TRAJECTORY ANALYSIS OF SOLAR SAIL SPACECRAFT CONSIDERING THE LARGE UNCERTAINTY OF SOLAR RADIATION PRESSURE

Tomohiro Yamaguchi(1), Yuya Mimasu(2), Yuichi Tsuda(3), Ryu Funase(3), Hirotaka Sawada(3), Osamu Mori(3), Mutsuko Y. Morimoto(3), Hiroshi Takeuchi(3) and Makoto Yoshikawa(3)

(1) Department of Space and Astronautical Science, The Graduate University for Advanced Studies, Sagamihara, Kanagawa 229-8510, Japan, +81-42-759-8691, yamaguchi.tomohiro@jaxa.jp
(2) Department of Aeronautics and Astronautics, Kyushu University, Fukuoka, Japan
(3) Japan Aerospace Exploration Agency, Sagamihara, Japan

ABSTRACT

This study investigates the trajectory analysis of small solar sail demonstration spacecraft IKAROS considering the uncertainty of solar radiation pressure. Estimation of solar sail force model in space is the key factor for successful solar sail navigation because the solar sail have large uncertainty due to the flexible membrane. Since the sail wrinkles after the deployment and its surface will suffer from degradation, the solar sail force model is difficult to develop on the ground. In this paper, a practical analysis of estimating the solar sail force model from Doppler and range observable is investigated. This is demonstrated by orbit determination including parameter estimation of solar sail model. Some examples are described to investigate better parameters to estimate the solar sail force model.

1. INTRODUCTION

Recently, the target of space explorations become further and further from Earth. For instance, the interest in small celestial bodies are going to move on main-belt and Trojan asteroids from near Earth objects. Since these space explorations require huge energy to achieve their objectives, high efficient propulsion system (i.e. electric propulsion, solar sail, etc.) have developed. Hayabusa [1], Dawn spacecraft are now cruising deep space with electric propulsion system. Although more efficient propulsion system are required to expand deep space explorations.

Solar sail have a capability to receive solar photon momentum and obtain acceleration without fuel. Many application for the innovative space mission are investigated and planned using solar sail [2-5]. Also several concept and project are going on. UrtraSail is the solar sail concept which has several “brades” of solar sail firm material with a microsattellite on the tip [6]. This system uses a solar sail firm of densities approaching 1 g/m^2 for a very large, and high acceleration due to 1 km^2 solar sail areas. Cosmos 1 was a project to test the solar sail technology, funded by Planetary Society [7]. Although, the 600 m^2 mylar sail was planed to launch into the Earth orbit with inclination of 80 deg by converted missile, the launch was failed and the spacecraft failed to reach the orbit.

Japan Aerospace Exploration Agency (JAXA) is planning “solar power sail”, which is a hybrid system using ion-propulsion engines and solar sail. JAXA is now studying two missions to demonstrate solar power sail. One is small-size solar sail demonstration

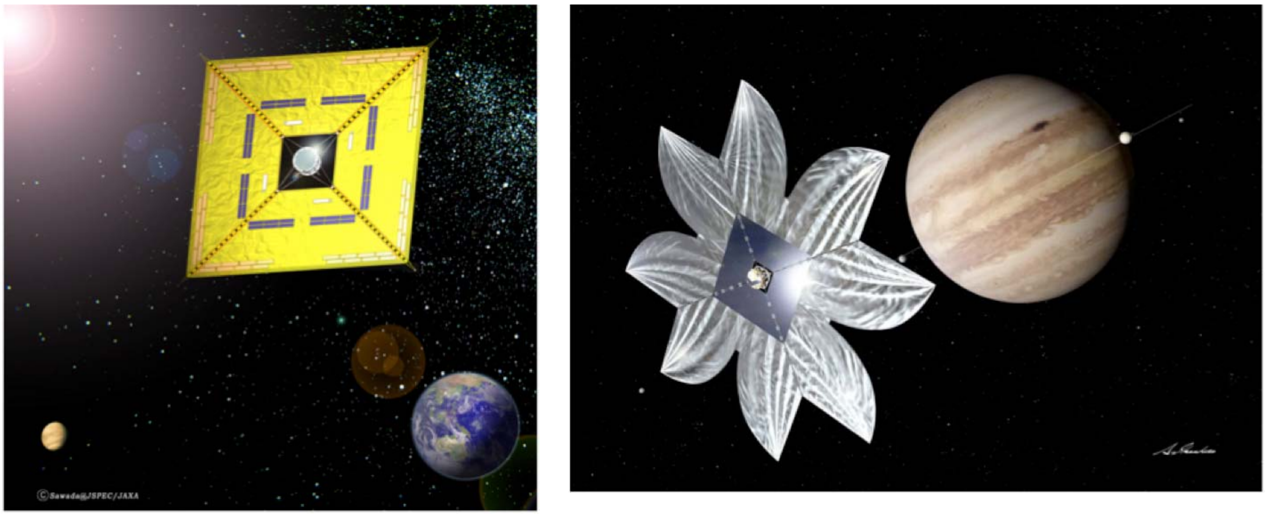

Fig. 1 JAXA solar power sail project

spacecraft IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun, Fig. 1), which has 20 m solar sail with solar cells on the membrane. The minimum success criteria of this spacecraft are the demonstration of the solar sail deployment system and power supply from the thin-film solar cell. The full success criteria of the spacecraft are acceleration and navigation using the solar sail in the interplanetary trajectory. Another spacecraft is the medium-sized solar power sail, which has ion propulsion system with solar sail of diameter 50 m. This spacecraft is planning the launch at mid-2010s and its destinations are the Jupiter and the Trojan asteroids.

