SUN INFLUENCE ON TWO-IMPULSIVE EARTH-TO-MOON TRANSFERS

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Abstract: *In the present work, the influence of the Sun on the fuel consumption of transfers from circular low Earth orbits (LEOs) to circular low Moon orbits (LMOs) is investigated. The class of two impulse trajectories is considered: a first accelerating velocity impulse tangential to the space vehicle velocity relative to Earth is applied at a circular low Earth orbit and a second braking velocity impulse tangential to the space vehicle velocity relative to Moon is applied at a circular low Moon orbit. The fuel consumption is equivalent to the total characteristic velocity which is defined by the arithmetic sum of velocity changes. Local optimal transfers are calculated through two different approaches: inner transfers and Belbruno-Miller transfers. In both cases, the optimization problem is solved by means of an algorithm based on gradient method in conjunction with Newton-Raphson method.*

Keywords: *Bicircular problem, local optimal trajectories, inner transfers, Belbruno-Miller transfers.*

1 Introduction

Since the remarkable success of the *Hiten* mission, Belbruno-Miller transfers have been considered an efficient way to explore the gravity of the Sun to reduce fuel consumption in lunar missions [1]. Using these type of transfers it is possible to safe up to 25% of on-board fuel when compared with Hohmann transfers, but at cost of a much higher time of flight[2].

In this work, the dynamical model used is the planar bicircular restricted four-body problem [4]. Local optimal transfers are calculated through two different approaches: inner transfers and Belbruno-Miller transfers. In both cases, the optimization problem is solved by means of an algorithm based on gradient method (Miele et al. [6]) in conjunction with Newton-Raphson method (Stoer and Bulirsch [7]).

Inner transfers and Belbruno-Miller transfers are compared in terms of fuel consumption and time of flight, considering several relative positions of the Sun with respect to the Earth-Moon axis.

Finally, in comparison to optimal local trajectories calculated using the planar restricted three-body problem (PCR3PB) it is shown that the inner transfers have a better performance when the presence of the Sun is included in the dynamical model. Recall that Belbruno-Miller transfers are designed only in the planar bicircular restricted four-body problem.

2 Problem formulation

Bicircular model

In the planar bicircular restricted four-body problem the Earth and the Moon revolve in circular orbits around their center of mass, and the Earth-Moon barycenter moves in a circular orbit around the center of mass of the Sun-Earth-Moon system [4]. A spacecraft of infinitesimal mass moves under the gravitational attraction of the Earth, Moon, and Sun. The orbits of the four bodies are in the same plane.

Let μ_E be the Earth gravitational parameter, μ_M the Moon gravitational parameter and μ_S the Sun gravitational parameter. Let $\mathbf{r}_P = (x_P, y_P)$ be the position of the spacecraft with respect to the barycenter (B) of the Earth-Moon system. The respective distance from B to Earth, Moon and Sun are denoted by a_E, a_M and a_S. The coordinates of Earth, Moon and Sun with respect to B are:

$$\begin{aligned} x_E &= -a_E \cos(\omega t + \theta_0), \quad x_M = a_M \cos(\omega t + \theta_0), \quad x_S = a_S \cos(\omega_S t + \theta_{S0}), \\ y_E &= -a_E \sin(\omega t + \theta_0), \quad y_M = a_M \sin(\omega t + \theta_0), \quad y_S = a_S \sin(\omega_S t + \theta_{S0}), \end{aligned} \tag{1}$$

where ω is the angular velocity of the Earth and Moon around B; θ_0 is the respective initial phase; ω_S is the angular velocity of the Sun around B; θ_{S0} is the initial phase of the Sun.

In the barycenter Earth-Moon reference frame, the equations of motion are given by:

$$\begin{aligned} \ddot{x}_P &= -\mu_E \frac{(x_P - x_E)}{r_{EP}^3} - \mu_M \frac{(x_P - x_M)}{r_{MP}^3} - \mu_S \frac{(x_P - x_S)}{r_{SP}^3} - \frac{\mu_S}{a_S^2} \cos(\omega_S t + \theta_{S0}), \\ \ddot{y}_P &= -\mu_E \frac{(y_P - y_E)}{r_{EP}^3} - \mu_M \frac{(y_P - y_M)}{r_{MP}^3} - \mu_S \frac{(y_P - y_S)}{r_{SP}^3} - \frac{\mu_S}{a_S^2} \sin(\omega_S t + \theta_{S0}), \end{aligned} \tag{2}$$

where r_{EP}, r_{MP} and r_{SP} denote the distance from de spacecraft to Earth, Moon and Sun, respectively. See Figure 1.

