SPIN STABILIZED SATELLITE'S ATTITUDE ANALYTICAL PREDICTION

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Abstract: *An analytical approach for spin stabilized attitude propagation is presented, considering the coupled effect of the aerodynamic torque and the gravity gradient torque. A spherical coordination system fixed in the satellite is used to locate the satellite spin axis in relation to the terrestrial equatorial system. The spin axis direction is specified by its right ascension and the declination angles and the equation of motion are described by these two angles and the magnitude of the spin velocity. An analytical averaging method is applied to obtain the mean torques over an orbital period. To compute the average components of both aerodynamic torque and the gravity gradient torque in the satellite body frame reference system, an average time in the fast varying orbit element, the mean anomaly, is utilized. Afterwards, the inclusion of such torques on the rotational motion differential equations of spin stabilized satellites yields conditions to derive an analytical solution. The pointing deviation evolution, that is, the deviation between the actual spin axis and the computed spin axis, is also availed. In order to validate the analytical approach, the theory developed has been applied for spin stabilized Brazilian satellite SCD1, which are quite appropriated for verification and comparison of the data generated and processed by the Satellite Control Center of the Brazil National Research Institute (INPE). Numerical simulations performed with data of Brazilian Satellite SCD1 show the period that the analytical solution can be used to the attitude propagation, within the dispersion range of the attitude determination system performance of Satellite Control Center of the Brazilian Research Institute.*

Keywords: *spin axis, spin velocity, external torques, analytical propagation, pointing deviation.*

1 Introduction

The goal of this paper is to analyses the rotational motion dynamics of spin stabilized Earth artificial satellites, through derivation of an analytical attitude prediction. Emphasis is placed on modeling the aerodynamic torque and gravity gradient torque, as well as their coupled effects on the satellite spin velocity and space orientation.

A spherical coordinate system fixed in the satellite is used to locate the spin axis of the satellite in relation to the terrestrial equatorial system. The directions of the spin axis are specified by the right ascension (α) and the declination (δ) as represented in the Fig. 1. In this paper the satellite body frame reference system is called *satellite system*, their unit vectors are ($\hat{i},\ \hat{j}, \hat{k}$) and the axis z is along the direction of the spin velocity vector.

The gravity gradient torque is created by the difference of the Earth gravity force direction and intensity acting on each satellite mass element [5,8]. This torque is inversely proportional to the cube of the satellite geocentric distance. Therefore it decreases when the altitude increases. The aerodynamic torque is created by the interactions of rarefied air particles with the satellite surface [5,8]. This torque is predominant in satellites with low altitude, because it depends on the quantity of air molecules in the Earth atmosphere. Calculation of aerodynamic torques for realistic spacecraft is not very accurate because of existing uncertainties in the atmospheric density and in drag coefficient. In this paper TD-88 model is used to describe the atmospheric density.

The pointing deviation evolution, that is, the deviation between the actual spin axis and the computed spin axis, is also availed in this paper. In order to validate the analytical approach, the theory developed has been applied for spin stabilized Brazilian satellite SCD1, which are quite appropriated for verification and comparison of the data generated and processed by the Satellite Control Center (CCS) of the Brazil National Research Institute (INPE).

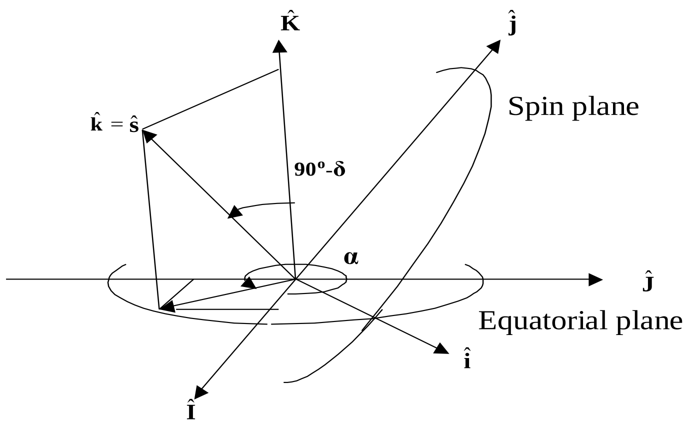

Figure 1 - Orientation of the spin axis ($\hat{s}$): Equatorial System ($\hat{I}, \hat{J}, \hat{K}$), satellite body frame reference system ($\hat{i},\ \hat{j}, \hat{k}$), right ascension ($\alpha$) and declination ($\delta$) of the spin axis.

It is assumed two approaches to examine the influence of these two external actuating during the evolution of rotational motion of the satellite. In the first approach the attitude and orbit data are updated every 24 hours. In the second approach the computed attitude and orbit data aren't updated in order to determine the validate period of the analytical solution. In all numerical simulation the orbital elements are updated taking in account the main influence of the Earth oblateness.

