A BATCH-SEQUENTIAL FILTER FOR THE BEPICOLOMBO RADIO SCIENCE EXPERIMENT

Antonio Genova (1), Manuela Marabucci (1), Luciano Iess (1)
(1) Dipartimento di Ingegneria Meccanica e Aerospaziale, Università di Roma La Sapienza, Via Eudossiana 18 – 00184 Rome, (+39) 06 44 585 976
antonio.genova@uniroma1.it, manuela.marabucci@uniroma1.it, luciano.iess@uniroma1.it

Abstract: *Precise trajectory reconstruction of an orbiting spacecraft is inherently related to the estimation of the gravity field of the central body. Gravity not only allows a good orbit determination, but also provides crucial information on the interior structure of the planet, and therefore constitutes an important scientific objective in many planetary missions such as BepiColombo, the ESA mission to Mercury. One of BepiColombo's investigations is the Mercury Orbiter Radioscience Experiment (MORE), whose main objective is the accurate estimation of Mercury's gravity field. This task will be accomplished by means of range rate measurements accurate to 0.003 mm/s (at 1000 s integration times), enabled by highly stable, multi-frequency radio links in X and Ka band. After an introduction to the mission and the MORE experiment, we report on numerical simulations aiming at a realistic assessment of the attainable accuracy in the determination of Mercury's gravity field. The best results are obtained with a batch-sequential filter, which proves to cope well the complexity of the noise and dynamical models.*

Keywords: *Orbit determination, radio science, gravity field, BepiColombo, batch-sequential*

1. Introduction

Deep-space tracking systems exploit microwave links for obtaining crucial navigation data, such as the frequency and the propagation time of radio signals. These quantities are used by orbit determination codes to estimate the spacecraft state and of other parameters of the dynamical model (such as the Stokes coefficients of the gravity field).

This work reports on numerical simulations of the gravity experiment of BepiColombo, the cornerstone mission to Mercury of the European Space Agency (ESA). Our main goal is an assessment of the uncertainties in the spacecraft state vector and the gravity field coefficients of the planet. The simulations are carried out assuming the latest mission scenario, and include the effects of the frequent desaturation events of the onboard attitude wheels.

2. The mission BepiColombo

Although Mercury plays a fundamental role in constraining and testing the competing theories explaining dynamical and compositional aspects of the formation and evolution of the terrestrial planets, it is still one of the least known planets of the solar system. In addition to its paramount importance for planetary science, Mercury has since long ago raised the interest of the fundamental physics community. Indeed, Mercury's proximity to the sun entails significant relativistic effects, providing therefore unique possibilities for testing Einstein's theory of general relativity and alternative theories of gravity.

Only two spacecraft visited Mercury in the past, namely NASA's Mariner 10 and MESSENGER. Mariner 10, a pioneering mission in many aspects, provided the first close images of the planet in the 1974-1975, and made the puzzling discovery of an internally generated magnetic field. The more recent MESSENGER mission was undertaken by NASA 30 years after Mariner 10 (launch in

2004). So far MESSENGER has completed three Mercury's flybys (January and October 2008, September 2009), and is ready for orbit insertion around the planet (March 2011).

The scientific relevance of Mercury to planetary science and fundamental physics was soon recognized also by other space agencies. In 2001 ESA and JAXA (the Japanese Space Agency) jointly started the BepiColombo project, an ambitious dual-spacecraft mission for the exploration of the planet. The mission consists of two spacecraft, the Mercury Planetary Orbiter (MPO), lead by ESA, devoted to remote sensing observations of the planet, and the Mercury Magnetospheric Orbiter (MMO), build by JAXA, for the investigation of Mercury's magnetosphere [1]. Because of the proximity to the Sun and the harsh thermal environment, the spacecraft design had to face several challenges. In fact, the spacecraft have to reach and survive in a really hard environment.

BepiColombo is endowed with a solar electric propulsion (SEP) system. The interplanetary cruise phase exploits a combination of low-thrust arcs and flybys at Earth, Venus and Mercury itself to reach the planet with low relative velocity. The mission scenario entails launch by an Ariane 5 (nominally on 19 July 2014) of a spacecraft composite made up by the MPO, MMO and a propulsion element, the Mercury Transfer Module (MTM). The lift off mass will be 4.2 metric tons (including a launch vehicle adapter), of which a large fraction (about 32%) is propellant. The duration of the cruise phase will be about 6 years for a planned arrival at Mercury in the second half of 2020.

The heliocentric low-thrust arcs reduce the approach velocity at Mercury arrival to low values such that, when the spacecraft passes the Sun-Mercury Lagrange L_1 and L_2, it is weakly captured in a highly eccentric orbit around Mercury. After the approach through a weak stability boundary of the planet, the spacecraft requires only a small velocity change for a firm capture. An additional thrust phase will insert the MMO into its operational orbit and the MPO into its lower orbit. The baseline lifetime of the spacecraft in Mercury orbit is one Earth year.

3. The MORE experiment

MORE (Mercury Orbiter Radio science Experiment) is the radio science experiment of the mission BepiColombo. Its flight hardware, a transponder enabling high accuracy range and range rate measurements at Ka-band (34 GHz uplink, 32.5 GHz downlink), is hosted on board the MPO. MORE has been designed to determine the planet's gravity field to degree and order 25 and carry out improved tests of relativistic gravity.

The measurements undertaken by MORE address three different areas:

- reconstruction of the planet's gravity field, the coefficients of the spherical harmonics expansion up to degree and order 25, and the Love number k_2 (gravimetry experiment). The expected accuracies range from signal to noise ratio of 10^4 for degree 2, to SNR of 10 for degree 20.
- estimation of the rotational state of Mercury by means of obliquity and amplitude of the physical librations in longitude (rotation experiment). These measurements, carried out in collaboration with the camera team, provide the moments of inertia of the whole planet and its mantle.
- determination of the post-Newtonian parameters, the mass and the oblateness of the Sun, and the upper limits to the temporal variation of the gravitational constant G (relativity experiment).

The crucial onboard elements of the MORE experiments are:

- the Ka/Ka transponder (KaT), provided by the Italian Space Agency
- the TLC/TCM deep space transponder (DST)
- the 1.2 m high gain antenna (HGA)
- the high sensitivity Italian Spring Accelerometer (ISA)

In addition, the final global fit and orbit reconstruction will incorporate also laser-altimetric and optical observables provided by the onboard laser altimeter (BELA) and high-resolution camera (SIMBIO-SYS). The KaT and the DST are particularly important because they enable a multi-frequency radio link at X (7.2 GHz uplink/8.4 GHz downlink) and Ka band (34/32.5 GHz), a configuration already exploited by the Cassini mission [2], [3]. Thanks to this configuration, range rate measurements will attain accuracies of 3 micron/s (at 1000 s integration time) at nearly all solar elongation angles. In the geometric optics limit, the use of multiple frequencies allows a complete cancellation of plasma noise, the dominant noise source in S and X band radio links [4]. A novel wideband ranging system (WBRS), based upon a pseudo-noise modulation scheme at 24 Mcps, will provide observables accurate to 20 cm (two-way) [5].

The effects of non-gravitational accelerations on the spacecraft dynamics (quite large in the harsh hermean environment) will be removed to a large extent thanks to the ISA accelerometer. These instrument readouts will be sent to ground in the telemetry stream and referenced to the phase center of the high gain antenna. The orbit determination code will then use a smoothed version of the accelerometer measurements to integrate the equation of motion, effectively realizing a software version of a drag-free system.

Such a complex experimental setup, implemented for the first time in a planetary mission, will be used not only for the reconstruction of the Mercury's gravity field, but also for a precise reconstruction of the spacecraft orbit. Accuracies of 0.1-1 m in the radial position seem attainable. The position of the MPO in the hermean frame (whose origin is defined by zeroing the dipole terms in the harmonic expansion of the gravitational potential) will be used for the appropriate referencing of the laser altimetric measurements and the images from the high-resolution camera. The combination of altimetric and gravity measurements will provide the topographic heights, a crucial information to determine the structure of Mercury's crust and outer mantle.