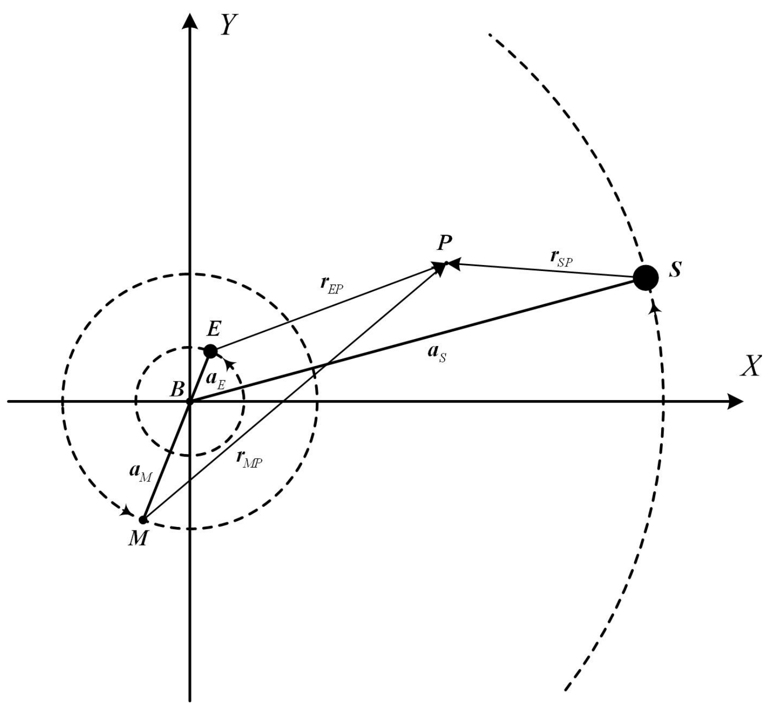


Figure 1: Bicircular model.

The initial conditions of the system of differential equations (2) correspond to the position and velocity vectors of the space vehicle after the application of the first impulse. The initial conditions ($t_0 = 0$) can be written as follows:

$$
\begin{aligned}
x_P(0) &= r_{EP}(0)\cos\theta_{EP}(0) + x_E(0),\\
y_P(0) &= r_{EP}(0)\sin\theta_{EP}(0) + y_E(0),\\
\dot{x}_P(0) &= -\left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_1\right]\sin\theta_{EP}(0) + \dot{x}_E(0),\\
\dot{y}_P(0) &= \left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_1\right]\cos\theta_{EP}(0) + \dot{y}_E(0),
\end{aligned}
\tag{3}
$$

where Δv_1 is the velocity change at the first impulse; $\theta_{EP}(t)$ is the angle which $\mathbf{r}_P$ forms with $\mathbf{r}_E$. It should be noted that $\mathbf{r}_{EP}(0)$ and $\mathbf{v}_{EP}(0)$ are orthogonal, because the impulse is applied tangentially to LEO, assumed circular.

The final conditions of the system of differential equations (2) correspond to the position and velocity vectors of the space vehicle before the application of the second impulse. The final conditions ($t_f = T$) can be put in the form:

$$
\begin{aligned}
&(x_P(T) - x_M(T))^2 + (y_P(T) - y_M(T))^2 = (r_{MP}(T))^2,\\
&(\dot{x}_P(T) - \dot{x}_M(T))^2 + (\dot{y}_P(T) - \dot{y}_M(T))^2 = \left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_2\right]^2,\\
&(x_P(T) - x_M(T))(\dot{y}_P(T) - \dot{y}_M(T)) - (y_P(T) - y_M(T))(\dot{x}_P(T) - \dot{x}_M(T))\\
&= \mp r_{MP}(T)\left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_2\right].
\end{aligned}
\tag{4}
$$

where Δv_2 is the velocity change at the second impulse. The upper sign refers to clockwise arrival to LMO and the lower sign refers to counterclockwise to LMO.