An analytical averaging method is applied to obtain the mean torques over an orbital period. To compute the average components of both aerodynamic torque and the gravity gradient torque in the satellite body frame reference system, an average time in the fast varying orbit element, the mean anomaly, is utilized. This approach involves several rotation matrices, which are dependent on the orbit elements, right ascension and declination of the satellite spin axis. Afterwards, the inclusion of such torques on the rotational motion differential equations of spin stabilized satellites yields conditions to derive an analytical solution.

Numerical simulations performed with data of Brazilian Satellites SCD1 show the period that the analytical solution can be used to the attitude propagation, within the dispersion range of the attitude determination system performance of Satellite Control Center of the Brazilian Research Institute.

2. Gravity Gradient Torque Model

The gravity gradient torque [3,7,9] for a spin stabilized satellite in a satellite system ($\hat{i},\ \hat{j}, \hat{k}$) can be expressed by:

$$\vec{N}_g = N_{gx}\hat{i} + N_{gy}\hat{j} + N_{gz}\hat{k}, \tag{1}$$

with

$$N_{gx} = 3\frac{\mu}{r^3}\left[a_{21}\,a_{31}\left(I_z - I_y\right)\cos\theta - a_{11}\,a_{31}\left(I_x - I_z\right)sen\theta\right],$$

$$N_{gy} = 3\frac{\mu}{r^3}\left[a_{21}\,a_{31}\left(I_z - I_y\right)sen\theta + a_{11}\,a_{31}\left(I_x - I_z\right)\cos\theta\right], \qquad (2)$$

$$N_{gz} = 3\frac{\mu}{r^3}\left[a_{11}\,a_{21}\left(I_y - I_x\right)\right],$$

where μ (3.986 x 10^{14} m^3/s^2) is the Earth gravitational parameter, r is the satellite geocentric distance, a_{11}, a_{21} and a_{31} are the direction cosines which relate the orbital system and the satellite fixed system (the latter being associated with the principal moments of inertia axes of the satellite), I_x, I_y, I_z are the Principal Moments of Inertia of the satellite and θ is the angle between the satellite principal axis of inertia x and the satellite axis x, defined in each instance by the product of the spin velocity w and the time t. The elements a_{11}, a_{21} and a_{31} depend on the orbital elements (orbit inclination, true anomaly, longitude of the ascending node and argument of the perigee), the angle θ and the right ascension and declination of the spin axis [3,7]. Equation 2 shows that this torque decreases with the cube of the altitude and depends on the shape, dimension and mass distribution of the satellite. If the satellite has a uniform mass distribution and the principal moments of inertia are equal, this torque vanishes.

3. Aerodynamic Torque Model

In this paper it will adopt the following model to represent the aerodynamic torque [2,7,9]:

$$\overrightarrow{N_A} = \overrightarrow{m_e} \times \vec{D}, \qquad (3)$$

Where $\overrightarrow{\boldsymbol{m_e}}$ is the position vector between the center of pressure and the center of mass of the satellite, the $\vec{\boldsymbol{D}}$ is the drag force (the influence of the lift force in the aerodynamic torque is negligenciable) and in the satellite system it is given by [2]:

$$\vec{D} = D_x\,\hat{i} + D_y\,\hat{j} + D_z\,\hat{k}\,, \qquad (4)$$

with

$$D_x = -D\left[a_{11}\cos(\gamma_S) + a_{21}sen(\gamma_S)\right], \qquad (5)$$

$$D_y = -D\left[a_{12}\cos(\gamma_S) + a_{22}sen(\gamma_S)\right], \qquad (6)$$

$$D_z = -D\left[a_{13}\cos(\gamma_S) + a_{32}sen(\gamma_S)\right], \qquad (7)$$

$$D = \frac{1}{2}\rho v^2\, S\, C_D \;\; . \qquad (8)$$

where ρ is the local density, v represents the magnitude of the satellite's velocity relative to the atmosphere, S is a reference section area of the satellite, C_D is the Drag Coefficient, γ_S is the angle between the position vector and the orbital velocity vector and a_{ij} , i=1,2,3, j=1,2, are the direction cosines which relate the orbital system and the satellite system and depend on the orbital elements, right ascension and declination of the spin axis and the angle γ_S [2].

