "Molniya" type orbit parameters $i, \omega, \Omega$ and $r_\pi$ tend to a secular variation. Thus, depending on initial value $\Omega'$ rates of this changing will be different, but, as a whole, at recalculation for a year for the case of the basic orbit they are in the limits of effective range presented in Tab. 2.

Table 2. Limits of secular drifts of parameters of the "Molniya" type orbit due to luni-solar perturbations

| Perturbing body | Rates of secular drift for orbital parameters | | | |
|---|---|---|---|---|
| | $\delta i$, deg/year | $\delta\omega$, deg/year | $\delta\Omega$, deg/year | $\delta r_\pi$, km/year |
| The Moon | -0.01÷0.40 | -1.62÷0.79 | -2.09÷0.10 | -263÷416 |
| The Sun | 0.02÷0.17 | -0.66÷0.30 | -0.96÷-0.13 | -96÷177 |

### 2.2.2 ***Long-term perturbations***

Long-term oscillations of elements of SC orbit due to the gravitational attraction of the Moon and the Sun $-\ \{\delta_l q_k\}$ have frequency $2\lambda_\alpha$ and period $\frac{P_\alpha}{2}$, where $\lambda_\alpha$ is an angular velocity and $P_\alpha$ - a period of revolution of a perturbing body in its motion around the Earth. Amplitudes of these oscillations depend on the SC orbital parameters and the parameters of perturbing bodies. In particular, for an estimation of the amplitude *E* of long-term oscillations of the eccentricity inducing respective variations of perigee distance $r_\pi$, the following formula [5] can be applied:

$$E = \frac{15}{16} \cdot C_\alpha \cdot D(\tilde{\omega}, \tilde{i}) \cdot e \cdot \sqrt{1-e^2}, \quad (\alpha = L, S)$$

where

$$C_L = \frac{\mu_L}{\mu_0 + \mu_L} \cdot \frac{P}{P_L} = \frac{1}{82{,}3} \cdot \frac{P}{P_L}, \quad C_S = \frac{P}{P_S}, \quad P = 2\pi \sqrt{\frac{a^3}{\mu_0}},$$

$$D(\tilde{\omega}, \tilde{i}) = \sqrt{4 \cdot cos^2 \tilde{i} \cdot cos\, 2\tilde{\omega} + \left(1 + cos^2 \tilde{i}\right)^2 \cdot sin^2\, 2\tilde{\omega}}.$$

The character of luni-solar perturbations, including their secular and long-term component, in an example of parameter $r_\pi$ for the case of basic "Molniya" orbit at initial $\Omega = 280^\circ$ is shown in Fig. 10. Here the variations of $r_\pi$ within one year interval due to the influence of the Moon and the Sun separately and due to joint perturbation effect of these bodies are shown.

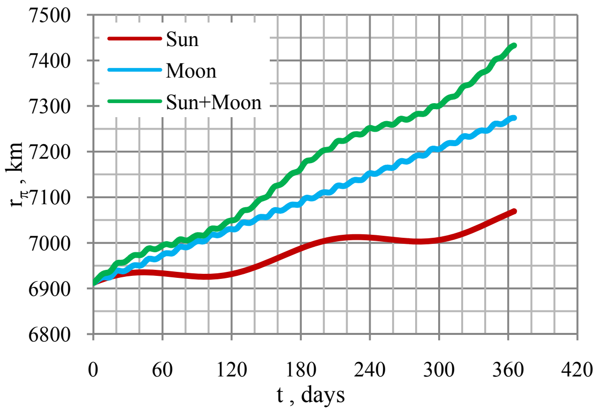



Fig. 10. Variations of perigee distance of the "Molniya" type orbit due to luni-solar perturbations

As it is seen from Fig. 10 for the basic orbit the value of $r_\pi$ may increase up to ~ 500 km within the period of a year due to luni-solar perturbations.

### 2.2.3 ***The influence of solar radiation pressure and atmospheric drag on the SC' "Molniya" orbit***

The pressure of solar radiation on a surface of the SC "Molniya" leads to short-term and long-term perturbations of its orbital elements. Thus, short-term perturbations are small enough. For example, for a semi-major axis they do not exceed ~ 10 m. Long-term perturbations are present in all elements, except a semi-major axis.