Local optimal trajectories

Local optimal inner transfers and Belbruno-Miller transfers are designed as follows. For a fixed $\theta_{S0}(0)$, the problem defined by equations (1)-(4) involves four unknowns Δv_1, Δv_2, T and $\theta_{EP}(0)$ that must be determined in order to satisfy the three final conditions (4). So, the problem has one degree of freedom and a minimization of the fuel consumption can be made. The optimization problem can be formulated as follows: determine Δv_1, Δv_2, T and $\theta_{EP}(0)$ which satisfy the final constraints (4) and minimize the total characteristic velocity $\Delta v_{Total} = \Delta v_1 + \Delta v_2$.

A similar optimization problem, considering a simplified model of the planar circular three body problem, was solved by Miele and Mancuso [5] using the sequential gradient-restoration algorithm for mathematical programming problems developed by Miele et al. [6]. In this paper, the optimization problem described above is solved by means of an algorithm based on gradient method [6] in conjunction with Newton-Raphson method [7]. The angle $\theta_{EP}(0)$ has been chosen as the iteration variable in the gradient phase with Δv_1, Δv_2 and T calculated through Newton-Raphson method.

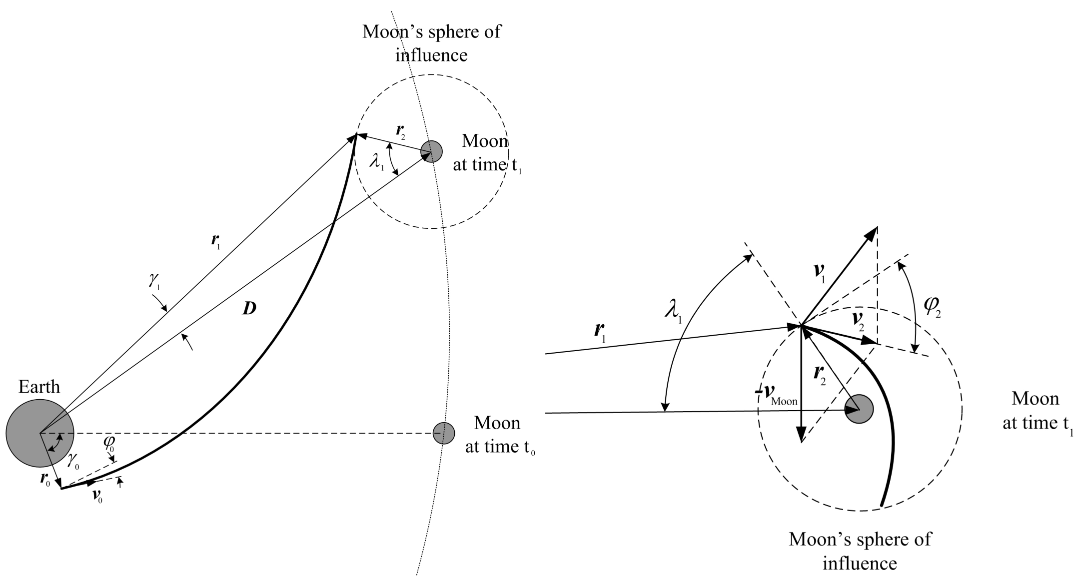
Inner transfers

The initial guess to determine local optimal inner transfers using the bicircular model is given by an optimized version of the patched-conic approximation [3], briefly described in the next paragraphs.

The well-known patched-conic approximation has two distinct phases: geocentric and selenocentric trajectories. The geocentric phase corresponds to the portion of the trajectory which begins at the point of application of the first impulse and extends to the point of entering the Moon's sphere of influence. The selenocentric phase corresponds to the portion of trajectory in the Moon's sphere of influence and ends at the point of application of the second impulse. In each one of these phases, the space vehicle is under the gravitational attraction of only one body, Earth or Moon (see Fig. 2).

In the patched-conic approximation, an Earth-Moon trajectory is then completely specified by four quantities: r_0 - radius of circular LEO; v_0 - velocity of the space vehicle at the point of application of the first impulse after the application of the impulse; φ_0 - flight path angle at the point of application of the first impulse and γ_0 - phase angle at departure. These quantities must be determined such that the space vehicle is injected into a LMO with specified altitude after the application of the second impulse. It is particularly convenient to replace γ_0 by the angle λ_1 which specifies the point at which the geocentric trajectory crosses the Moon's sphere of influence.