Then by substituting the Eq. 3 in Eq. 2, the aerodynamic torque in the satellite system is given by:

$$\vec{N}_A = \left[D_z m_{ey} - D_y m_{ez}\right]\hat{i} + \left[D_x m_{ez} - D_z m_{ex}\right]\hat{j} + \left[D_y\, m_{ex} - D_x\, m_{ey}\right]\hat{k}\ . \tag{9}$$

In order to estimate the influence of the aerodynamic torque magnitude in the rotational motion, in this paper some simplifications are done and the thermosphere model TD-88 is used for the atmospheric density [6,9]. The velocity *v* is assumed equal to the orbit velocity and the drag coefficient is fixed.

4. Mean Gravity Gradient and Aerodynamic Torques

In order to obtain the mean gravity gradient and aerodynamic torques, it is necessary to integrate the instantaneous torques $\vec{N}_g$ and $\vec{N}_a$, given in Eqs. (1) and (9) respectively, over one orbital period T:

$$\vec{N}_{gm} = \frac{1}{T}\int_{t_i}^{t_i+T} \vec{N}_g\, dt \qquad \text{and} \qquad \vec{N}_{am} = \frac{1}{T}\int_{t_i}^{t_i+T} \vec{N}_a\, dt\ , \tag{10}$$

where: t is the time, t_i the initial time and T the orbital period. Changing the independent variable to the fast varying true anomaly, the mean gravity gradient and aerodynamic torque can be obtained by [9,10]:

$$\vec{N}_{gm} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi} \vec{N}_g \frac{r^2}{h}\, d\upsilon \qquad \text{and} \qquad \vec{N}_{am} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi} \vec{N}_a \frac{r^2}{h}\, d\upsilon\ , \tag{11}$$

where υ_i is the true anomaly at instant t_i, r is the geocentric distance and h is the specific angular moment of orbit.

To evaluate the integrals of Eqs.(11) we can use spherical trigonometry properties, rotation matrix associated with the references systems and the elliptic expansions of the true anomaly in terms of the mean anomaly [10,11], including terms up to the first order in the eccentricity (*e*). Without losing generality, for the sake of simplification of the integrals, we consider the initial time for integration equal to the instant that the satellite passes through perigee. After extensive but simple algebraic developments, the mean gravity gradient and aerodynamic torques can be expressed by [2,3]:

$$\vec{N}_{gm} = N_{gxm}\,\hat{i} + N_{gym}\,\hat{j} + N_{gzm}\,\hat{k} \qquad \text{and} \qquad \vec{N}_{am} = N_{axm}\hat{i} + N_{aym}\hat{j} + N_{azm}\,\hat{k} \tag{12}$$

with

$$\begin{aligned} N_{axm} &= D_{zm} m_{ey} - D_{ym} m_{ez}\ , \\ N_{aym} &= D_{xm} m_{ez} - D_{zm} m_{ex}\ , \\ N_{azm} &= D_{ym} m_{ex} - D_{xm} m_{ey}\ . \end{aligned} \tag{13}$$

$$\begin{aligned} D_{xm} &= \Psi \cos(\omega)[\cos i \cos(\Omega-\alpha) + sen(\Omega-\alpha)], \\ D_{ym} &= \Psi \cos(\omega)[-sen\delta \cos(\Omega-\alpha) + \cos i\, sen\delta\, sen(\Omega-\alpha) + sen i \cos\delta], \\ D_{zm} &= \Psi \cos(\omega)[\cos\delta \cos(\Omega-\alpha) + \cos i \cos\delta\, sen(\alpha-\Omega) + sen i\, sen\delta], \end{aligned} \tag{14}$$

$$\Psi = -\left(\frac{e}{4a^{3/2}}\right)\rho\ SC_D \mu\, p^{1/2} \quad , \tag{15}$$

and N_{gxm}, N_{gym}, N_{gzm} are presented in reference [1]. It t is important to observe that the mean components of these torques depend on the attitude angles $(\delta,\ \alpha)$ and the orbital elements (orbital major semi-axis - a, orbital eccentricity - e, longitude of ascending node - Ω, argument of perigee - ω, orbital inclination - i) .

4. Analytical Solution for the Equations of Rotational Motion

The variations of the spin velocity, the declination and the ascension right of the spin axis for spin stabilized artificial satellites are given by the Euler equations in spherical coordinates [10]:

$$\frac{dW}{dt} = \frac{1}{I_z} N_z , \tag{16}$$

$$\frac{d\delta}{dt} = \frac{1}{I_z\ W} N_y , \tag{17}$$

$$\frac{d\alpha}{dt} = \frac{1}{I_z\ W\ Cos\,\delta}\ N_x , \tag{18}$$

where I_z is the moment of inertia along the spin axis, N_x, N_y, N_z are the components of the external torques in the satellite system and here given by the sum of the gravity gradient torque and the aerodynamic torque.