The along- and across-track position of the spacecraft is crucial for the rotation experiment, aiming to determine the rotational state of the planet by means of optical tracking of surface landmark. The pole position and physical librations in longitude will be obtained from a precise georeferencing of high-resolution images (5m pixel size at pericenter). The final accuracy of this experiment rests not only upon an accurate knowledge of the spacecraft position, but also on the quality of the attitude reconstruction. The onboard star trackers and gyroscopes should allow an accuracy of 1-2 arcsec. In addition, the spacecraft design ensures a high stability of the optical alignment between the star trackers and the camera.
Although MORE will make use of laser altimetric and optical images to stabilize the global orbital solution, the crucial observable quantities are range and range rate. These quantities are generated at the ground station after establishing a two-way coherent link. The core element of the tracking system is the reference oscillator, a H-maser with a frequency stability of one part in 10^{15} over time scales of 1000 s. The orbital solution is obtained from the observable quantities by means of a least squares fit, where the state vector of the spacecraft and the parameters of the dynamical model are jointly derived.

In the case presented in this work, a single global fit (for the entire mission) doesn't appear a practical solution because of model errors, the large amount of data and free parameters and the lack of dynamical coherence of the trajectory. The main difficulties are the uncompensated non-gravitational accelerations and the periodic desaturations of the momentum wheels employed in the attitude control system. Both effects destroy the dynamical coherence of the trajectory and lead to a degraded estimation of the gravity field. In order to cope with these problems, we have used a batch-sequential approach, described in the next sections. This method appears adequate to the processing of the large amount of data generated by MORE, and capable of providing the required accuracies both in the determination of the gravity coefficients and the spacecraft orbit.

4. Reaction wheels desaturation maneuvers and their consequences in the estimation problem

The current spacecraft design entails a single solar panel located asymmetrically with respect to the center of mass. Due to the large solar and IR flux, any offset between the center of mass and center of pressure gives rise to a significant torque. In addition, the tight mass constraint limits the mass of the wheels and therefore the angular momentum stored onboard. According to current estimates, momentum-dumping maneuvers may occur as frequently as 12 hours, rather than once a day, as initially required. The MORE team has requested that maneuvers take place outside tracking passes from the primary ground station of the experiment. (This station is the one that will enable the multi-frequency link; in the current plan it is DSS 25, a 34 m antenna of NASA's Deep Space Network located in Goldstone, California.) The sequence of maneuvers and tracking passes (approximately 8 h long) is shown in Figure 1. The Ka-band/multifrequency tracking period is followed by the unobservable arc (~16 hours), during which the wheels off loading maneuvers (DSM1 and DSM2) will occur.

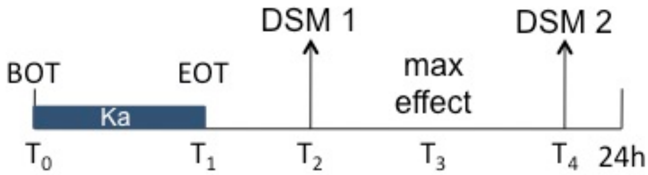

Figure 1 - Tracking period and wheel desaturation maneuvers (DSM)

The desaturation maneuvers of the MPO will be carried out by using two complementary sets of thrusters. Unfortunately the thrusters are unbalanced, so that the torque is associated to a quite significant thrust. The ensuing, large velocity variations (nearly 6 cm/s) degrade the knowledge of the spacecraft state and adversely affect the estimation of the gravity field. The knowledge of the delta-V associated to desaturation maneuvers is expected to be in the range of 2-5% (1.2-3 mm/s), a value sufficiently large to cause substantial errors in the state propagation.

Preliminary simulations of the radio science experiment [6] indicated that the spacecraft position with respect to Mercury's barycenter shall be known with an accuracy of 10-30 m at the beginning of the tracking pass, where the integration is typically restarted. Meeting this condition in the presence of desaturation events every 12 hours is not straightforward, especially if the knowledge of the actual delta-V is poor. The BepiColombo project and the prime contractor (Astrium Germany) estimate that each maneuver will result in the following velocity nominal variations:

$$
\begin{aligned}
|\Delta V_x| &= 0\, mm/s \\
|\Delta V_y| &< 17\, mm/s \\
|\Delta V_z| &= 42\, mm/s
\end{aligned}
\qquad (1)
$$

where the x, y and z directions are respectively along track, radial nadir pointing and out of plane. The maximum effect in the propagation of the uncertainties occurs in the middle of the unobserved arc (T_3 in Figure 1). As a first approximation, the displacement in the three directions can be computed using perturbative equations. A ΔV event changes the mean motion by an amount

$$\Delta n = -\frac{3}{a\sqrt{1-e^2}}\left(-e\Delta V_y \sin f + \left(1-e\cos f\right)\Delta V_x\right) \tag{2}$$

so that the along-track displacement is:

$$\Delta x = a\Delta\lambda = a\left(T_3 - T_2\right)\Delta n \cong 3e\Delta V_y\left(T_3 - T_2\right) \cong 176m \tag{3}$$

The change in eccentricity

$$\Delta e = -\frac{\sqrt{1-e^2}}{na}\left(-\Delta V_y \sin f + \left(\cos E + \cos f\right)\Delta V_x\right) \cong \frac{\sqrt{1-e^2}}{na}\Delta V_y \sin f \tag{4}$$

produces variations in the along-track and radial directions, by an amount

$$\begin{aligned} \Delta x &\cong 2a\Delta e \cong 46m \\ \Delta y &= a\Delta e \cong a\frac{\Delta V_y}{V} \cong 23m \end{aligned} \tag{5}$$

Each event changes also the inclination and node longitude:

$$\begin{aligned} \Delta i &= \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\Delta V_z \cos(f+\omega) \cong \frac{1-e\cos M}{na}\Delta V_z \cos(f+\omega) \\ \Delta\Omega &= \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\Delta V_z \frac{\sin(f+\omega)}{\sin i} \cong \frac{1-e\cos M}{na}\Delta V_z \sin(f+\omega) \end{aligned} \tag{6}$$

The resulting out of place displacement is:

$$\Delta z \cong a\Delta i \cong a\Delta\Omega \cong 54m \tag{7}$$

Assuming a knowledge to a 5% accuracy of the ΔV the resulting displacement is about 11m in radial direction, 1m along y-axis and 3m out of plane.

Although in principle these estimates indicate that the orbital perturbations are still within acceptable limits, better results may be obtained using the ISA accelerometer. Indeed, preliminary estimates indicate that ISA may improve the knowledge of the velocity increments to a level of 0.15 mm/s. This capability requires a new operational mode that changes the dynamic range of the instrument, could prove very useful to increase the dynamical coherence to the orbit through several tracking passes.

5. Orbit and gravity field determination

The dedicated instrumentation of radio science experiments may provide an improved reconstruction of the spacecraft orbit. Accuracies in the range of 0.1-1 m in the radial position with respect to the central body can be obtained for orbiting spacecraft. In the case of the mission BepiColombo, such improved orbit determination is possible thanks to the multi-frequency radio link and the wideband ranging system (WBRS). Range and range rate measurements attain a precision of the order of cm and μm/s, respectively. These measurement accuracies are possible thanks to the implementation of a plasma noise cancellation system, already used for the Cassini mission [4].

Deep space probes are visible from Earth stations for several hours per day, but in general are supported by only one station. Tracking data are then processed through an orbit determination code to update the orbital reconstruction. In a classical approach, all collected data are processed through a batch filter, in order to improve the knowledge of the state vector and of free parameters. In spite of sophisticated tracking system, this kind of data filters can provide orbital reconstruction to the tens-of-meter level because of non-accurate force models governing the spacecraft motion and the large number of solved-for parameters. The errors due to dynamical models can be drastically reduced by means of stochastic accelerations but they are not appropriate when the force models (i.e. gravity field) must be evaluated with high accuracy.