The optimization problem based on patched-conic approximation can be formulated as follows: for specified initial parameters r_0 and $\varphi_0 = 0$ (the impulse is applied tangentially to the space vehicle velocity relative to Earth) determine v_0 and λ_1 to minimize the total characteristic velocity Δv_{Total}, such that the final condition $r_f = r_{MP}$ is satisfied. This problem is solved using the same method described in the preceding section. The angle λ_1 has been chosen as the iteration variable in the gradient phase with v_0 calculated through Newton-Raphson method.



(a) Geocentric phase.

(b) Selenocentric phase.

Figure 2: Patched conic approximation.

Belbruno-Miller transfers

The initial guess to determine local optimal Belbruno-Miller transfers is described as follows. For fixed $\theta_{EP}(0)$ and θ_{S0}, find the values of Δv_1, Δv_2, and T, such that solutions of the system given by (1)-(2) satisty the boundary conditions (3)-(4), with an extra constraint: the spacecraft is sent outside the Earth's sphere of influence. Given the initial conditions with respect to the Earth, the Newton-Raphson method is used to perform a target search to the final desired LMO. The first shoot to solve the boundary value problem comes from the elliptic approximation of the trajectory joining the LEO and the apogee where the spacecraft returns to the Earth-Moon system (see Fig. 3). So, an impulse is applied to inject the spacecraft into a high eccentric orbit which reaches an apogee between 1.0 and 1.5 million km. At this position the spacecraft starts to fall back to the Earth-Moon system and is eventually captured by gravitational field of the Moon.

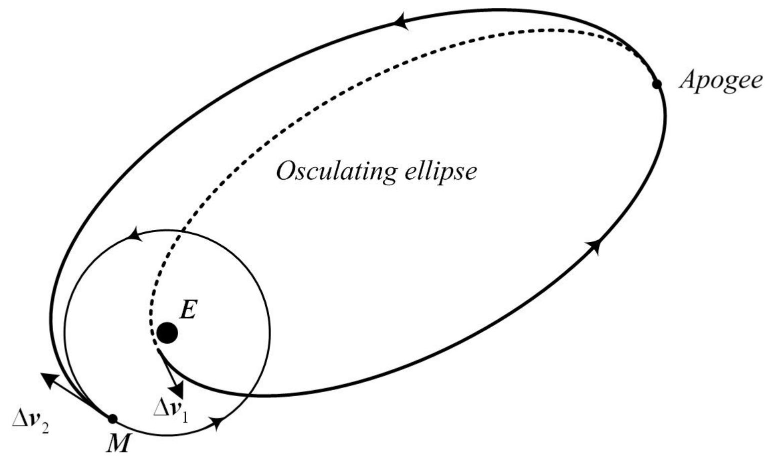


Figure 3: Belbruno-Miller trajectory.

3 Results

The results for lunar missions using local optimal trajectories are presented in Tab. 1, for three different initial phase of the Sun, θ_{S0}. The major parameters are: Δv_1, Δv_2, Δv_{Total}, T, $\theta_{EP}(0)$, E_K and $\theta_S(0)$. E_K is the spacecraft Keplerian energy with respect to the Moon at $t = T$, before the application of the second impulse. For $\Delta v_2^* = 0.6765$ km/s, $E_K = 0$; i. e., the lunar approximation trajectory is parabolic. All missions consider counterclockwise LEO and LMO. The following data are used:

$R_E = 6378\,\text{km}$ (Earth radius);
$R_M = 1738\,\text{km}$ (Moon radius);
$\mu_E = 3.986 \times 10^5\,\text{km}^3/\text{s}^2$;
$\mu_M = 4.903 \times 10^5\,\text{km}^3/\text{s}^2$;
$\mu_S = 1.327 \times 10^{11}\,\text{km}^3/\text{s}^2$;
$a_E = 4.678 \times 10^3\,\text{km}$;
$a_M = 3.803 \times 10^5\,\text{km}$;
$a_S = 1.497 \times 10^8\,\text{km}$;
$\omega = 2.659 \times 10^{-6}\,\text{rad/s}$;
$\omega_S = 1.989 \times 10^{-7}\,\text{rad/s}$;
$h_{LEO} = 167\,\text{km}$ (LEO altitude);
$h_{LMO} = 100\,\text{km}$ (LMO altitude);
$\theta_0 = 0\,\text{deg}$.