By substituting $\vec{N}_g$ and $\vec{N}_a$, given in Eqs. (12), in equations Eqs. (16), (17) and (18), the equations of motion are:

$$\frac{dW}{dt} = \frac{N_{gzm} + N_{azm}}{I_z} , \tag{19}$$

$$\frac{d\delta}{dt} = \frac{N_{gym} + N_{aym}}{I_z W} , \tag{20}$$

$$\frac{d\alpha}{dt} = \frac{N_{gxm} + N_{axm}}{I_z W \cos\delta} . \tag{21}$$

The differential equations of Eqs. (19) - (21) can be integrated assuming that the orbital elements (I, Ω, w) are held constant over one orbital period, and that all other terms on right-hand side of equations are equal to initial values.

For one orbit period the analytical solutions of Eqs. (19) - (21) for the spin velocity, declination and right ascension of spin axis respectively can simply be expressed as:

$$W = k_1 t + W_0 , \qquad \delta = k_2 t + \delta_0 \quad \text{and} \quad \alpha = k_3 t + \alpha_0 \tag{22}$$

with:

$$k_1 = \frac{N_{gzm} + N_{azm}}{I_z}, \qquad k_2 = \frac{N_{gym} + N_{aym}}{I_z\ W_o} \qquad \text{and} \qquad k_3 = \frac{N_{gxm} + N_{axm}}{I_z\ W_o \cos\delta_0}, \tag{23}$$

where W_0, δ_0 and α_0 are the initial values for spin velocity, declination and right ascension of Spin Axis.

The solutions presented in the Eqs. (21), (22) and (23), for the spin velocity magnitude, declination and right ascension of the spin axis respectively, are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth oblateness. With this approach, the analytical theory will be close to the real attitude behavior of the satellite.

5. Applications

The theory developed has been applied to the spin stabilized Brazilian Satellite SCD1 for verification and comparison of the theory against data generated by the Satellite Control Center (SCC) of INPE. Operationally, SCC attitude determination comprises: sensors data pre-processing, preliminary attitude determination and fine attitude determination. The pre-processing is applied to each set of data of the attitude sensors collected from every satellite that pass over the ground station. Afterwards, from the whole pre-processed data, the preliminary attitude determination produces estimative to the spin velocity vector from every satellite that pass over a given ground station. The fine attitude determination takes (one week) a set of spin velocity vector and estimates dynamical parameters (spin velocity vector, residual magnetic moment and Foucault parameter). Those parameters are further used in the attitude propagation to predict the need of attitude corrections. Over the test period there isn’t attitude corrections. In all numerical simulation the orbital elements are updated, taking into account the main influences of the Earth oblateness.

For the tests it is important to observe the deviation between the real attitude data supplied by INPE and the computed attitude. for each satellite. Here this deviation is called pointing deviation and given by the angle θ between the actual spin axis $\hat{k}$ and the computed spin axis $\widehat{k_c}$. It can be computed by [10,11]:

$$\cos\theta = \hat{k} \cdot \widehat{k_c}\ , \tag{24}$$

where ($\cdot$) indicates the scalar product. The unit vectors $\hat{k}$ and $\widehat{k_c}$ can be obtained using the right ascension and declination of the spin axis as:

$$\hat{k} = \cos\alpha_{INPE} \cos\delta_{INPE}\ \hat{I} + \sin\alpha_{INPE} \cos\ \delta_{INPE}\ \hat{J} + \sin\delta_{INPE}\ \ \hat{K}, \tag{25}$$

$$\widehat{k_c} = cos\ \alpha_c\ cos\ \delta_C \hat{I} + sin\ \alpha_c\ cos\ \delta_C\ \hat{J} + sin\ \delta_C\ \ \hat{K}, \tag{26}$$

with α_{INPE} and δ_{INPE} supplied by INPE and α_c and δ_c computed by the presented theory.

Two approaches are presented. In the first one the propagated attitude is daily updated with the help of real satellite data, supplied by INPE. In the second approach the daily updates of the attitude data has not been performed in the propagation process. In both approaches is assumed that $\overrightarrow{m_e}$ is fixed and aligned along the z-axis [2], then the m_{ex} and m_{ey} are vanishes and N_{zm} is zero.

The initial conditions of attitude had been taken for date of July, 24th, 1993 at 00:00:00 GMT, supplied by the INPE’s Satellite Control Center (SCC) for 40 days and are presented in the Tab. 1.

Results for the first approach: daily updated data

The results for the first approach are shown in Tab. 2 and Fig. 2 to Fig. 7. The results for the deviation between the computed values and real values and the mean values for right ascension, declination and spin velocity and the pointing deviation are shown in Tab.2. In Fig. 2 to Fig. 4 are presented the results for temporal behavior of the spin velocity, right ascension and declination of spin axis. Figure 5 and Fig. 6 represent the deviation between the computed values and real values of the attitude variables. The behavior of the pointing deviation is presented in Fig.7 for this approach.