5.1. Classical batch estimation

In the general orbit determination problem, both the dynamics and the measurements involve nonlinear relationships. We write the dynamics-observation equations as:

$$\begin{aligned} \dot{X} &= F(X,t) & X(t_k) &\equiv X_k \\ Y_i &= G(X_i,t_i)+\varepsilon_i & i &= 1,\dots,l \end{aligned} \tag{8}$$

where X_k is the unknown n-dimensional vector of solved-for parameters (spacecraft position and velocity components and free parameters of the dynamical model) at the time t_k, and Y_i for $i=1,\dots,l$, is a p-dimensional set of observations that are used to obtain the best estimate of the unknown vector X_k, according to an optimum criterion. These equations are linearized in terms of the state deviation and observation deviation vectors, obtained from a priori estimates of the state parameters:

$$\begin{aligned} x(t) &= X(t)-X^*(t) \\ y(t) &= Y(t)-Y^*(t) \end{aligned} \tag{9}$$

where $(\,)^*$ indicates the reference solution. The linearized problem can be written as:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) \\ y_i &= \tilde{H}_i x_i + \varepsilon_i \end{aligned} \qquad i = 1,\dots,l \tag{10}$$

where:

$$\begin{aligned} A(t) &= \left[\frac{\partial F(t)}{\partial X(t)}\right]^* \\ \tilde{H}_i(t) &= \left[\frac{\partial G(t)}{\partial X(t)}\right]_i^* \end{aligned} \tag{11}$$

Most filters used for interplanetary orbit determination problem are based on a least squares solution, which selects the best estimate of x by minimizing the sum of the squared residuals. The well-known weighted LS solution with a priori information is:

$$\hat{x}_k = \left(H^T W H + \bar{W}_k\right)^{-1} \left(H^T W y + \bar{W}_k \bar{x}_k\right) \tag{12}$$

where $\bar{x}_k$ and $\bar{W}_k$ represent respectively the *a priori* estimate and covariance matrix of x_k. The iteration of this process allows obtaining the best estimate of x in the case of planetary flybys or trajectories of few days. Nevertheless, this solution could not be sufficient when the dynamics of the spacecraft is quite complicated and when there are a large number of solving-for parameters. In such case, the trajectory could be subdivided into small arcs (i.e. single day). It allows obtaining the continuously convergence of the single arc estimation and using the multi-arc Approach that improves considerably the estimation of the global parameters.

5.2. Multi-arc method

In several space geodesy experiments the number of parameters to be determined is quite large. This problem is in many cases severe, to the point that a single batch estimation becomes impractical. In addition, deterministic models are unable to reliably account for non-gravitational perturbations. In many cases a multi-arc method alleviates these problems, although the presence of disturbances may end up in a divergence in the estimation process. In a multi-arc approach the orbit of the spacecraft is fragmented into different arcs, corresponding to non-overlapping time intervals. Each arc is completely independent from the others, because the initial conditions of the state vector of the space vehicle do not depend in any way on the orbital propagation of the preceding arcs. The estimated parameters are divided into two categories: the *local* parameters, pertaining to the single arc (i.e. state vector, manoeuvres and periodic acceleration), and *global* parameters, independent of time and pertaining to all arcs (the mass and the gravitational harmonics of the body, for example). The main goal of this over-parameterization is to absorb the effects and errors caused by unmodelled dynamics, errors that accumulate with time. Therefore, the length of every arc should be chosen so as to maintain the errors in the dynamical model at a level compatible with the observation errors. The arc length must be chosen judiciously: if too long, the errors accumulate, if too short the solution becomes unstable. For instance, arcs characterized by favourable geometrical conditions are to be privileged, whereas those during which the spacecraft performs manoeuvres are to be disregarded.

In a classical multi-arc estimation, the solution vector $x = [g;l]$ is split into the vectors g and l of global and local parameters. The observations and the corresponding residuals are partitioned into n arcs, in such a way that $z = [z_1;z_2;\ldots;z_n]$. The vector l is further split into vectors l_i, one for each arc. Each sub-vector l_i is associated to the arc with the same index, in such a way that the residuals from one arc do not depend upon the local parameters of another arc:

$$H_g^{(j)} = \frac{\partial z_j}{\partial g} \qquad H_{l_i}^{(j)} = \frac{\partial z_j}{\partial l_i} = 0 \qquad \text{for} \quad i \neq j \tag{13}$$

The mapping matrix H is structured as follows:

$$H=\begin{pmatrix} H_{l_1}^1 & 0 & \cdots & 0 & H_g^1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & H_g^{n-1} \\ 0 & \cdots & 0 & H_{l_n}^n & H_g^n \end{pmatrix} \quad (14)$$

Therefore, the observables of each arc depend only on the local parameters of the same arc and on the global parameters. The contributions of each arc to the overall normal equation are:

$$C_{l_i l_j} = \left(H_{l_i}^{(i)}\right)^T H_{l_j}^{(j)} = C_{l_j l_i}^T = 0 \qquad \text{for} \quad i \neq j$$
$$C_{g l_i} = \left(H_g^{(i)}\right)^T W_{l_i} H_{l_i}^{(i)} = C_{l_i g}^T \qquad C_{gg} = \sum_{i=1}^{n} \left(H_g^{(i)}\right)^T W_{l_i} H_g^{(i)} = C_{gg}^T \quad (15)$$

where

$$W_{l_i} = \begin{pmatrix} 1/\sigma_1^2 & 0 & \cdots & 0 \\ 0 & 1/\sigma_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1/\sigma_n^2 \end{pmatrix} \quad (16)$$

is the $n \times n$ data weighting matrix of the single arc and σ_j is the error affecting the observation point z_j. The normal matrix C has an arrow-like structure:

$$C = \begin{pmatrix} \left(C_{gg} + \Gamma_g^{-1}\right) & C_{gl_1} & \cdots & C_{gl_{n-1}} & C_{g1_n} \\ C_{l_1 g} & \left(C_{l_1 l_1} + \Gamma_{l_1}^{-1}\right) & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ C_{1_{n-1} g} & \vdots & \ddots & \left(C_{l_{n-1} l_{n-1}} + \Gamma_{l_{n-1}}^{-1}\right) & 0 \\ C_{l_n g} & 0 & \cdots & 0 & \left(C_{l_n l_n} + \Gamma_{l_n}^{-1}\right) \end{pmatrix} \quad (17)$$

where Γ_g and Γ_l are the a priori covariance matrix respectively for global and local parameters. The contributions to the right hand side D of the normal equation are:

$$D = \left[D_g; D_l\right] = \left[D_g; D_{l_1}; \ldots; D_{1_n}\right]$$
where $$D_g = -\sum_{i=1}^{n} \left(H_g^{(i)}\right)^T z_i \qquad D_{l_i} = -\left(H_{l_i}^{(i)}\right)^T z_i \quad (18)$$

Then the normal equation is a system of two vector equations [8]:

$$\begin{cases} C_{gg}\Delta g + C_{gl}\Delta l = D_g \\ C_{lg}\Delta g + C_{ll}\Delta l = D_l \end{cases} \tag{19}$$

given by the minimization of the least-squares cost function.

One of the most serious problems of the multi-arc approach is the stability of the solution. Attaining convergence can prove very difficult when in the single arcs, the number of parameters to be estimated is large, as in the case of the MORE experiment with the MPO. Single arc estimation over a few hours relies heavily on a tight a priori covariance matrix, which may result in a bias of the solution, that the multi-arc cannot retrieve. Convergence and orbital solutions improve considerably by using sequential updates of the global parameters using a batch-sequential filter.