A precise solar sail force model is the key factor of successful navigation for the solar sail spacecraft. Many prelaunch experiments have held to confirm the performance of solar sail deployment and stability [8,9]. Although conventional spacecraft model can be based on ground-based measurements, solar sail force model needs to be refined after launch. Since a practical solar sail requires large area to obtain a force from solar radiation pressure, a sail needs to be deployed in space. The deployment may occur some wrinkles and deformation from nominal shape that expected before launch. Also the uncertainty becomes much larger since the sail surface will suffer from degradation. Rios-Reyes and Scheeres investigated a methodology for estimating the force and moment generated by arbitrary shape using generalized sail model based on in-flight data [10]. However they assumed ideal in-flight data measured using inertial measuring units.

This paper investigates the simulation of solar sail force estimation using measurement using Doppler and range measurement based on realistic sequence of IKAROS. Since an actual navigation data is not available right now, pseudo navigation data is generated using the multi plane model that considers the deformation and detail optical properties of the solar sail. Several simulations are demonstrated to investigate the better parameter for estimating the solar sail force model for IKAROS.

2. SOLAR SAIL MODEL ESTIMATION

Estimation of solar sail force means a parameter estimation of solar sail model. Since Ikaros's solar sail is composed by flexible membrane, the sail may have a large uncertainty in its performance. A complex solar sail model with many parameters is required to simulate all the uncertainty and effect due to the geometry and solar sail condition. Although some of the parameters need to estimate after launch because of the space environment and sail's

deployment uncertainty, the complex model required a lot of high quality measurement.
The main measurement to estimate solar sail model is radiometric observable (range and Doppler observable), and those observables detect small maneuvers due to attitude control and trajectory correction. Since those small maneuvers are required during the whole mission sequence, it's difficult to obtain long period tracking data without any maneuvers.
In this study, the estimation of solar radiation pressure (SRP) force is implemented by two stages. In first stage, SRP acceleration is estimated using simple solar sail model with short period orbit determination (OD). Short period tracking data without any maneuvers is available to estimate SRP force with simple solar sail model, but it doesn't have enough measurement to estimate with complex solar sail model which can simulate whole solar sail force. In next stage, precise solar sail model called generalized sail model is estimated using the results of several SRP accelerations from short period OD. This two-stage estimation allows us to exclude the bad noise due to small maneuvers from the tracking data, and to obtain a useful reference of solar sail model for future solar sail missions.

2.1. Single plane solar sail model

The single plane solar sail model is implemented to estimate SRP acceleration with short period OD. This model assumes a solar sail as a plane composed by a unique material. The acceleration is modeled as following equation, considering absorbing and reflecting component of SRP.

$$\ddot{r}_{SRP} = -P\frac{1AU^2}{r^2}\frac{A}{m}\cos\theta\left[(1-\rho s)\boldsymbol{e}_{SUN} + \left(2\rho s\cos\theta + \frac{2}{3}(1-s)\rho\right)\boldsymbol{n}\right] \tag{1}$$

where P is the solar radiation pressure at sun distance 1AU , r is sun-spacecraft distance, A is the effective cross section area, m is the spacecraft mass, ρ is the reflectivity, s is the fraction of specular reflectioin. $\boldsymbol{n}$ is the normal vector of the solar sail and $\boldsymbol{e}_{SUN}$ is the unit vector of the sun direction with respect to the spacecraft, θ is called as solar cone angle and calculated by $\cos\theta = \boldsymbol{n}^T\boldsymbol{e}_{SUN}$, and those vectors are described in Fig. 2. The spacecraft bass area is neglected assuming the solar sail area is larger enough than the area of the spacecraft bass system. In this model, spacecraft area and specular and diffusive reflectivity can be estimated parameter, because those parameters may have large uncertainty due to the deployment or degradation. However it is difficult to estimate all those parameters in short period OD, because it requires a measurement with various solar cone angles to distinguish those parameters. Therefore we

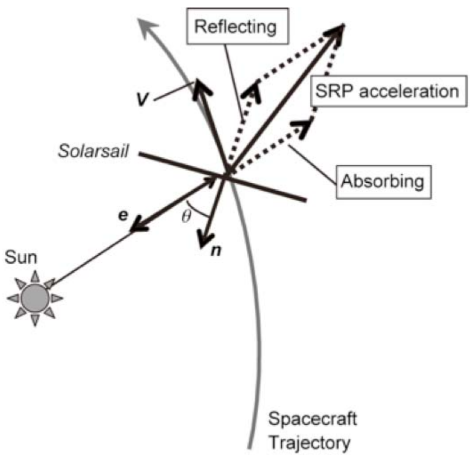

Fig. 2 Simple plane solar sail model

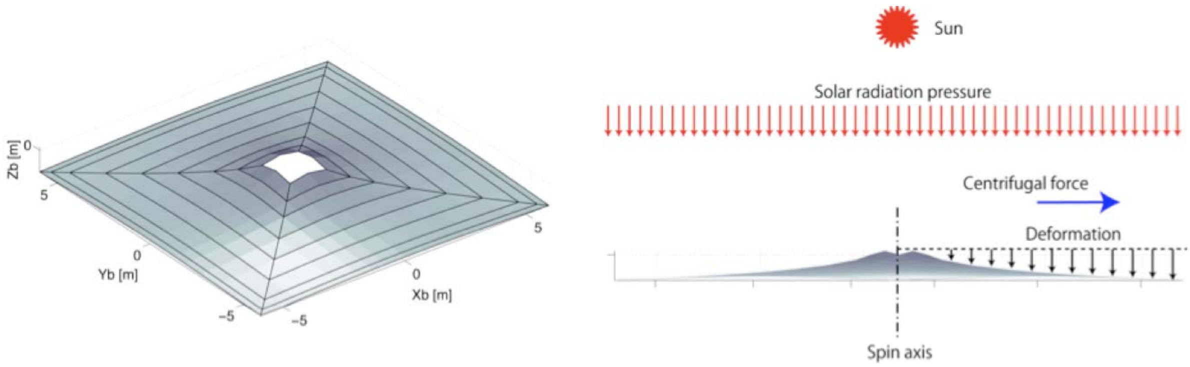

Fig. 3 Solar sail deformation model

need to select best parameter in terms of estimating SRP force. In this research, specular reflectivity or spacecraft area are selected as an estimate parameter.