For a "Molniya" type SC with the standard mass-dimensional specifications the value of orbital parameters variations throughout enough long-term time periods (several years) due to solar

radiation pressure are estimated as follows:
$\left|\Delta a\right| < 10\,м$, $\left|\Delta e\right| < 4 \cdot 10^{-5}$, $\left|\Delta i\right| < 0.002^{\circ}$, $\left|\Delta\omega\right| < 0.007^{\circ}$, $\left|\Delta\Omega\right| < 0.007^{\circ}$.

The amplitude of parameter $r_\pi$ oscillations does not exceed 1 km. There by solar radiation pressure does not take any significant influence on evolution of the “Molniya” type orbits from the point of view of our investigations.

Moving along the high elliptical orbit, SC “Molniya” is situated within the atmosphere spreading area only small part of its orbit revolution time. So for the nominal orbit with a period ~ 12 hours and the minimum altitude $H_{min}$ ~ 500 km SC “Molniya” will have the altitude $H$ <1000 km for no longer than 15 min within one revolution and the atmospheric drag in this flight part will be rather insignificant in the value, and its integral effect on variation of SC orbital parameters will be small in comparison with the influence of the Moon and the Sun.

At perigee rising, the influence of atmosphere on the “Molniya” orbit evolution will weaken even more. However, at lowering $r_\pi$ the atmospheric drag around the perigee area will increase due to the atmospheric density increment, and at the reaching of $r_\pi$ some threshold values the braking effect of the atmospheric drag will lead to the appreciable and even to the significant variations of SC orbital elements. First of all, it will concern a semi-major axis *a* and an eccentricity *e*. And even if in the course of the subsequent changing due to the influence of other factors $r_\pi$ will exceed that threshold value, and the SC will prolong the orbital flight the parameters of its orbit (*a* and *e)* will be already essentially changed. In the cases when the minimum altitude $H_{min}$, reduced up to a critical threshold will remain lower this level during certain time the strong atmospheric drag effect around the perigee area will lead to rather fast reduction of a semi-major axis and an eccentricity of an orbit. As the result, the effect of the aerodynamics resistance can become a dominant one for the termination of the SC lifetime.

### 2.3 <u>Combined effect of various perturbing factors on sc’ lifetime on the “Molniya” type orbits</u>

On the basis of the above-stated it is possible to conclude that the SC’ lifetime on the “Molniya” type orbits will be determined by a combined effect of a non-central part of the Earth gravity field, the luni-solar perturbations and the atmospheric drag.

The carried out calculated estimations and results testify that:

- long-term variations of eccentricity *e*, and perigee distance $r_\pi$ due to the non-central part of the Earth gravity field, considerably less than the variations of these parameters induced by the luni-solar attraction;
- the character and velocity of changing $r_\pi$ will depend on current values $\Omega$ and $\omega$. So for each concrete value $\omega$ (from the range of possible values of this parameter) the perigee distance of an orbit at the given values $\Omega$ will increase, and at others − on the contrary decrease, thus the velocity of changing $r_\pi$ will depend on this concrete value $\omega$ (see Fig. 11);
- the gravitational attraction from the Moon and the Sun can lead to such variations of perigee distance (minimum altitude) of an orbit, that only due to this reason further SC orbital motion becomes impossible. At the same time, the influence of the atmospheric drag will promote in these cases an acceleration of process of finishing a SC flight, and in some cases this perturbing factor can become even a principal reason of termination of a space vehicle life in an orbit.

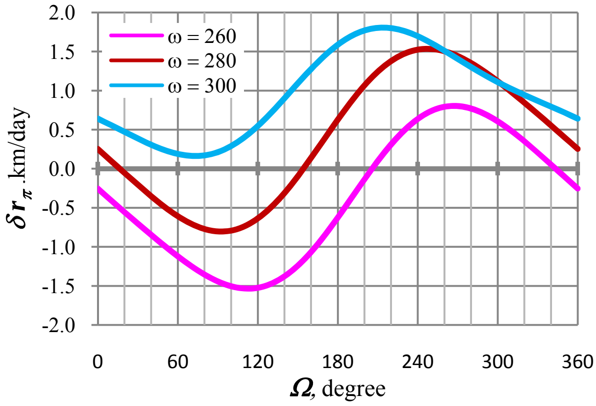



Fig. 11. Velocity of changing of parameter $r_\pi$ of the "Molniya" type orbit depending on ascending node longitude Ω and argument of perigee ω due to joint influence of the Moon and the Sun (for the case $I_L = \varepsilon$)