5.3. Batch-Sequential estimation

A batch-sequential filter has been developed in the context of the numerical simulations of the Mercury Orbiter Radio Science Experiment (MORE) of the ESA mission BepiColombo. The filter was devised in order to cope with the uncertainties in the dynamical models (associated to wheel off-loading manoeuvres, accelerometer bias and drift, etc.). The accumulation of the errors due to unmodelled effects leads to a divergence in the trajectory reconstruction, making the multi-arc method ineffective.

With the batch-sequential method the dynamical model is updated and improved as additional, short data batches included in the solution. The sequential processing of short batches improves the estimation of the state vector as the mission goes on. The choice of the batch duration is particularly important for a fast convergence and a good accuracy of the final orbital solutions. Its length is a trade off between the estimation accuracy of the global and local parameters.

A single batch (Figure 2) is processed following three essential steps:

- Single arc estimation of a limited number of arcs
- Multi-arc estimation with initial conditions and a priori uncertainties respectively equal to the estimated values (weighted mean for the global parameters, with formal uncertainties as weights) and a priori uncertainties of the single arc estimation
- Update of the dynamical model with the estimated parameters for the next batches

With this procedure the processing of the following arcs benefits from an improved dynamical model, thus allowing a better estimate of the state vector. In addition, this sequential updating leads also to improved estimates of the global parameters, which attain convergence after a limited number of batches. However, the results from this first iteration may still be improved, both for the state vector and global parameters estimation. The initial batch processing is only the starting point of a two-step estimation process. The first step just described has two important tasks, namely to prepare the initial conditions of each single arc and improve the knowledge of the dynamical model. The second step uses the improved dynamical model to initialize the global multi-arc estimation. The batch-sequential estimation provides the nominal values and the a-priori uncertainties for global multi-arc setup. Whereas the global parameters are initialized with the estimation of the last batch, the initial values of local parameters are obtained from the single arc estimation. This second step can hence provide an improved estimate of the global parameters. Once the dynamical model is recovered to the required accuracy, the orbit reconstruction can be obtained by means of a final single arc estimation.

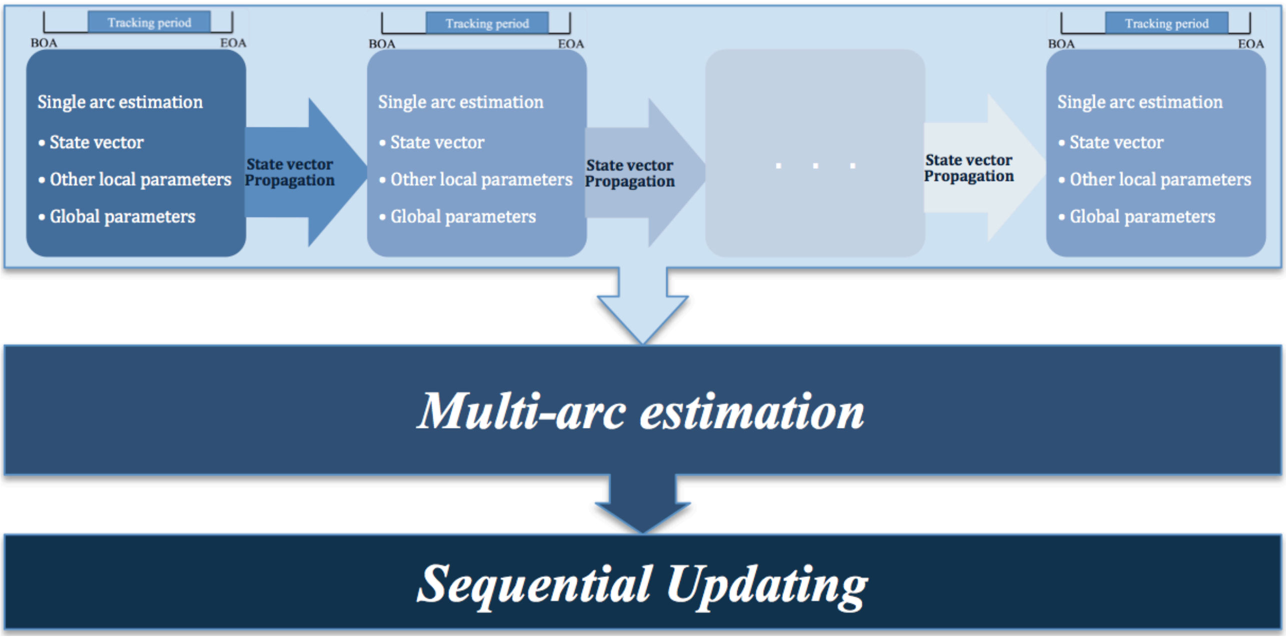

Figure 2. General scheme of a single batch

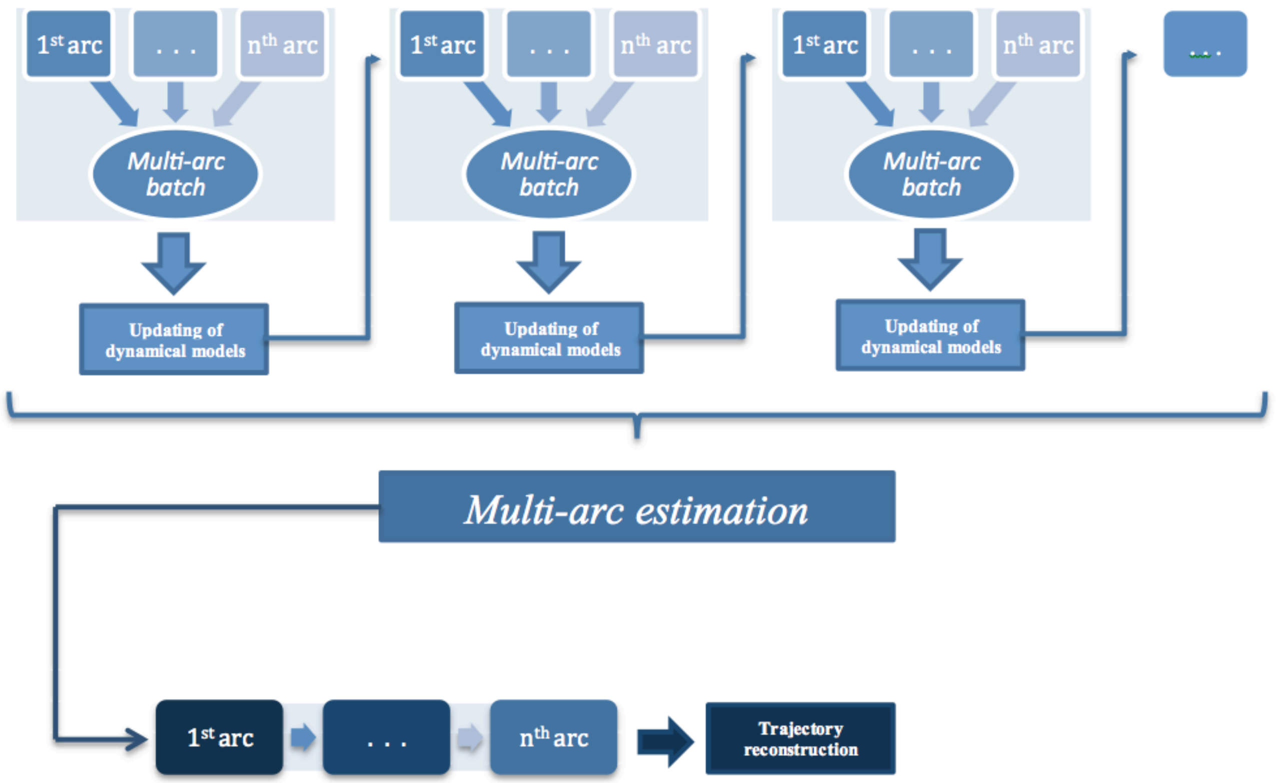

Figure 3. Flux diagram of the batch sequential method

6. Simulation setup

The performances of the batch-sequential filter described above have been assessed by means of a full numerical simulation of the BepiColombo radio science experiment. The initial MPO orbit around Mercury is polar with pericenter and apocenter altitudes respectively at 400 km and 1500 km. The simulation covers a full terrestrial year, i.e. the duration of the nominal mission. Support from ground has been limited to only one station, with Ka-band uplink and multi-link capabilities.