The covariance of the SRP acceleration $\boldsymbol{P}_{SRP}$ is quantified by covariance of estimate parameter $\boldsymbol{P}_X$ using following equation.

$$\boldsymbol{P}_{SRP} = \left(\frac{\partial \ddot{r}_{SRP}}{\partial X}\right) \boldsymbol{P}_X \left(\frac{\partial \ddot{r}_{SRP}}{\partial X}\right)^T \tag{2}$$

where, $\partial \boldsymbol{r}_{SRP}/\partial X$ is a partial derivative of SRP acceleration with respect to a estimate parameter X.

2.2. Multi plane solar sail model

Multi plane solar sail model is implemented as a “true” trajectory of the solar sail spacecraft. This model considers the sail’s deformation and several optical properties due to multiple materials. The deformation is caused by SRP and centrifugal force. Tsuda[11)] developed the analytical solution of deformation for rotating membrane, and described as following equation.

$$z_b = \frac{4P}{(3+\nu)\mu\Omega^2} \log\left(\frac{\sqrt{x_b^2 + y_b^2}}{r_a}\right) \tag{3}$$

where x_b, y_b, z_b are the components of the solar sail fixed coordinate (X_b, Y_b, Z_b), respectively. X_bY_b plane is the solar sail plane without deformation, and Z_b axis is the direction of angular momentum vector. ν is Poisson’s ratio, μ is the plane density, Ω is the rotation rate of the spacecraft, and r_a is the inner radius (the boundary between the membrane and the spacecraft hub). In this research, poisson’s ratio ν is 0.3, the plane density ν is 0.01 kg/m2, rotation rate Ω is 1 rpm, inner radius r_a is 0.75 m and deformation is described in Fig. 3. The out-plane deformation is about 10 cm.

The multi plane model is implemented by 3 categories, 48 nodes. 3 categories indicate aluminized membrane, solar cells and liquid crystal. Each category has own optical

Table. 1. Optical properties for multi plane solar sail model

Device	Specular reflectivity	Diffusive reflectivity	Absorbing coefficient
Aluminized membrane	0.819	0.062	0.119
Solar cell	0.035	0.099	0.866
Luquied crystal	0.010	0.550	0.440

parameters (specular and diffusive reflectivity, absorbing coefficient) and each node has own area and normal vector considering the deformation. The optical properties of multi plane solar sail model are summarized in Table 1.
The acceleration of multi plane solar sail model is calculated as a summation of single plane model and described as follows:

$$\ddot{r}_{SRP} = -P\frac{1AU^2}{r^2}\frac{1}{m}\sum_{k=1}^{N}\left\{A_k\cos\theta_k\left[(1-\rho_k s_k)\boldsymbol{e}_{SUN} + \left(2\rho_k s_k\cos\theta_k + \frac{2}{3}(1-s_k)\rho_k\right)\boldsymbol{n}_k\right]\right\} \tag{4}$$

where N is the number of the node. In this study, N is 48 and each node has each normal vector.

2.3. Generalized sail model

Generalized sail model (GSM) is implemented to estimate the "global" parameters of SRP acceleration. GSM is developed by Rios-Reyes [12], and allows for the analytic computation of forces and moments acting on a solar sail of arbitrary fixed shape. These forces and moments are computed analytically using a set of coefficients, assuming the sail remains fixed with attitude. GSM is a generalization of the flat sail model for arbitrary shapes and 19 coefficients in three tensors are used to compute the solar sail force. The deviation from flat plate model is described in the tensors obtained by integrating the differential forces generated by differential areas. In the integration, it is assumed that the sail shape remains fixed and no self-shadowing occurs. Then, the acceleration of solar sail acceleration using GSM is calculated as follows:

$$\ddot{r}_{SRP} = P\frac{1AU^2}{r^2}\frac{A}{m}\left\{\boldsymbol{J}^2\cdot\hat{\boldsymbol{r}} - 2\hat{\boldsymbol{r}}\cdot\boldsymbol{J}^3\cdot\hat{\boldsymbol{r}} - \left(\boldsymbol{J}^1\cdot\hat{\boldsymbol{r}}\right)\hat{\boldsymbol{r}}\right\} \tag{5}$$

where $\hat{\boldsymbol{r}}$ is the sun's unit position vector from the sun into the sail. $\boldsymbol{J}^m$ is a rank m force tensor defined as integrals over a sail's surface as follows:

$$\boldsymbol{J}^1 = \frac{1}{A}\int_A a_3\hat{\boldsymbol{n}}dA \tag{6}$$

$$\boldsymbol{J}^2 = \frac{1}{A}\int_A a_2\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}dA \tag{7}$$

$$\boldsymbol{J}^3 = \frac{1}{A}\int_A \rho s\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}dA \tag{8}$$

where $a_2=2(1-s)\rho/3$, $a_3=1-\rho s$, and $\hat{\boldsymbol{n}}$ is the normal vector of sail surface elements, which is a function of location on the sail. The coefficients $\boldsymbol{J}$ also capture the arbitrary optical parameters of the sail surface including them in the integrals. Since the tensors are defined as integrals of outer-products with same vector $\hat{\boldsymbol{n}}$, they are symmetrical in all their components. Therefore independent parameter of GSM for arbitrary shape of the solar sail becomes 19.
The GSM parameters reduce if we consider symmetric sail shapes. For the spinning solar sail, the shape may describe that the sail has continuous axis of symmetry. Using the symmetric relationship of spinning solar sail, the independent coefficients of GSM reduce to 5 (J_3^1, J_{11}^2, J_{33}^2, J_{113}^3, J_{333}^3).