The results show that the region where the analytical solution is closer to the real data corresponds to the smallest decay of the spin velocity (around 0.08 rpm/day) in the last 8 days of the simulation period. In other periods the spin velocity decays around 0.1rpm/day. Over the test period the difference between theory and real data has mean error deviation in right ascension, in declination and in spin velocity of 0.1596^o, 0.0214^o and 0.1409rpm respectively. Then the mean error deviation for the right ascension is bigger than the INPE required precision during more than 70% of the time simulation.

The mean pointing deviation for the period test was 0.4377^o, which is within the dispersion range of the attitude determination system performance by SCC. For 18 days the values of the pointing deviation are bigger than INPE precision required (0.5^o) and this period is associated with the period that the deviation of the right ascension is also bigger than 0.5^o

If it is considered only the last 18 days of the simulation, from August, 15th, 1993 until September, 1st, 1993, in which one of the values of pointing deviation are less than 0.5^o , then all the values are within of INPE required precision, and the mean values are -0.4544^o for right ascension, 0.2344^o for the declination, 0.1107 rpm for spin velocity and 0.3179^o for the for pointing deviation.

Table 1 – Data supplied from INPE's CSS.

SCD1	α_{INPE}(°)	δ_{INPE}(°)	W_{INPE}(rpm)
07/24//93	234.1000	77.3000	90.8100
07/25/93	233.7400	77.6900	90.7100
07/26/93	233.5400	78.0900	90.6200
07/27/93	233.5300	78.5000	90.5200
07/28/93	233.7300	78.9300	90.4200
07/29/93	234.1400	79.3500	90.3300
07/30/93	234.8300	79.7800	90.2300
07/31/93	235.8000	80.2000	90.1200
08/01/93	237.1200	80.6000	90.0200
08/02/93	238.8200	80.9900	89.9100
08/03/93	240.8900	81.3400	89.8100
08/04/93	244.0400	81.8600	89.5400
0805/93	246.6200	82.1200	89.3500
08/06/93	249.5300	82.3300	89.1600
08/07/93	252.7400	82.4800	88.9700
08/08/93	256.1500	82.5800	88.7900
08/09/93	259.7000	82.6000	88.5900
08/10/93	263.2000	82.5600	88.4100
08/11/93	266.5500	82.4400	88.2200
08/12/93	269.7000	82.2800	88.0300
08/13/93	272.5400	82.0600	87.8500
08/14/93	275.7500	81.8500	87.6100
08/15/93	277.4500	81.6200	87.4200
08/16/93	278.9000	81.3700	87.2400
08/17/93	280.0900	81.1000	87.0600
08/18/93	281.0100	80.8200	86.8800
08/19/93	281.7400	80.5300	86.7100
08/20/93	282.2400	80.2300	86.5400
08/21/93	282.5700	79.9300	86.3700
08/22/93	282.7000	79.6400	86.2100
08/23/93	282.6700	79.3500	86.0400
08/24/93	283.5000	79.2200	85.8800
08/25/93	283.0100	78.9500	85.8000
08/26/93	282.4300	78.7000	85.7300
08/27/93	281.7600	78.4800	85.6600
08/28/93	281.0100	78.2700	85.5800
08/29/93	280.1800	78.0800	85.5100
08/30/93	279.2900	77.9100	85.4400
08/31/93	278.3400	77.7800	85.3700
09/01/93	277.3600	77.6700	85.3100

Table 2– Deviation between computed and real values, with the daily updated data.