A daily tracking period of approximately 8 hours has been assumed. Loss of the radio link due to occultations has been also been accounted for.

In the simulation setup (the "truth") the hermean gravity field is expanded in spherical harmonics up to degree 30 and the coefficients are calculated by means of the Kaula's rule

$$C_l^2 = A_k \frac{10^{-10}}{l^4} \quad (20)$$

where the constant A_k is assumed equal to 9 for Mercury [6]. Tidal effects have been considered, with a dynamic Love number k_2 set to 0.3.

Non-gravitational accelerations, such as direct solar radiation and thermal emission from the planet, have not been modeled, because they have been assumed to be removed by the accelerometer. In the orbit determination process the accelerometer transforms de facto the real spacecraft in a virtual, drag-free test particle [7]. However, residual accelerometer biases and drifts, mostly due to changes in the thermal environment, have been simulated through an acceleration noise correlated over time scales of the orbital period [6].

7. Results

The numerical simulations of the MORE experiment have been an excellent test bench for the batch-sequential filter. As today the hermean gravity field is almost unknown, a classical batch estimation is insufficient to satisfy the goals in the orbit reconstruction and gravity field determination. The graphs reported in the Figure 4 and 5 show the estimation errors and the formal uncertainties of the state vector by means of the classical batch estimation. Since day 40, the estimation errors (defined as "truth" minus "estimate") quickly exceed the formal uncertainty, an unacceptable situation. In particular the position error reaches 4-5 km, while the formal uncertainty is in the order of meters. A similar behaviour is found for the velocity, where an average error of 1m/s has to be compared with formal uncertainty as small as 8 mm/s. This poor orbit determination leads to unacceptable results for Mercury's gravity field coefficients in a multi-arc analysis.

Much better results have been found with the batch-sequential method. The selected batch length spans over twenty days, i.e. a quarter of Mercury's revolution period around the Sun. In this way, the frequency of dynamical model updating is maximized because twenty days is the minimum time period that allows leading to coherent results for the global parameters. Furthermore, the estimation of the state vector can be considerably improved (see Figure 6 and 7). The state vector still shows estimation errors beyond the formal uncertainties, but after day 40 such errors are prominently lower than in the classical batch estimation. Both position and velocity vectors exhibit a maximum error around day 40, with values around 1 km and 40 cm/s. Therefore, the estimation errors of the state vector decrease thanks to the improvement of the knowledge of the dynamical model.

The plots in Figure 6 and 7 show how the updating of the global parameters improves the orbital reconstruction. In particular, estimation errors smaller than 10 m in position and 10 mm/s in velocity are obtained in the last arcs, and this time the associated formal uncertainties are larger than the errors. The estimation of the global parameters converges to a stable solution after a transient phase, which is characterized by the oscillatory trend of the real errors of the state vector. At the end of the batch-sequential estimation the gravity field coefficients are coherently estimated (see Figure 8). They are used as nominal values for the following step.

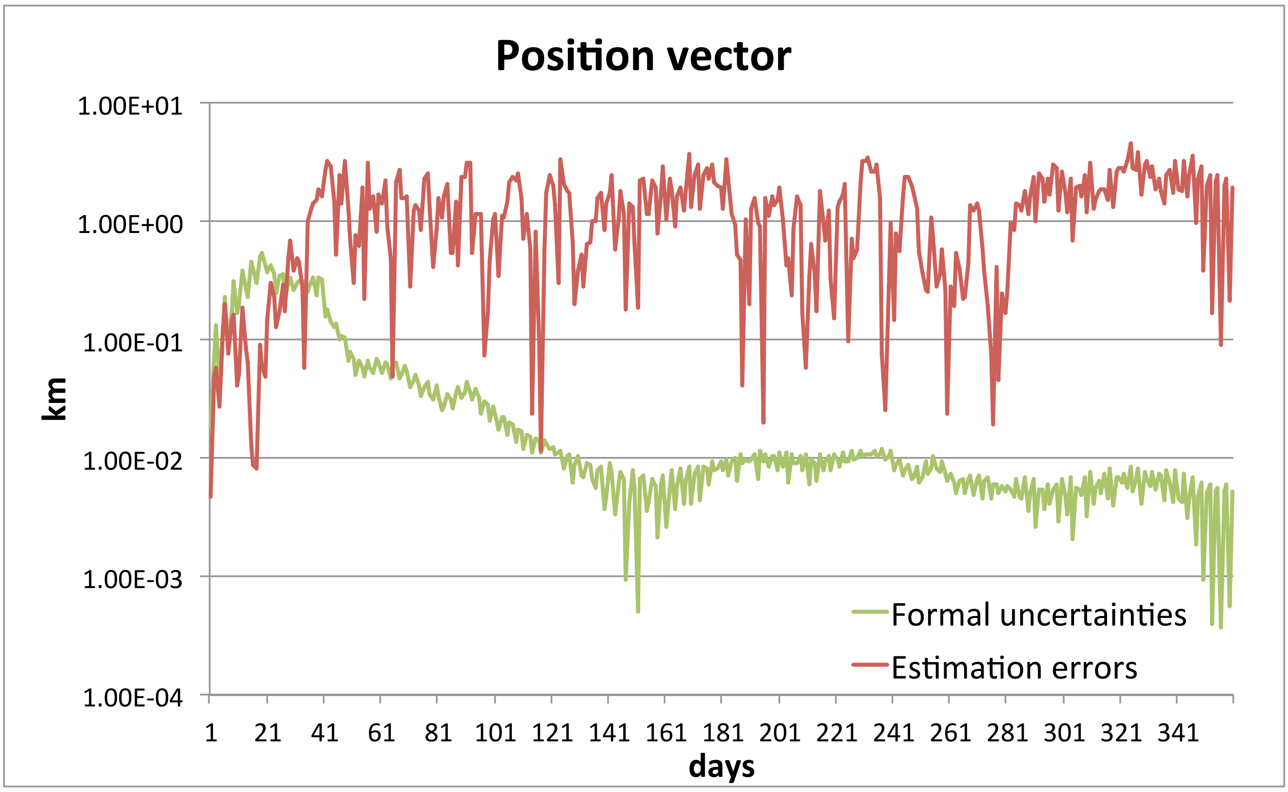

Figure 4. Real errors and formal uncertainties in the estimation of the spatial components of the state vector, for a classical batch estimation. The y-axis shows the norm of the position error vector.