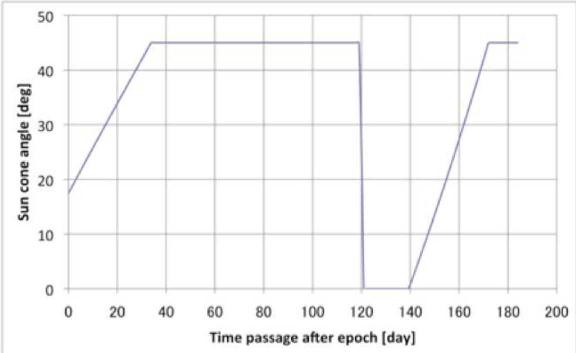

Fig. 4. Attitude of solar sail during interplanetary trajectory

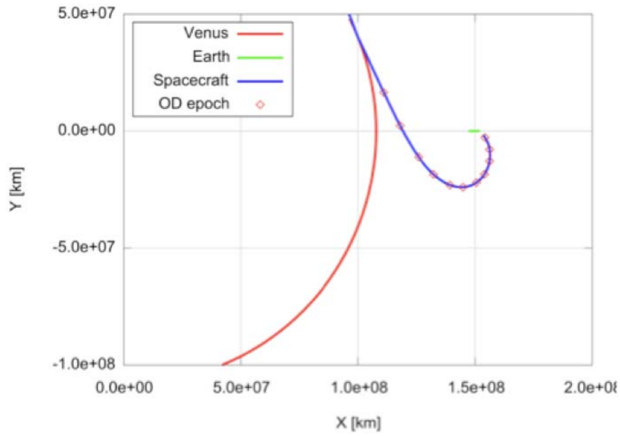

Fig. 5. Interplanetary trajectory of IKAROS (Sun-Earth line fixed coordinate)

2.4. Difference of single and multi plane solar sail model

The difference of the two models is calculated assuming the transfer trajectory to Venus from Earth. The material of the single plane model is assumed as aluminized membrane. The normal vector of the sail is laid on the trajectory plane of the spacecraft and the solar cone angle of solar sail during the interplanetary space is described in Fig. 3. Since the angle varies only between 0 to 45 degrees, we cannot detect the characteristics of the solar sail in case of shallow insertion of solar radiation pressure. The plot of trajectory is described in Fig. 5 with Sun-Earth line fixed coordinate.

The acceleration due to solar radiation pressure is shown in Fig. 6. The components in J2000.0EQ are described separately. Comparing the acceleration of multi plane solar sail model with single plane solar sail model, the tendency of the acceleration is same, though the maximum of the acceleration is different. Also the time that gives the largest difference varies depending on the components. The acceleration difference between the two models is about 0 to 10 %. The effect of the deformation will effect as scaling down of the area, because we assume only symmetric deformation. However the deformation of the sail is small (see previous section), the main factor of this difference in acceleration is the difference of optical properties. The order of the “natural” acceleration to the spacecraft is described in Fig. 7, and

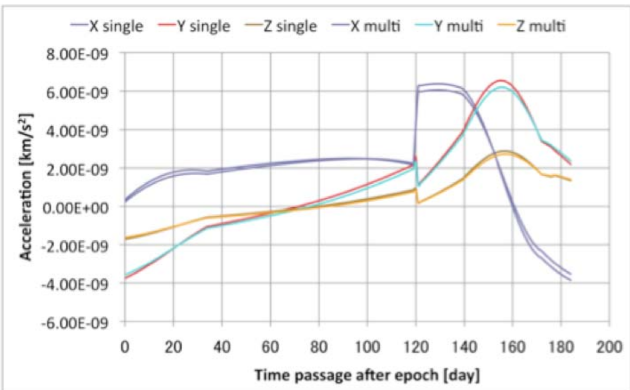

Fig. 6. Acceleration due to solar radiation pressure

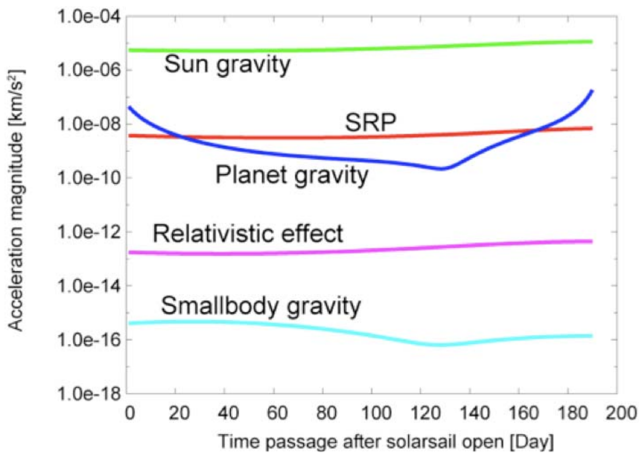

Fig. 7. Order of acceleration during the interplanetary space

indicates that the SRP acceleration is the second largest force during the interplanetary. This order of acceleration is 2-order larger than a conventional spacecraft like Hayabusa.