Day	Right ascension (°)	Declination (°)	Spin velocity (rpm)	Point deviation(°)
07/24//93	0.0000	0.0000	0.0000	0.0000
07/25/93	0.2164	-0.4157	0.1658	0.4184
07/26/93	0.0083	-0.4444	0.1531	0.4444
07/27/93	-0.2677	-0.4778	0.1599	0.4809
07/28/93	-0.5155	-0.4996	0.1531	0.5098
07/29/93	-0.9203	-0.5202	0.1378	0.5486
07/30/93	-1.2224	-0.5198	0.1397	0.5654
07/31/93	-1.6279	-0.5119	0.1421	0.5855
08/01/93	-2.2547	-0.4995	0.1248	0.6263
08/02/93	-2.4730	-0.4617	0.1271	0.6089
08/03/93	-3.2954	-0.4178	0.1111	0.6576
08/04/93	-4.1386	-0.5634	0.2758	0.8272
0805/93	-3.6918	-0.2798	0.1922	0.5860
08/06/93	-4.1569	-0.2013	0.1904	0.5968
08/07/93	-4.0976	-0.1390	0.1901	0.5586
08/08/93	-4.4731	-0.0490	0.1811	0.5815
08/09/93	-4.2721	0.0172	0.2026	0.5497
08/10/93	-4.1366	0.0869	0.1842	0.5394
08/11/93	-3.8845	0.1669	0.1950	0.5322
08/12/93	-3.5859	0.1934	0.1944	0.5134
08/13/93	-3.2529	0.2351	0.1825	0.5012
08/14/93	-3.6183	0.2248	0.2391	0.5535
08/15/93	-2.1789	0.2219	0.1835	0.3840
08/16/93	-2.0105	0.2271	0.1691	0.3744
08/17/93	-1.7181	0.2655	0.1648	0.3729
08/18/93	-1.7288	0.2292	0.1604	0.3560
08/19/93	-1.3248	0.2851	0.1472	0.3569
08/20/93	-1.2870	0.2762	0.1444	0.3502
08/21/93	-1.0851	0.2880	0.1428	0.3434
08/22/93	-0.6929	0.3003	0.1325	0.3245
08/23/93	-0.7862	0.2888	0.1428	0.3224
08/24/93	-1.2360	0.1545	0.1347	0.2767
08/25/93	0.0009	0.2959	0.0548	0.2959
08/26/93	0.2646	0.2807	0.0485	0.2854
08/27/93	0.5270	0.2501	0.0525	0.2709
08/28/93	0.6413	0.2394	0.0662	0.2720
08/29/93	0.8883	0.2104	0.0602	0.2781
08/30/93	1.0626	0.1804	0.0633	0.2852
08/31/93	1.2010	0.1287	0.0662	0.2838
09/01/93	1.2842	0.0969	0.0583	0.2898
Mean value	-1.5960	-0.0214	0.1409	0.4377

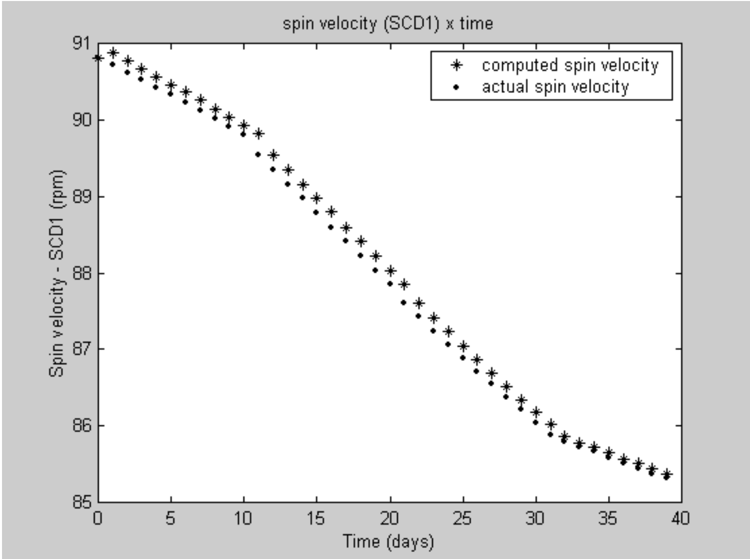

Figure 2. Temporal Behavior of the spin velocity, with daily updated data.

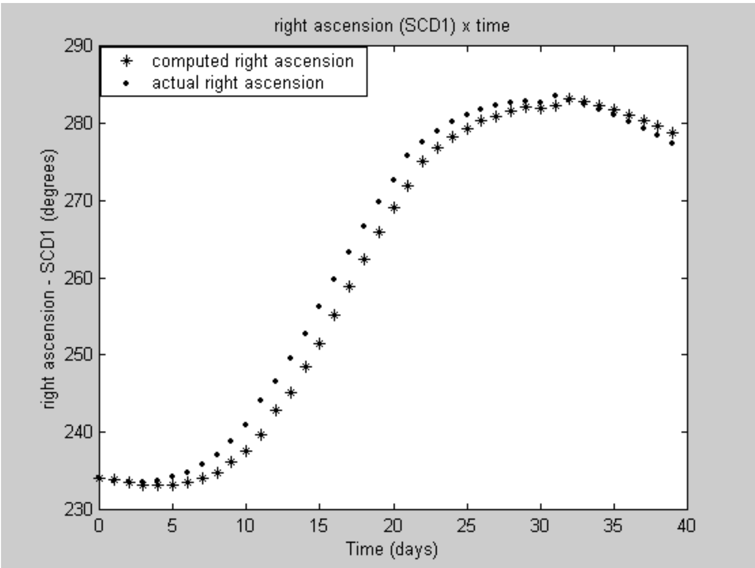

Figure 3 – Temporal Behavior of the right ascension, with daily updated data.