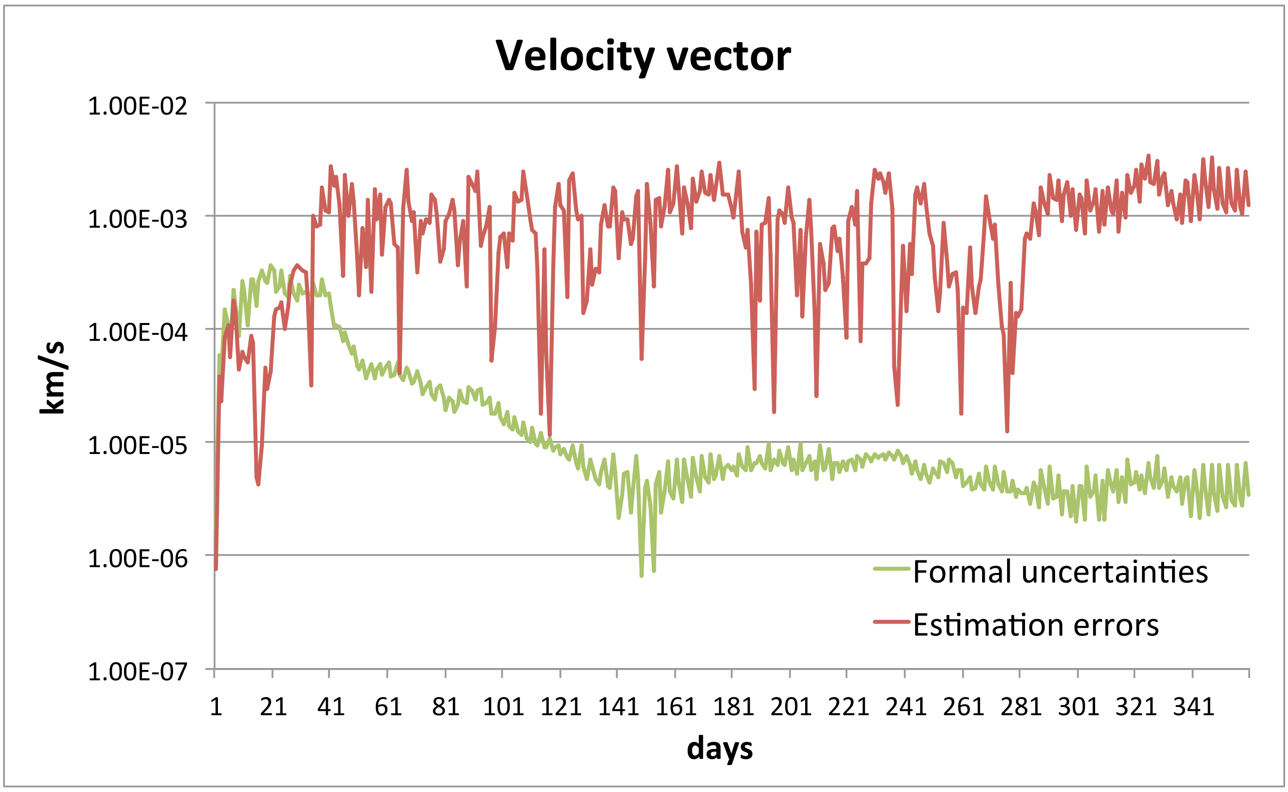

Figure 5. Real errors and formal uncertainties in velocity vector estimation by means of classical batch estimation.

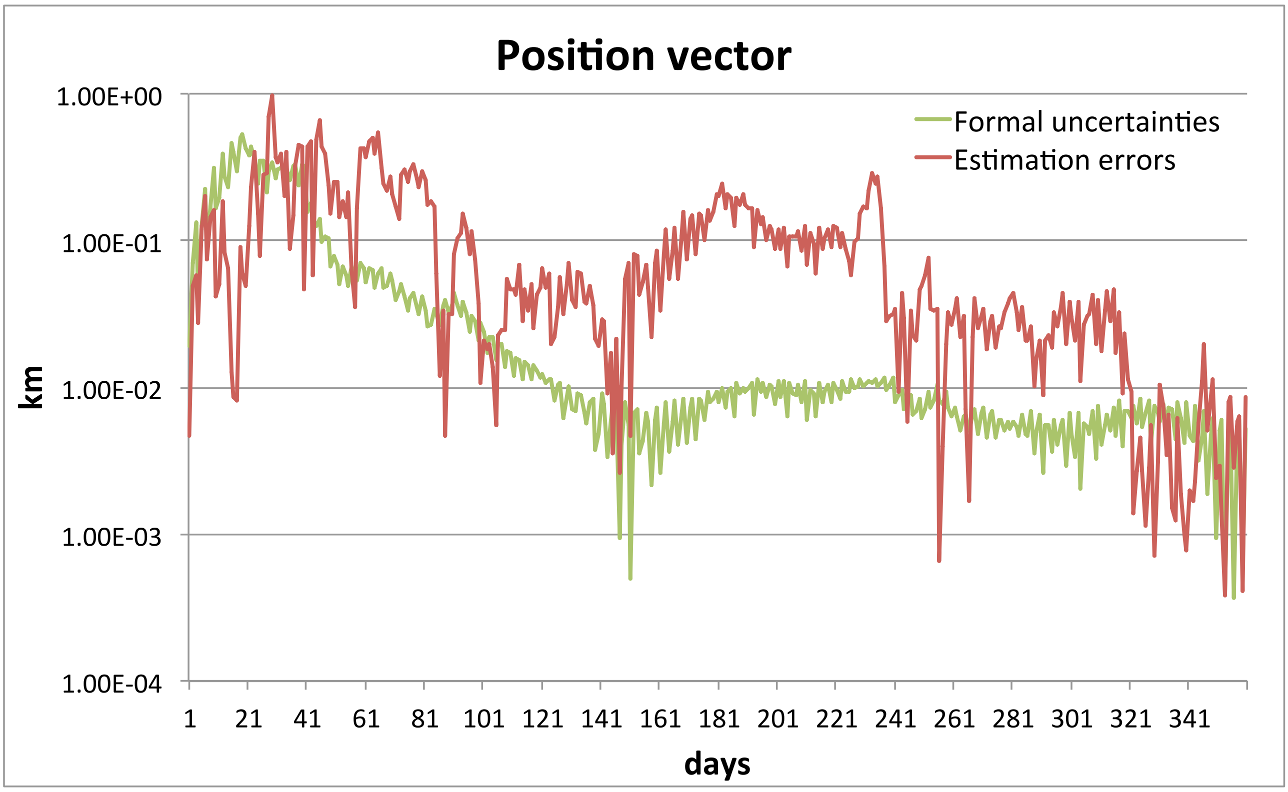

Figure 6. Estimation errors and formal uncertainties in position for the batch-sequential filter.