3. SOLAR SAIL FORCE ESTIMATION USING SHORT PERIOD ORBIT DETERMINATION

The short period orbit determination is simulated using pseudo tracking data generated by multi plane model. The purpose of this estimation is to get a state vector fit within the short period navigation data.

3.1. Condition of pseudo tracking data

The pseudo tracking data is generated using multi plane solar sail model. The epoch of the trajectory is assumed as a direct transfer to Venus. The tracking station is assumed as Usuda deep space center 64 m antenna. A set of short period tracking data is generated by 3 days observation. 5 hours operation is assumed per day and 5 hours Doppler and an hour range observable is obtained in the operation. A set of tracking data is obtained every 2 weeks, and total 11 sets of tracking data is calculated for this research. The geometry of OD epoch is shown in Fig. 5. The 1-sigma white noise is assumed as 0.5 mm/s and 10 m for Doppler and range observable, respectively.

3.2. Condition of estimation

The estimation method is conventional weight least squares. Planetary perturbation using JPL ephemeris DE405 is considered for trajectory propagation. Reference frame model is compliant with IAU 2000A CIO based. Single plane solar sail model is implemented for SRP acceleration. Although GSM could describe the solar sail force more precisely, it doesn't have enough sensitivity in short period as 3 days. Three cases of estimated parameter are selected. First combination is state vector and specular reflectivity, and second combination is state vector and solar sail area. The third combination is state vector, specular reflectivity and solar sail area. The better combination of estimate parameter can be evaluated by comparing these two cases. Considering that a priori information of solar sail condition is poorly known, the

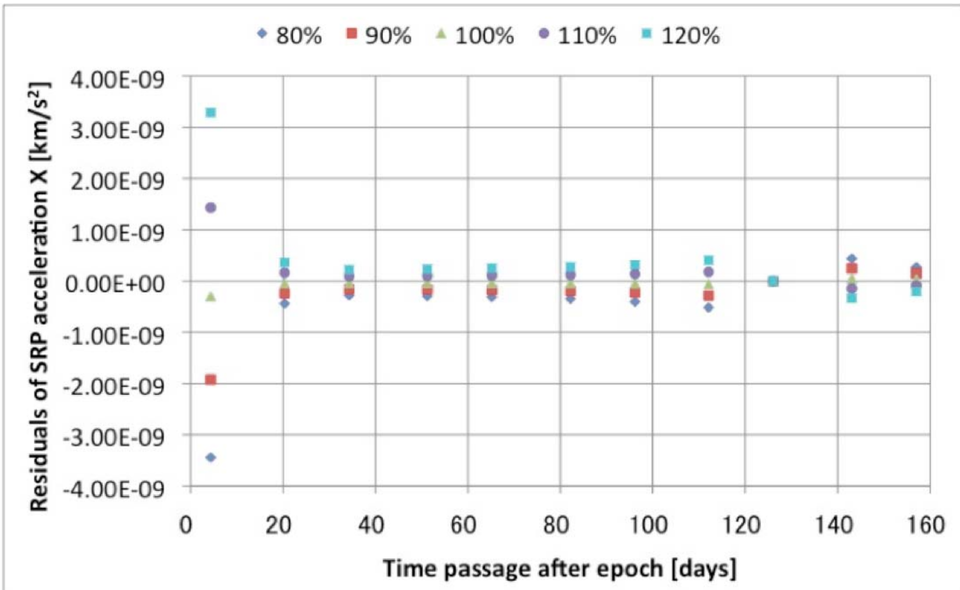

Fig. 8 Residuals of solar sail acceleration: Estimate reflectivity

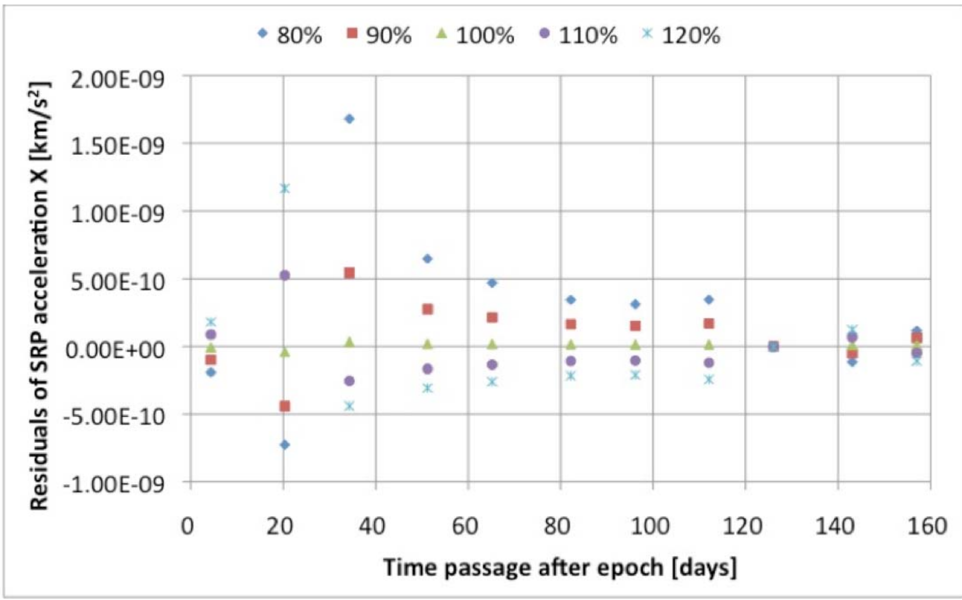

Fig. 9 Residuals of solar sail acceleration: Estimate area

uncertainty of specular reflectivity and solar sail area are assumed as 20 %.