At the same time it is necessary to notice, that due to evolution the values of orbital parameters $\Omega$ and $\omega$ will vary. These variations in appropriate way will change the character and value of perigee distance $r_\pi$. In this case the basic contribution to change the parameters $\Omega$ and $\omega$ (first of all − $\Omega$) will make the non-central part of the Earth gravitational field though luni-solar perturbations are capable to render appreciable influence on secular changing $\omega$. Thus, the direction and velocity of $\omega$ changing will depend on an orbit inclination and its eccentricity which also vary eventually in dependence on parameter $\Omega$ due to luni-solar perturbations.

In Fig. 12 a long-term variation of parameter $r_\pi$ for the basic orbit of SC "Molniya" is shown at the different initial values of longitude of ascending node $\Omega$. Evolution was calculated at taking into account all significant perturbing factors. The presented results show the character of the influence of parameter $\Omega$ on the change of $r_\pi$. As one can see from the figure, for the great number of considered variants of computation, evolution $r_\pi$ has led, eventually, to the termination of ballistic life of chosen virtual space vehicle though this finish needs a sufficiently long-duration time (~ from 12 till 19 years). Only in one of considered variants (at initial value $\Omega = 310°$) hypothetical SC has orbited within more than 25 years. But also in this case approximately in 21 years in the course of orbit evolution its minimum altitude decreased to the critical values ~ 130 km. In this time the SC underwent a strong enough effect of the atmospheric drag in the area of the low altitudes. And though at the expense of subsequent evolution $r_\pi$ (first of all − because of the luni-solar perturbations) a space vehicle has left dangerous band of flight, its orbit has already been essentially changed. Afterwards the perigee distance continued to vary and approximately in 22 years at next lowering $H_{min}$ to a critical threshold the object has finished its existence in the space.

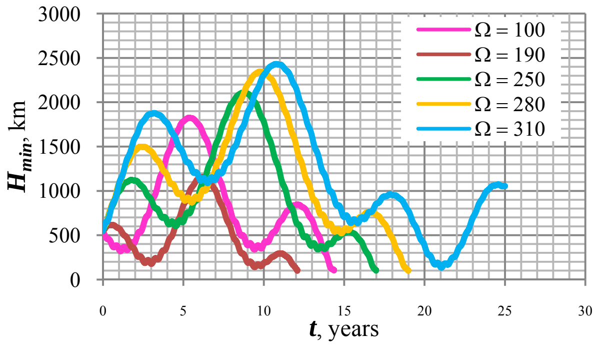


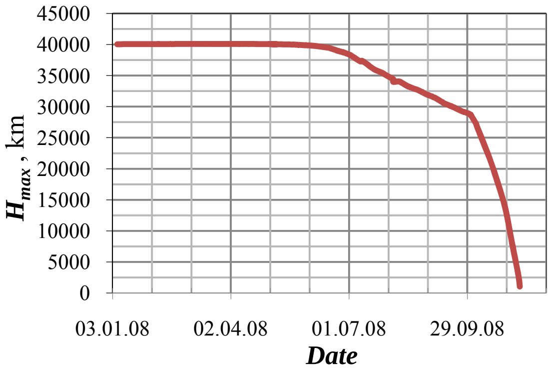

Fig. 12. Evolution of minimal altitude $H_{min}$ for the basic orbit of SC "Molniya" at different initial values Ω.

## 3. CONDITIONS OF THE FINAL PHASE OF A FLIGHT AND RE-ENTRY OF A "MOLNIYA" TYPE SATELLITE

If at the expense of luni-solar perturbations the perigee altitude of a high elliptical orbit of a "Molniya" type satellite decreased to a critical threshold and came into the atmosphere layers with high density, then within this area SC will be perturbed by a strong enough aerodynamic resistance of the atmosphere. Due to this perturbation, a semi-major axis and an eccentricity of an orbit will start changing (decreasing), that will cause the reduction of the maximal altitude ($H_{max}$) of flight over the Earth.