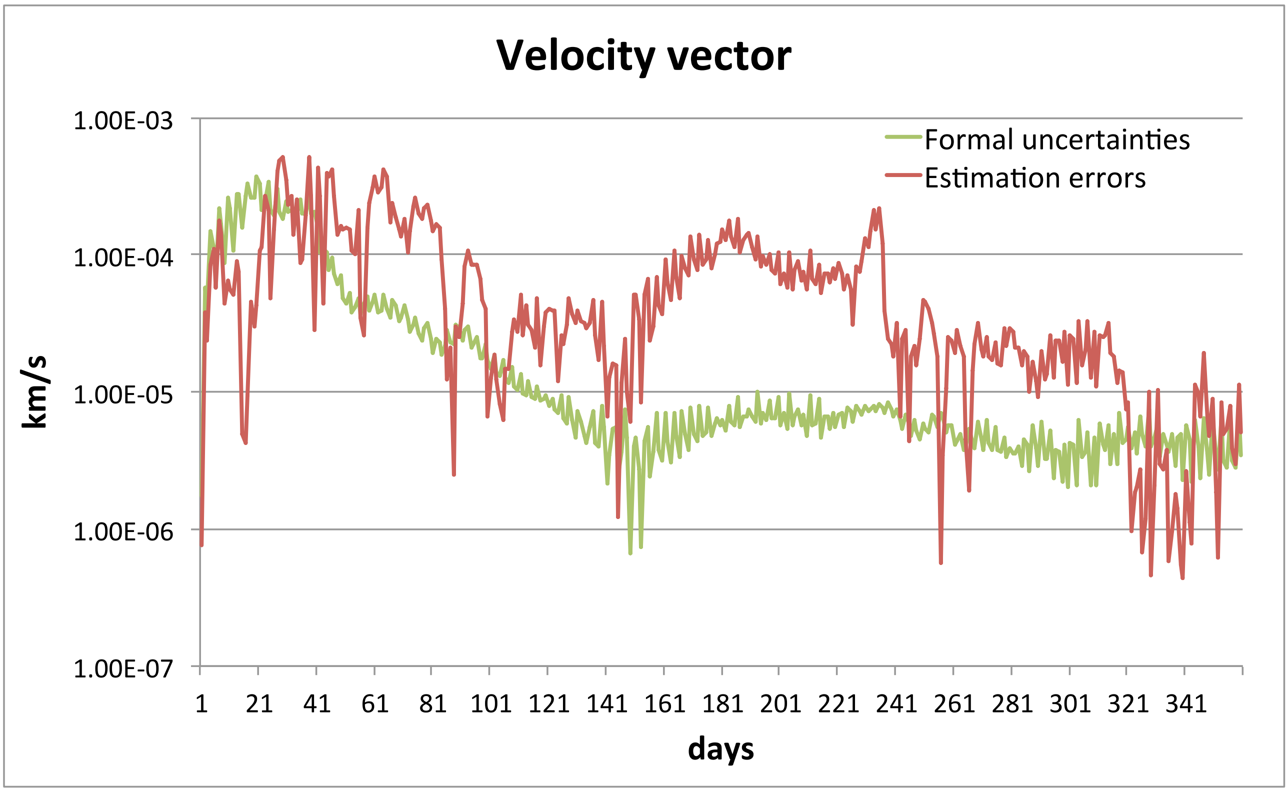

Figure 7. Estimation errors and formal uncertainties in velocity for the batch-sequential filter.

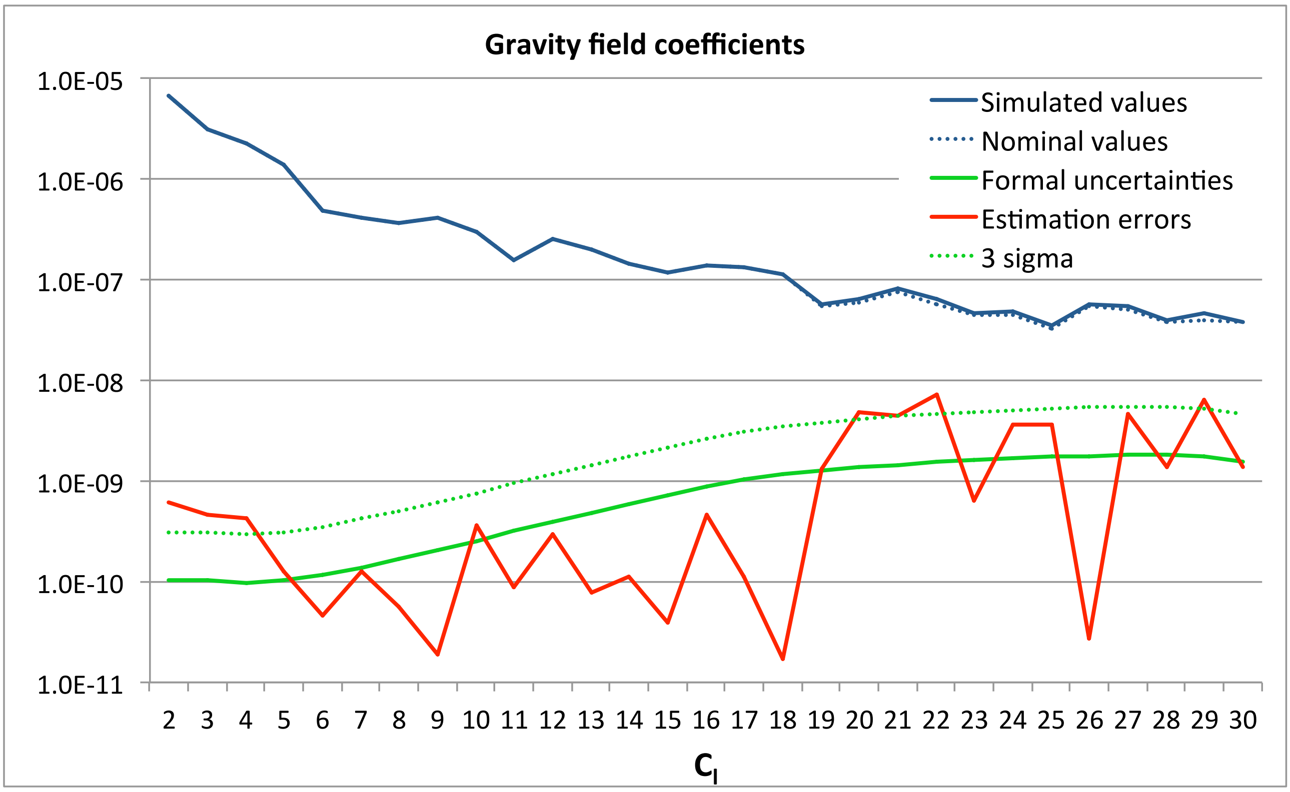

Figure 8. Estimation errors and formal uncertainties of gravity harmonics coefficients, from the batch-sequential filter.

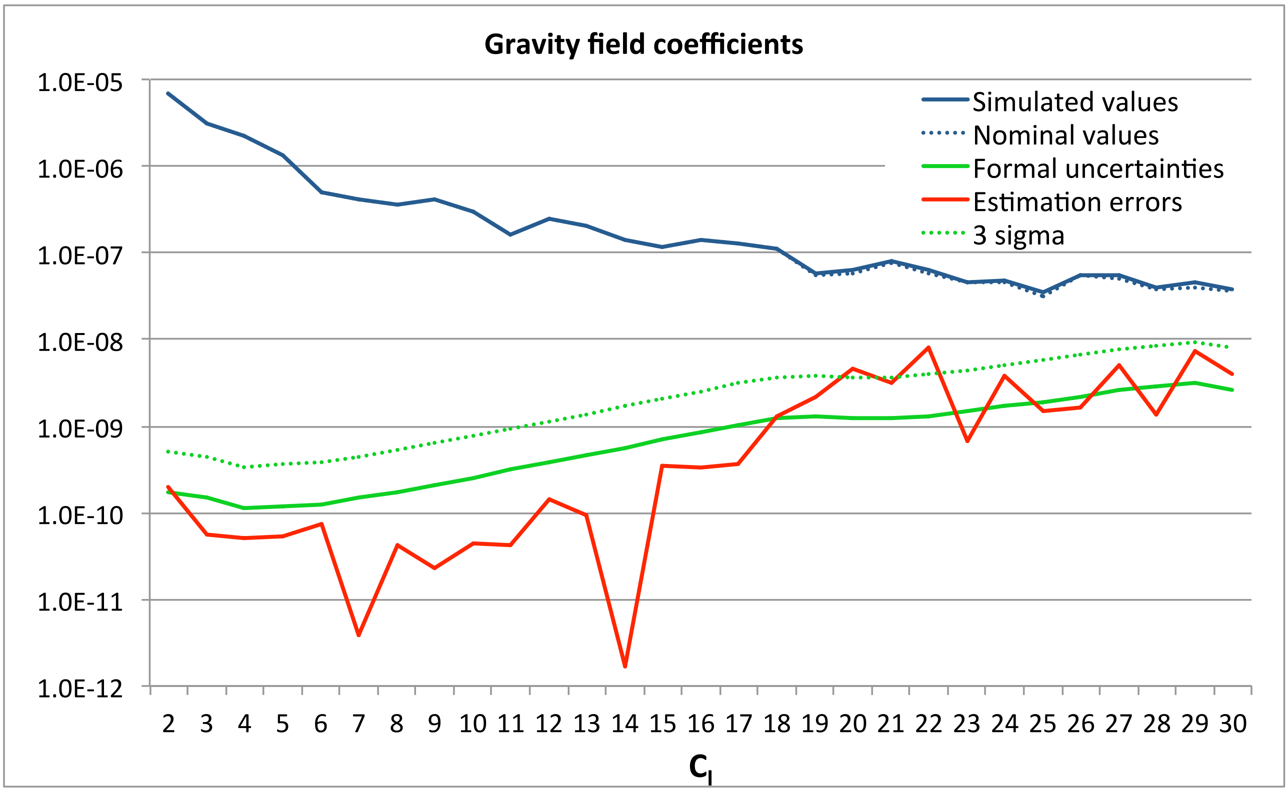

Figure 9. Final estimate of the gravity field coefficients, after multiarc step.