3.3. Results

The results from the estimations are shown in Fig. 8 to Fig. 10 using the residuals of SRP acceleration of *X* axis. Fig. 8 and 9 show the results of case 1 and case 2, respectively. In those cases the non-estimating parameter changes within +/- 20 % and these results are shown in same figures. The "range" of the residuals shows the impact of a priori uncertainty on the results. Comparing two estimation cases, it is found that the impact of a priori uncertainty varies due to the geometry of Sun, Earth and solar sail spacecraft. The large residual of SRP acceleration occurs when the direction of partial derivative of SRP with respect to specular reflectivity or sail's area and line of sight from Earth to solar sail are perpendicular to each other. In this case, the sensitivity of range and Doppler observable with respect to those two

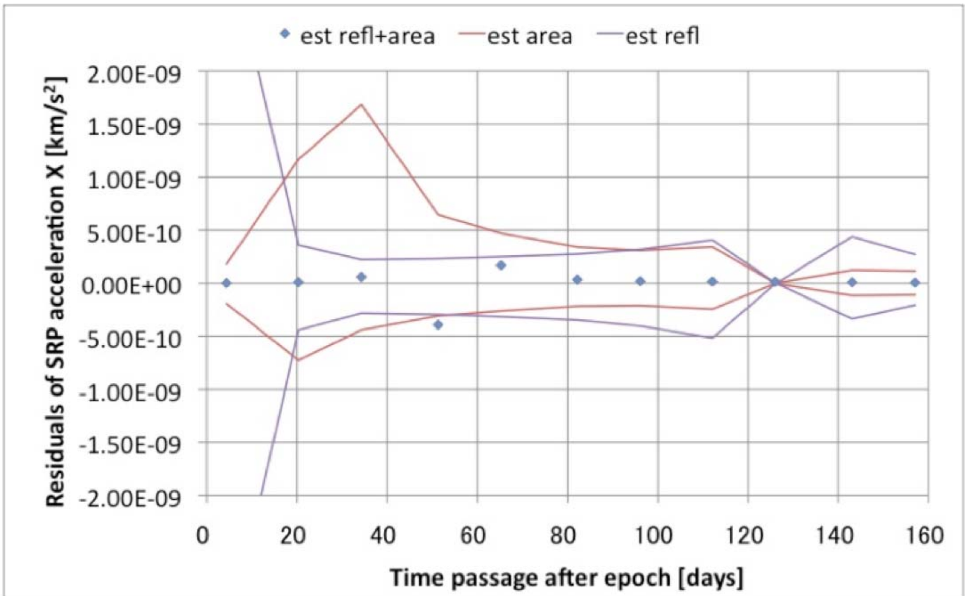

Fig. 10 Residuals of solar sail acceleration: Estimate both reflectivity and area

Table 2. Estimated area and reflectivity of case 3

Tp [day]	Estimated area [m^2]	Estimated reflectivity
4.33	181.01	0.6982
20.33	182.31	0.6990
34.29	190.55	0.6870
51.25	129.03	0.8812
65.21	203.26	0.6370
82.17	185.97	0.6820
96.13	183.89	0.6889
112.08	183.22	0.6912
126.04	290.11	0.0440
143.04	181.71	0.6907
157.00	181.62	0.6954

parameters becomes zero. The result from estimating both reflectivity and area are described in Fig. 10. It is found that this case shows better performance than case 1 and 2 in terms of residuals of SRP acceleration. Table 2 shows the estimated parameters of case 3 and found that the parameters vary due to the difference of attitude and sun distance.

4. ESTIMATION OF GLOBAL PARAMETER FOR SOLAR SAIL FORCE USING GENERALIZED SAIL MODEL

To obtain valuable information from actual solar sail flight data, the global parameters that describe the solar sail force needs to be estimated. Since section 3 shows that the solar sail parameters would vary from the tracking data of selected pass, those “local” solar sail parameters doesn’t become a good reference for future solar sail mission. This is because the actual solar sail force shows various deviations from simple plane model as a function of its attitude, sun distance and direction. The orbit determination with simple plane model could estimate its state and the parameters fitting in short period navigation data, however it cannot fit with long term tracking data that includes various attitude or sun locations. GSM could fit the tracking data in those situations, however some of GSM coefficients have not enough sensitivity within short term tracking data.

Therefore the estimation of the GSM coefficients for spinning spacecraft as global parameters is demonstrated using the results of section 3. This second stage estimation is performed using the 3 or 4 sets of 3 pass tracking data. The state vector of the head of 3 pass tracking data are given by the result of short period OD which described in section 3. The best-fitted GSM coefficients are estimated to fit the tracking data including various sail’s attitude and sun distance. The results from the estimation using 3 set of short period tracking data is shown in Fig. 11 with the residuals of 2-way range and 2-way Doppler observable. For the comparison, the result of residuals using single plane model is described in Fig. 12. It is shown that that residuals using GSM are varies around the zero, but the residuals using simple plane model include large extra residuals due to miss modeling of the solar sail. The results from the