Under the condition of a perigee staying in this specified area during a certain time a value $H_{max}$ will continue decreasing. Such changing of this high elliptical orbit degenerates it in a common elliptical orbit with such parameters, that the space object cannot continue orbiting and will terminate its ballistic life.

The change of orbital parameters of SC "Molniya" up to the critical ones and a condition of its re-entry are shown below in an example of SC "Molniya 3-43". In Fig. 13 changing of maximal ($H_{max}$) and minimal ($H_{min}$) altitudes of orbit during the last year SC flight is shown.



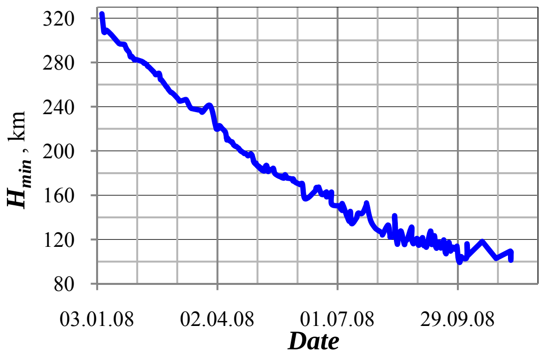




Fig. 13. Dynamics of the SC «Molniya 3-43» maximal and minimal altitudes decay during the last year orbiting.

In Fig. 14 dynamics of the SC "Molniya 3-43" orbit altitudes changing during the last two days is presented. As it follows from the figure, the maximal altitude of the satellite orbit has decreased during this time from ~ 4500 km to ~ 810 km, and parameter $H_{min}$ was practically invariable remaining within the limits of 100 km.

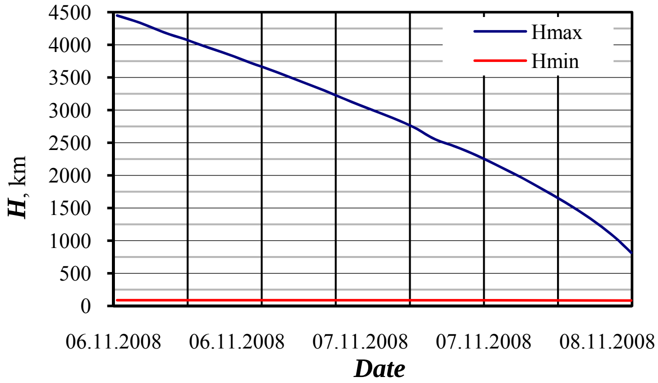



Fig. 14. Dynamics of the SC «Molniya 3-43» maximal and minimal altitudes decay
during the last two days orbiting

The dynamics of changing the motion parameters (altitude and velocity) in the area around the minimal distances from the Earth for the latest revolutions of "Molniya 3-43", directly before it's decaying is shown in Fig. 15.

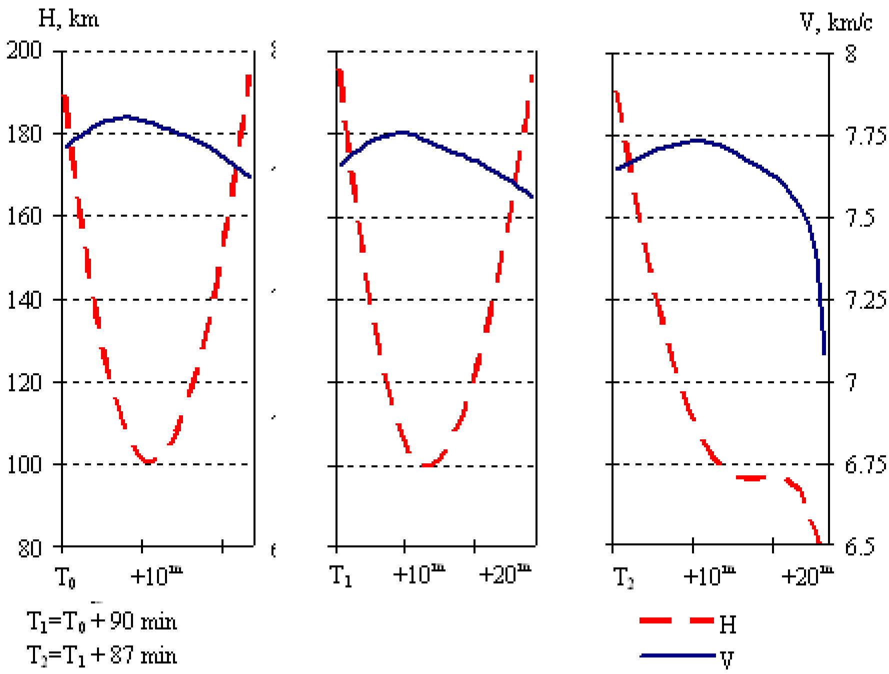



Fig. 15. Dynamics of changing the minimum altitude and velocity of the «Molniya 3-43»
on the latest revolutions