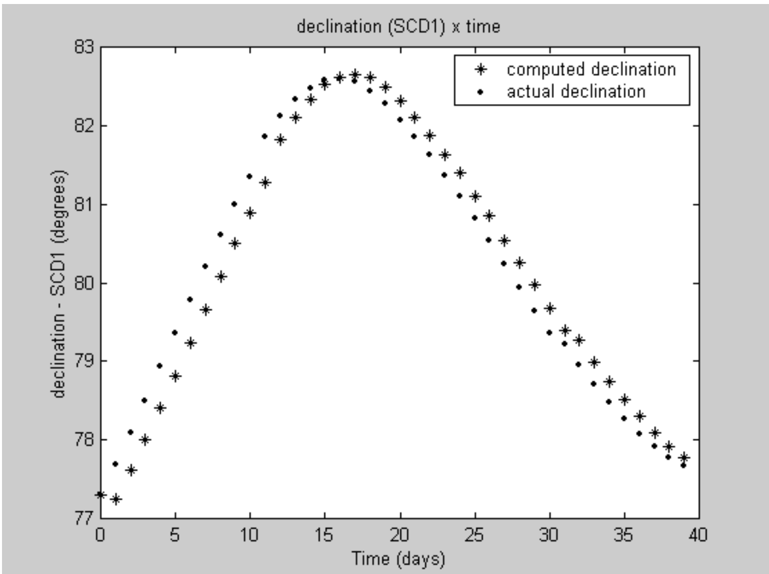

Figure 4 – Temporal Behavior of the declination, with daily updated data

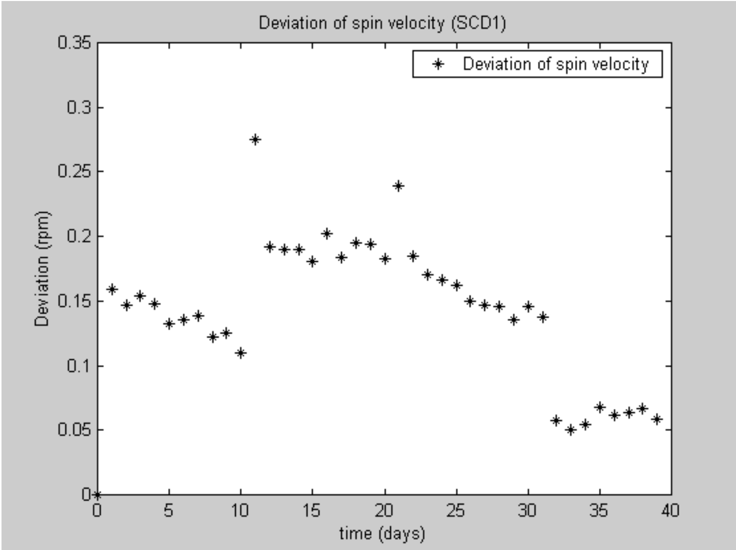

Figure 5 - Temporal behavior of the difference between the real and computed value of the spin velocity, with daily updated data.

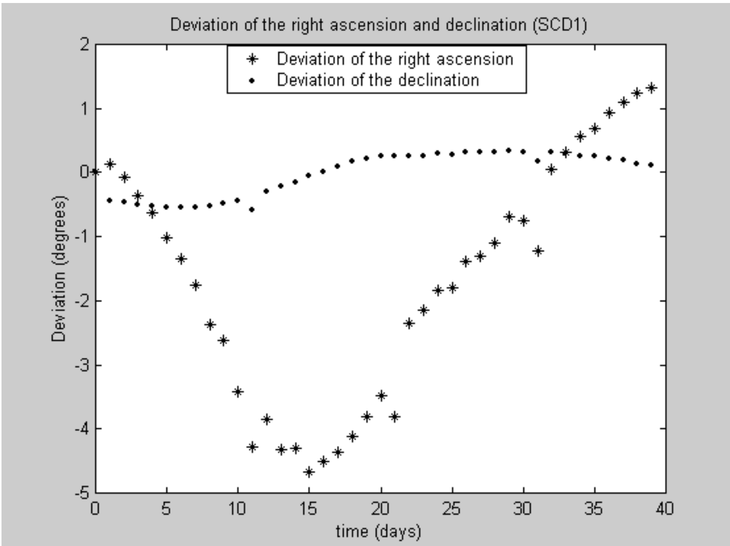

Figure 6 – Temporal behavior of the difference between the real and computed value of the right ascension and declination, with daily updated data.