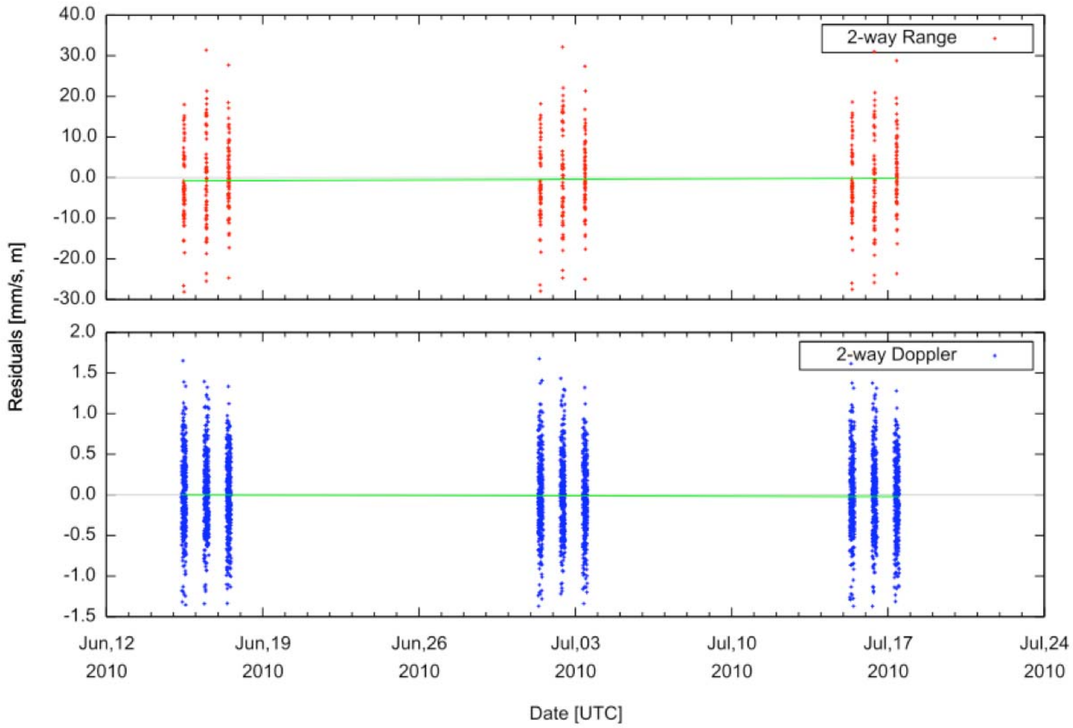

Fig. 11 Residuals of range and Doppler observable using GSM

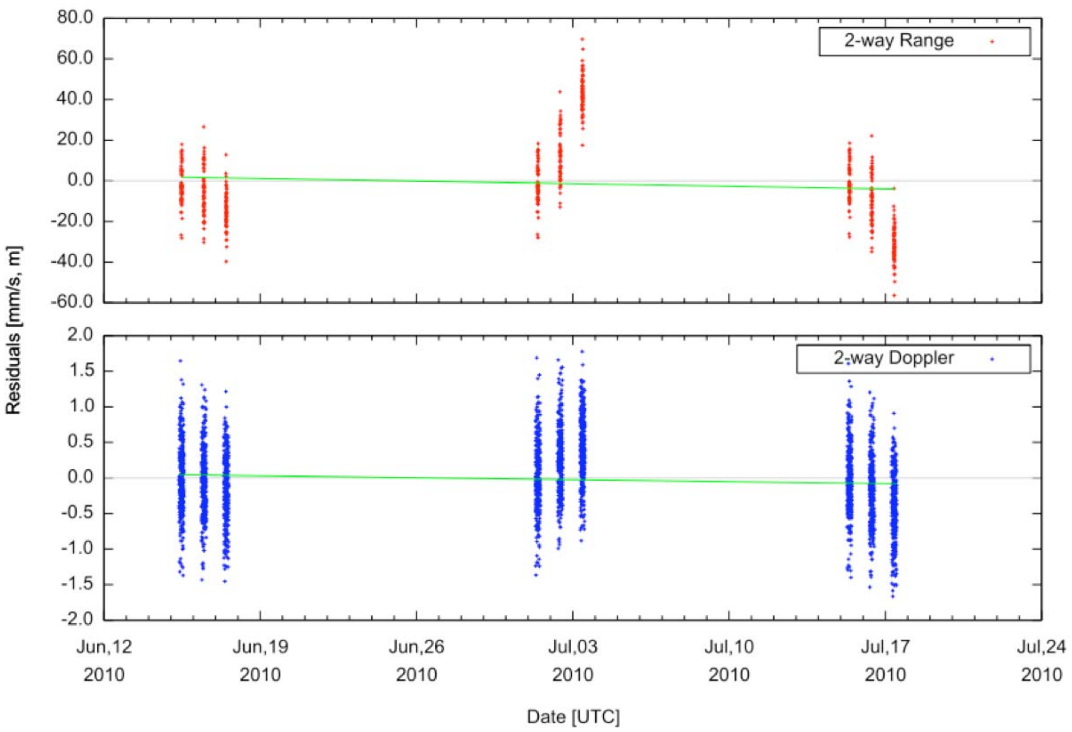

Fig. 12 Residuals of range and Doppler observable using simple plane model

estimation using 4 set of short period tracking data is described in Fig. 13, and the residuals are also varies around zero without any extra residuals. The estimated GSM coefficients and its covariance are summarized in Table 3 and Fig. 14. It is shown that the estimated GSM

