COMPARATIVE ANALYSIS OF CARTESIAN AND CURVILINEAR CLOHESSY-WILTSHIRE EQUATIONS

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ABSTRACT

A precise representation of relative motion of two or more satellites is a key enabler for formation flying missions. The classical and widely used equations describing relative flight dynamics of objects in orbit are the well known Clohessy-Wiltshire (CW) equations in Cartesian coordinates. Representations in relative orbital elements are also often used nowadays, but will not be discussed in this paper. The classical Cartesian CW equations work well for close formations in near circular orbits, but are increasingly erroneous for larger formations, or formations in eccentric orbits. In this paper, CW equations in curvilinear coordinates are derived and a new method of calculating the initial conditions for curvilinear CW equations is introduced. A comparative analysis between the Cartesian and curvilinear representation of CW equations is provided. It is shown that a curvilinear Clohessy-Wiltshire representation is superior for along-track formations, in particular for larger separations.

As satellites move along arcs curvilinear coordinates offer a more natural way to describe relative motion of satellites. This may be illustrated by e.g. an along-track formation of two satellites, Chief and Deputy, which are in exactly the same orbit, but separation by 10 km in the along-track direction. In Cartesian coordinates, the Deputy will have non-zero position coordinates in both x- and y-direction. Propagating the Clohessy-Wiltshire equations with these coordinates as initial conditions results in non-zero predicted velocity components in x- and y-direction and thus a movement of the Deputy with respect to the Chief. In reality however, the position of the Deputy will remain fixed with respect to the Chief. This error can be traced back to the linearization of the nonlinear equations of motion in Cartesian coordinates. This error is greatly reduced if curvilinear coordinates are used instead of Cartesian coordinates.

Using a first order linearization of the relative motion in curvilinear coordinates, similarly as in the derivation of Cartesian CW equations, a set of differential equations in curvilinear coordinates can be derived. These differential equations are equivalent to the Cartesian CW equations and will hereafter be named curvilinear CW equations. The key difference between Cartesian and curvilinear CW equations lies in the determination of the initial conditions. The determination of initial conditions for the curvilinear CW equations was based on a new geometric approach.

An analysis has been made comparing relative trajectories using Cartesian and curvilinear CW equations. The relative motion of two satellites in near-circular Keplerian orbits was considered for periods of 120 seconds and one orbit. The 120 seconds time window was chosen because an onboard orbit determination process on a small satellite with limited computational power may provide a very accurate update only every 30-120 seconds, in between which an orbit prediction algorithm takes over. As a reference, the nonlinear equations of motion were numerically propagated using a Runge-Kutta 4 integration scheme. The analysis considered pure Keplerian motion.

Two different cases have been examined, introducing respectively an along-track separation and a combination of an eccentricity and an inclination difference between Chief and Deputy. The relative motion prediction for an along-track formation, in the case where curvilinear coordinates were used, is close to the reference and clearly superior to Cartesian coordinates.

The analysis of the case of two satellites separated in the along-track direction was examined in more detail. This case has been chosen because trailing formations are of prime interest for Earth Observation missions. In Figure 1 the position error of respectively Cartesian and curvilinear representations is shown. From Figure 1 it is evident that the position error for Cartesian coordinates rapidly increases to over 250 meters in y-direction in one full orbit. The relative motion prediction for the curvilinear representation was actually expected to be free of errors for an along-track formation. The (negligible) error can be attributed to the numerical accuracy to which the inertial position and velocity vectors were determined from the Keplerian elements and to the accuracy of the numerical integration of the non-linear equations of motion.

The analysis of the curvilinear CW equations was part of a larger research on satellite formation flying in the presence of sensor and actuator errors. The derived curvilinear coordinates have been successfully applied in the design of formation controllers for continuous formation keeping control of small satellites in a near circular low Earth orbit. The controllers have been implemented in a complex simulation environment of an onboard formation flying guidance, navigation and control system. When the relative motion representations were used for short-time predictions of relative motion, the representation using curvilinear coordinates also lead to a superior performance of control which in turn resulted in a reduction of propellant use.

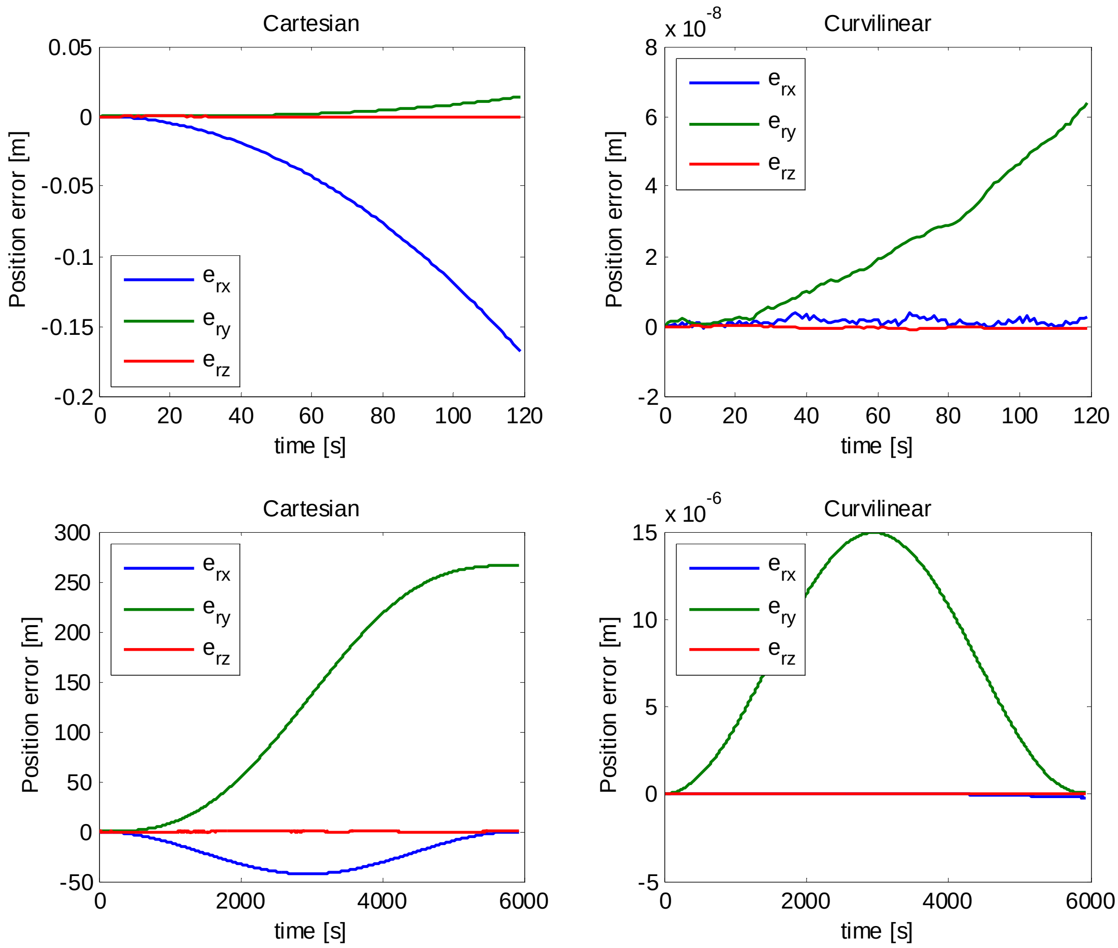

Figure 1. Short term (top) and long term (bottom) accuracy of relative motion prediction using Cartesian (left) and curvilinear (right) representation of relative motion