For $m_S = 0$, the equations (1)-(2) correspond to the classical planar circular restricted three body problem (PCR3BP). In this case, inner transfers can be calculated using the same method described previously. Belbruno-Miller maneuvers are not considered, since they are designed considering the presence of the Sun. The results are presented in Tab. 2.

From the results in Tab. 1 and Tab. 2, it can be concluded that:

Table 1: Lunar mission using the bicircular problem, major parameters.

Transfer	Δv_1 (km/s)	Δv_2 (km/s)	Δv_{Total} (km/s)	T (days)	$\theta_{EP}(0)$ (deg)	E_K (km^2/s^2)	$\theta_S(0)$ (deg)
	3.2003	0.6576	3.8579	86.254	59.734	-0.0435	70.0
B-M	3.2002	0.6571	3.8573	87.254	70.897	-0.0446	80.0
	3.2000	0.6662	3.8662	86.765	78.446	-0.0238	85.0
	3.1383	0.8123	3.9506	4.578	-116.642	0.3228	70.0
	3.1382	0.8050	3.9432	14.452	13.621	0.3050	
	3.1321	0.7714	3.9035	32.004	231.644	0.2236	
	3.1239	0.7264	3.8503	58.466	223.804	0.1164	
	3.1383	0.8118	3.9501	4.587	-116.560	0.3216	80.0
Inner	3.1382	0.8067	3.9449	14.440	13.523	0.3092	
transfer	3.1322	0.7695	3.9017	32.004	232.251	0.2191	
	3.1240	0.7295	3.8535	58.463	224.237	0.1238	
	3.1384	0.8116	3.9500	4.591	-116.524	0.3211	85.0
	3.1382	0.8076	3.9458	14.432	13.431	0.3114	
	3.1323	0.7692	3.9015	32.005	232.584	0.2184	
	3.1241	0.7318	3.8559	58.467	224.416	0.1292	

Table 2: Lunar mission using the PCR3BP, major parameters.

Transfer	Δv_1 (km/s)	Δv_2 (km/s)	Δv_{Total} (km/s)	T (days)	$\theta_{EP}(0)$ (deg)	E_K (km^2/s^2)
	3.1386	0.8133	3.9519	4.579	-116.382	0.3253
Inner	3.1377	0.8096	3.9473	14.317	12.259	0.3162
transfer	3.1323	0.7852	3.9175	31.906	232.447	0.2569
	3.1247	0.7504	3.8751	58.599	224.408	0.1734

1. All Belbruno-Miller transfers have quite similar first impulse and negative E_K, corresponding to $\Delta v_2 < \Delta v_2^*$. This means that the missions have elliptic lunar approximation trajectories.

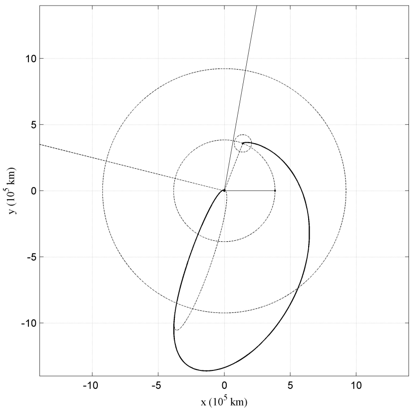
2. All inner transfers have positive E_K, corresponding to $\Delta v_2 > \Delta v_2^*$. This means that the missions have hyperbolic lunar approximation trajectories.

3. For inner transfers, the time of flight depends mainly on $\theta_{EP}(0)$.

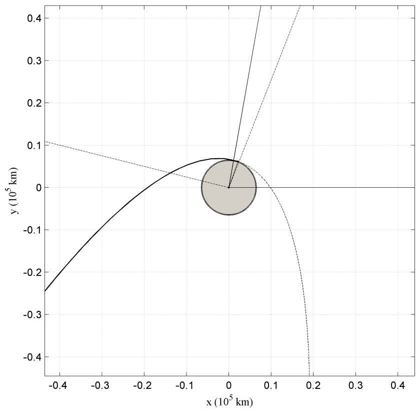
4. For inner transfers with time of flight about 58.5 days, fuel can be saved in comparison to Belbruno-Miller transfers.