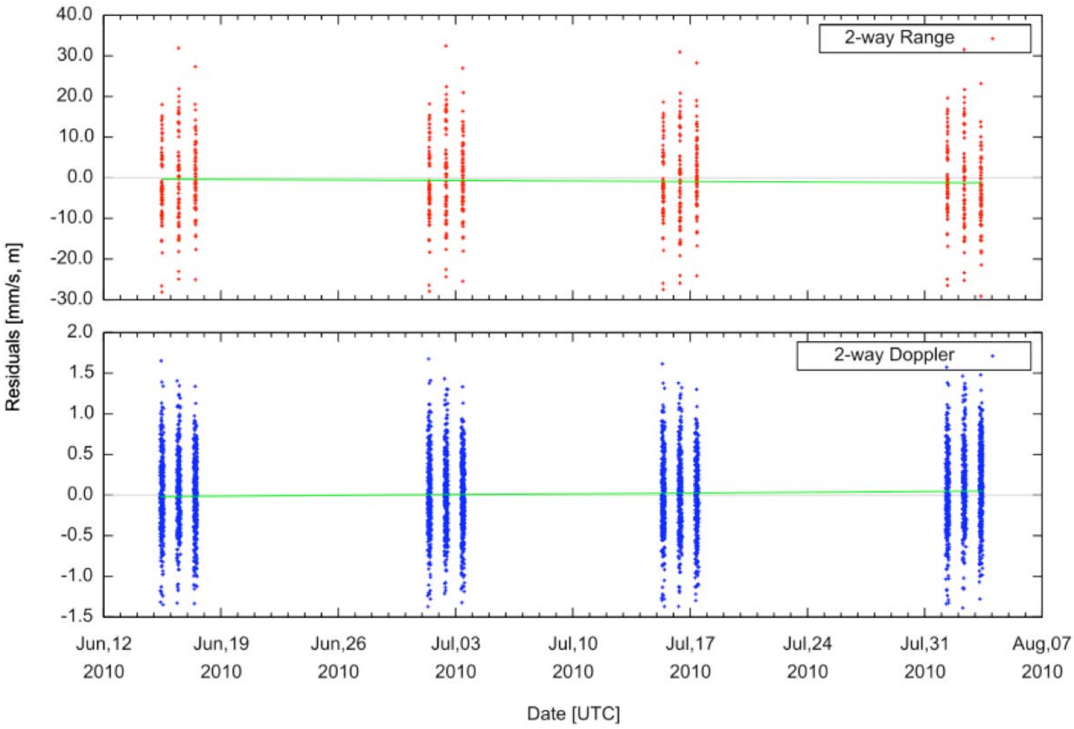

Fig. 13 Residuals of range and Doppler observable using GSM (4 set of shot period tracking data)

coefficient is stable and its covariance reduces as its tracking data becomes longer.

5. CONCLUSION

The trajectory analysis of small solar sail spacecraft IKAROS in terms of estimating solar radiation pressure force was investigated based on the force represented by multi plane solar sail model. The multi plane solar sail model presents the solar sail force including the impact of sail's deformation and non-uniform optical properties. The solar sail force estimation is demonstrated by two-stage estimation, which are short period orbit determination and long period generalized sail model coefficients estimation.

Short period orbit determination is discussed considering the mission sequence of IKAROS to find the better combination of the estimate parameter. It is found that short period orbit determination required estimating area and reflectivity due to poor a priori information of solar sail conditions.

Table 3 Estimated GSM coefficients

	GSM elements	3 set	4 set
1	J3	4.82E-01	4.56E-01
2	J11	7.52E-02	5.66E-02
3	J33	4.98E-01	4.12E-01
4	J113	-6.60E-02	-5.39E-02
5	J333	3.93E-01	4.49E-01

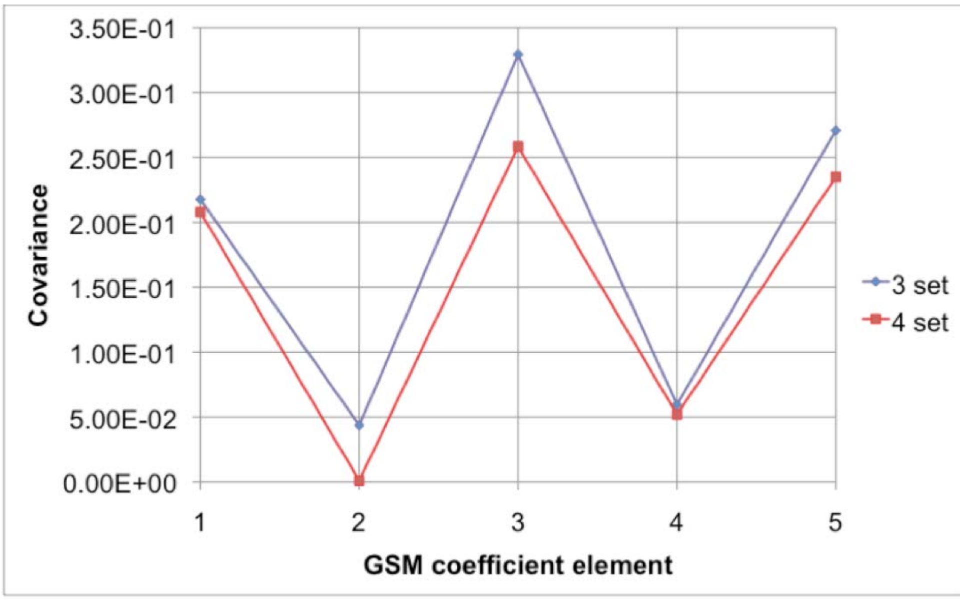

Fig. 14 Covariance of GSM coefficients

The analysis of long period estimation using the generalized sail model is investigated and showed its stable performance. However the long period estimation required accurate estimated state vector from short period orbit determination.

6. ACKNOWLEDGMENTS

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