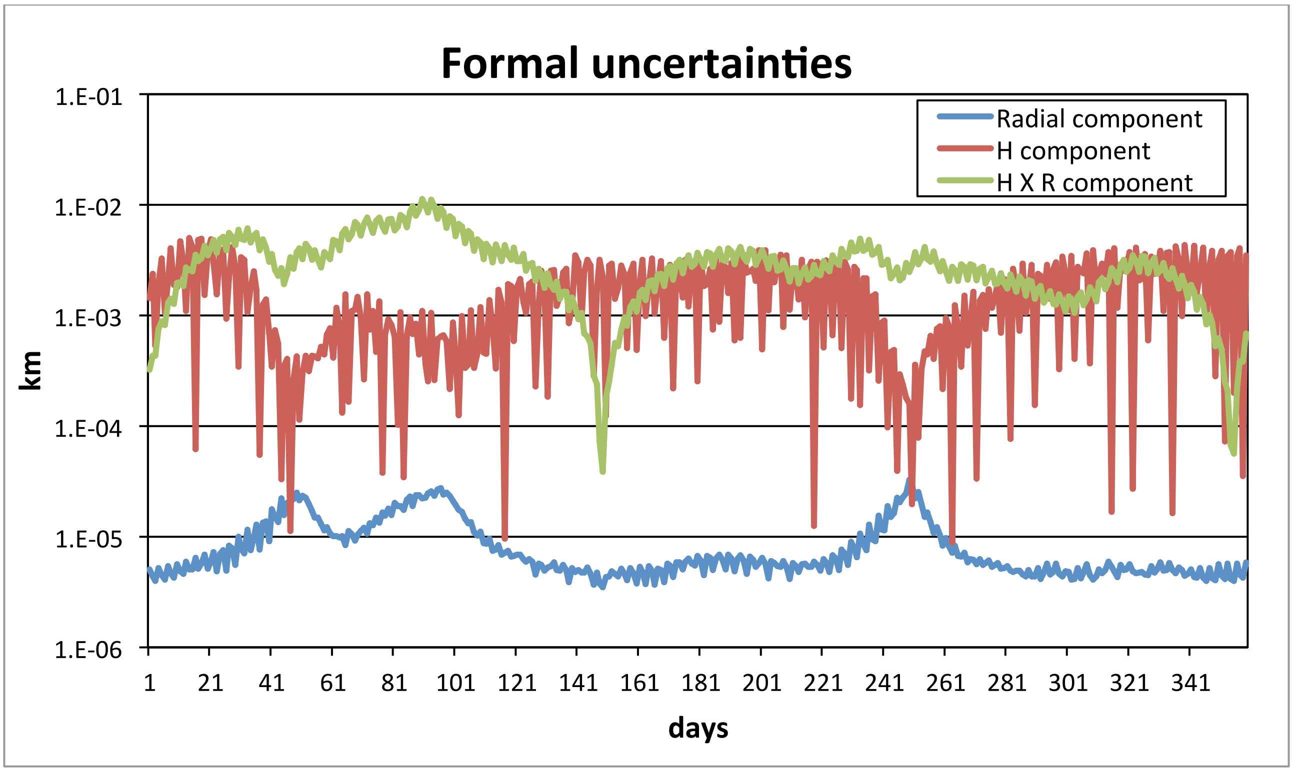

Figure 10. Uncertainties in position after final orbit reconstruction (in the orbital frame).

In the last step, a global multi-arc estimation provides the final improvement to the gravity field coefficients (Figure 9). The mission goals for the relative accuracy of the harmonic coefficients are fully met and the estimation is statistically coherent. Since the degree 20, the estimation error increases, but still remains in a range between one and three sigma and can therefore be considered as statistically acceptable.

In conclusion the improved dynamical model (gravity field coefficients, Love number k_2, manoeuvres and periodic accelerations) is exploited for a final single arc estimation that leads to the optimal orbit reconstruction (Figure 10).

8. Conclusions

Numerical simulations of the BepiColombo radio science experiment (MORE) show that a batch-sequential estimation is fully adequate to reach the mission requirements in orbit determination and gravity field reconstruction. The batch-sequential filter provides the advantages of a batch method (i.e. a-posteriori data processing and superior stability), while adding sequential updates of the dynamical model. In our approach, observation data are processed in three steps:

- Batch-sequential estimation (initialization of local and global parameters)
- Multi-arc estimation (improvement of the global parameters)
- Single arc estimation (trajectory reconstruction)

Position errors are consistently found below 10 m in the along and across track components, while much better accuracies are obtained for the radial component. The orbital reconstruction is therefore fully adequate to support the laser altimetric observations (accurate to 1 m in the nadir direction). In the remaining two components (along- and across-track) the orbit determination accuracies are significantly larger, but still compatible with the requirements of the libration experiment (2 arcsec for Mercury's librations in longitude). An improvement in the orbit determination is necessary for a measurement of physical librations below the arcsecond level.

Better results are expected if additional observations becomes available. In the current planning, BepiColombo's MPO will be tracked by two stations, namely ESA's 34 m antenna in Cebreros, supporting mission operations, and NASA's Deep Space Network antenna DSS 25 in Goldstone (California) for the radio science experiment. X-band Doppler data acquired at the Cebreros antenna may prove valuable for the estimation of the delta-Vs associated to desaturation manoeuvres, a major source of uncertainty in the orbital reconstruction.

Further investigations and refinements of the filter are certainly necessary. The simulation shows that the selection of the batch length depends on a trade-off between the estimation accuracy local and global parameters. A tuning of the batch duration according to some optimum criterion is desirable. A more realistic scenario where the spacecraft is tracked by two ground stations is expected to mitigate the effects of the wheel off-loading manoeuvres on the orbit determination. Finally, the evolution of the spacecraft state vector uncertainties still needs to be analysed. The propagation of the covariance matrix along the orbit is indeed required for an exhaustive assessment of the BepiColombo radio science experiment.

9. Acknowledgements

This work has been supported by the Italian Space Agency. We thank S. Asmar (JPL), A. Milani (Univ. of Pisa) and H.R. Schulte (Astrium) for helpful discussions.

10. References

[1] Schulz R., Benkhoff J., "BepiColombo: payload and mission updates", Advances in Space Research, Vol.38, 2006, pp. 572-577.

[2] Bertotti B., Iess L., Tortora P., "A test of general relativity using radio links with the Cassini spacecraft", Nature, Vol. 425, 2003, pp. 374-376.

[3] Tortora P., Iess L., Bordi J.J., Ekelund J.E., Roth D.C., "Precise Cassini navigation during solar conjunction through multifrequency plasma calibrations", J. Guidance Control Dyn., Vol. 27, 2004, pp. 251-257.

[4] Bertotti B., Comoretto G., Iess L. "Doppler tracking of spacecraft with multi-frequency links", Astronomy and Astrophysics, Vol. 269, 1993, pp. 608-616.

[5] Iess, L., Boscagli, G., "Advanced radio science instrumentation for the mission BepiColombo to Mercury", Planet. Space Sci., Vol. 49, 2001, pp. 1567-1608.

[6] Milani A., Rossi A., Vokrouhlick D., Villani D., Bonanno C., "Gravity field and rotation state of Mercury from the BepiColombo Radio Science Experiments", Planetary and Space Science, Vol. 49, 2001, pp. 1579-1596.

[7] Iafolla V., Nozzoli S., "Italian Spring Accelerometer (ISA) a high sensitive accelerometer for BepiColombo ESA Cornerstone", Planetary and Space Science, Vol. 49, 2001, pp. 1609-1617.

[8] Milani A., Gronchi G. F., "The theory of orbit determination", Cambridge University Press, 2010, pp. 311-314