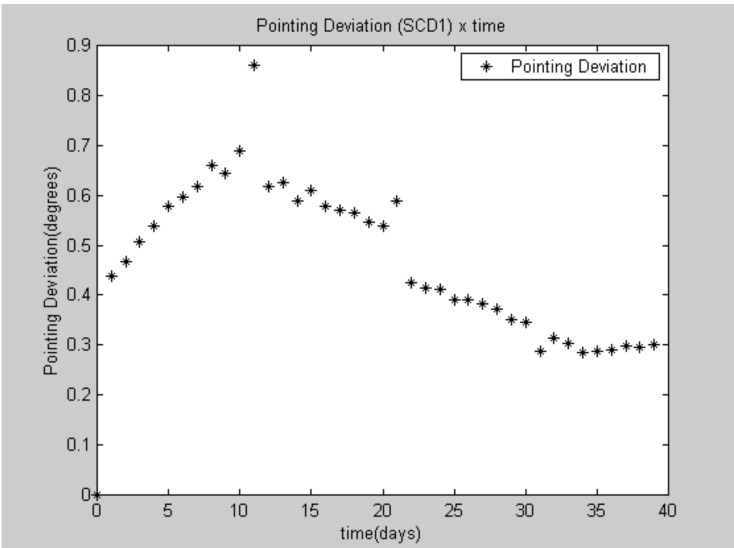

Figure 7 – Temporal behavior of the pointing deviation, with daily updated data

Results for the second approach: without daily updated data

Tables 3 and 4 present the results for this approach for 2 days. In Tab. 3 the values for right ascension, declination, spin velocity and the pointing deviation are presented. Table 4 shows the deviation between the computed values and real values and the mean values for the attitude variables. The simulations are interrupted in the 3^{dr} day because the deviations between the computed values and real values for all variables have been bigger than the INPE required precision for this satellite.

The mean pointing deviation was 0.4472^{o} and the mean of declination was -0.4469^{o}, which are close to the dispersion range of the attitude determination system performance of CCS. The mean for the right ascension (0.0634^{o}) and spin velocity (0.1548rpm) are within the INPE required precision, then it is possible to point out that the declination of the spin axis has great influence in the pointing deviation. The same observation can be applied for the first approach in 40 days simulation where mean pointing deviation was 0.4377^{o} and the mean of declination was -0.0214^{o}, both within the dispersion range required by INPE while the mean of right ascension (-1.596^{o}) did not satisfied the INPE required precision.

Other simulation were done for different initial data but in all of them the results were similar, which means that the computed values have a good agreement with the real data only for the 2 days simulations.

Table 3–Computed values for attitude variables, without the daily updated data.

	Right ascension (°)	**Declination (°)**	**Spin velocity (rpm)**	**Pointing deviation(°)**
07/24/93	234.1000	77.3000	90.8100	0
07/25/93	233.8741	77.2526	90.8690	0.4383
07/26/93	233.5962	77.1865	90.9253	0.9036

Table 4– Deviation between computed and real values and mean values without the daily updated data.

	Right ascension (°)	**Declination (°)**	**Spin velocity (rpm)**
07/24/93	0	0	0
07/25/93	0.1341	-0.4374	0.1590
07/26/93	0.0562	-0.9035	0.3053
Mean value	0.0634	-0.4469	0.1548

7. Conclusions

In this paper an analytical approach for the spin-stabilized satellite rotational motion was presented taking into account the influence of the aerodynamic torque and gravity gradient torque. The models for the gravity gradient and aerodynamic torques were discussed, considering the Earth atmosphere described by the model TD88.

The analytical solution shows that coupled effect of gravity gradient and aerodynamic torques cause linear variation in the spin velocity magnitude and the produces a precession and drift on the spin axis.

The theory was applied to the spin stabilized Brazilian's satellites SCD1. Results have shown the agreement between the analytical solution and the real satellite behavior for specific time simulation and two approaches were presented.

In the first one the attitude and orbital data are daily updated with real attitude data supplied by INPE. The results showed a good agreement between the computed and real data during 18 days. The mean pointing deviation was of 0.3179^o, which are within the dispersion range of the attitude determination system used for this satellite.

In the second approach the attitude and orbital data are not updated. The results presented a good agreement between the analytical solution and the actual satellite behavior only for two days simulation. For more than 2 days the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required for SCC (0.5º).

For both approaches it is possible to note the influence of the declination of the spin axis in the calculation of the pointing deviation.

In order to improve the results it is important to include the other external torques and to eliminate some simplifications in the aerodynamic torque. However the procedures are useful for modeling the dynamics of spin stabilized satellite attitude perturbed by gravity gradient and aerodynamic torques.

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