From Fig. 15 follows, that on the final phase of orbital flight the SC "Molniya 3-43" descended on each revolution up to minimal altitude ~ 100 km, i.e. practically to the very dense layers of atmosphere, and further lifted up to heights of the order of several hundred kilometers. Thus on the last revolution the maximal velocity of SC in pericenter made up 7,7 km/s, and the value of the atmosphere entry angle made up ~ -0,6°. For matching in Fig. 16 the character of motion parameters changing for the SC "Coronas-F" on final revolutions of its flight is shown. It is possible to consider the conditions of the final phase of flight of this SC to be typical for the majority of satellites re-entering from the near-circular orbits in an uncontrollable condition.

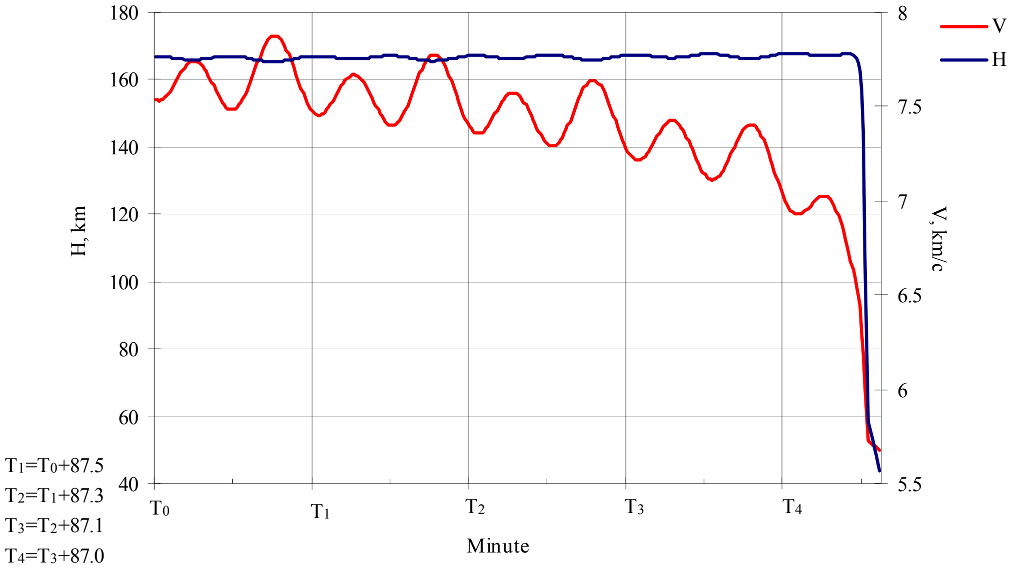



Fig. 16. Dynamics of changing the minimum altitude and velocity of the «Coronas-F» on the latest revolutions

## 4. CONCLUSION

The introduced methodology and procedure as well as the presented calculation results allow to estimate the influence of the various significant perturbing factors and their joint effect on the long-term evolution of the orbits and on the lifetime of the "Molniya" type space vehicle. It is found out, that the major factor influencing the lifetime of a space vehicle in an orbit is the gravitational attraction from the Moon and the Sun. The lifetime of SC "Molniya" depends on the initial values of the longitude of ascending node $\Omega$ and also on the value of argument of perigee $\omega$ and on eccentricity $i$ of an orbit.

The results characterizing the final phase of a flight of the "Molniya" type satellite and the re-entry conditions for similar space objects descending from their orbits in an example of SC "Molniya 3-43" are presented.

The carried out examinations allow to draw a conclusion, that as a result of an orbit evolution due to the influence of the considered perturbing factors all the "Molniya" type SCs sooner or later will terminate their orbital flight.

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