5. The presence of the Sun causes a small perturbation on inner transfers with a better performance in terms of fuel consumption for correspondent time of flight.

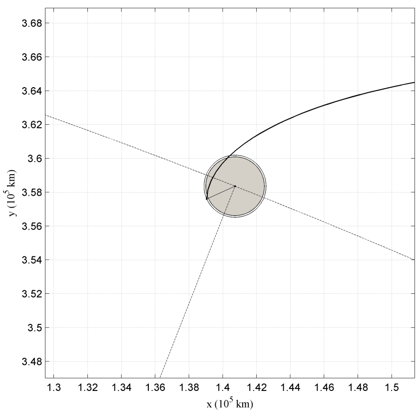
The Figs. 4-7 illustrate some missions. Note that the Moon is represented at its final position and there is no collision between the spacecraft and the Moon surface. All trajectories are plotted with respect a reference frame centered in the Earth.



(a) Earth-Moon trajectory.

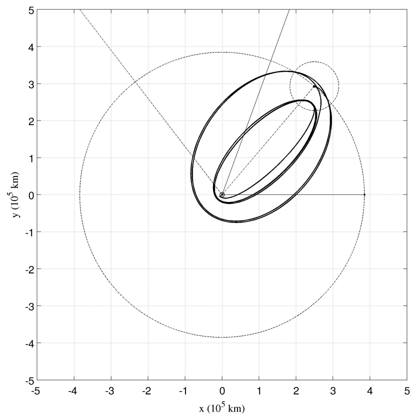


(b) LEO departure.

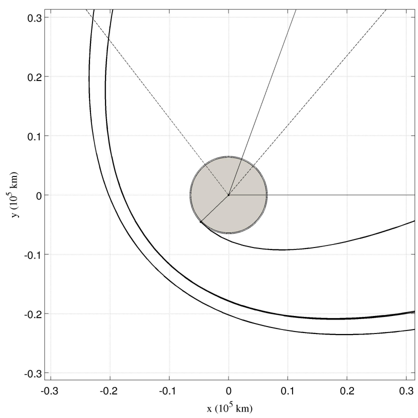


(c) LMO arrival.

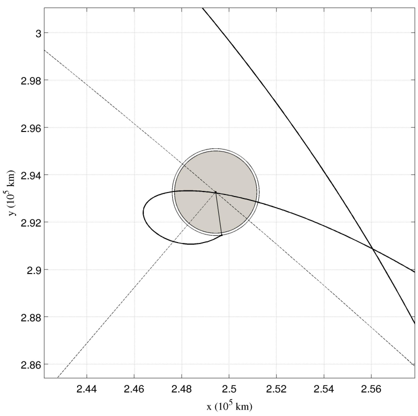
Figure 4: B-M transfer, where $\Delta v_1 = 3.2002$ (km/s); $\Delta v_2 = 0.6571$ (km/s); $T = 87.254$ (days); $\theta_{EP}(0) = 70.897$ (deg); $\theta_S(0) = 80.0$ (deg).



(a) Earth-Moon trajectory.

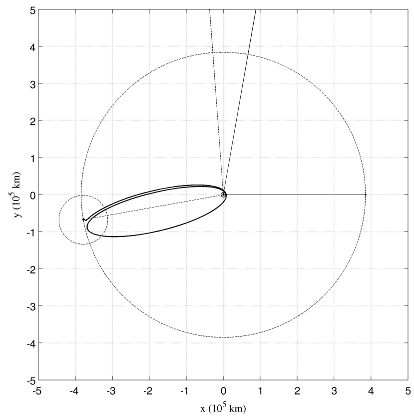


(b) LEO departure.

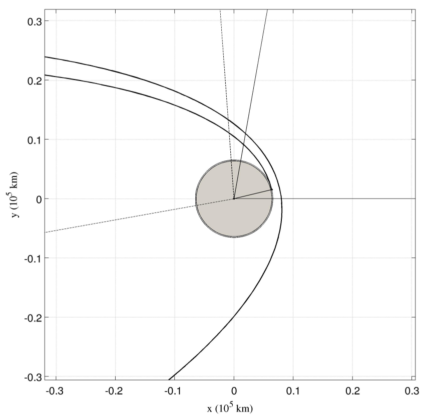


(c) LMO arrival.

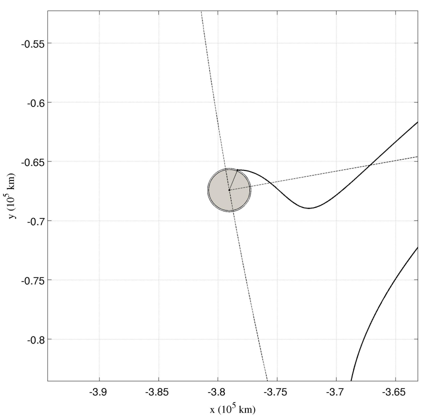
Figure 5: Inner transfer, where $\Delta v_1 = 3.1239$ (km/s); $\Delta v_2 = 0.7264$ (km/s); $T = 58.466$ (days); $\theta_{EP}(0) = 223.804$ (deg); $\theta_S(0) = 70.0$ (deg).



(a) Earth-Moon trajectory.

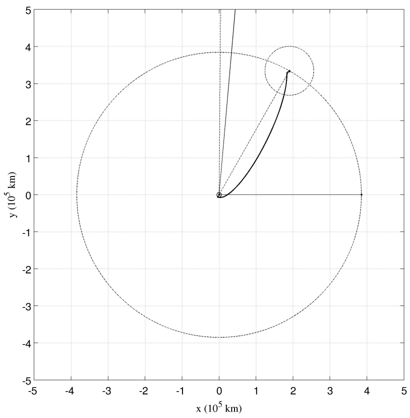


(b) LEO departure.

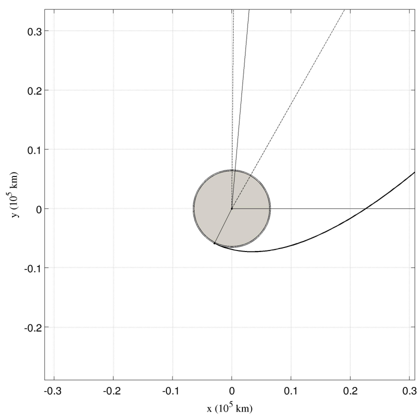


(c) LMO arrival.

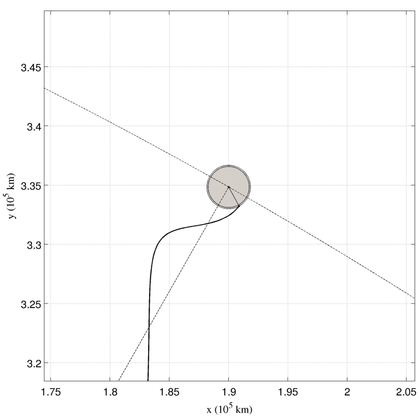
Figure 6: Inner transfer, where $\Delta v_1 = 3.1382$ (km/s); $\Delta v_2 = 0.8067$ (km/s); $T = 14.440$ (days); $\theta_{EP}(0) = 13.523$ (deg); $\theta_S(0) = 80.0$ (deg).



(a) Earth-Moon trajectory.



(b) LEO departure.



(c) LMO arrival.

Figure 7: Inner transfer, where $\Delta v_1 = 3.1384$ (km/s); $\Delta v_2 = 0.8116$ (km/s); $T = 4.591$ (days); $\theta_{EP}(0) = -116.524$ (deg); $\theta_S(0) = 85.0$ (deg).

4 Conclusions

In this work local optimal transfers are calculated for lunar missions and a comparison between the performance of Belbruno-Miller transfers and inner transfers is presented. In both approaches, the optimization problem has been calculated using a gradient algorithm in conjunction with Newton-Raphson method. It was shown that inner transfers with time of flight about 58.5 days have a better performance in comparison to Belbruno-Miller transfers. Finally, it is verified that the presence of the Sun improves slightly the fuel consumption for inner transfers with correspondent time of flight.

Acknowledgements

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