PRECISE DOPPLER MEASUREMENTS FOR NAVIGATION AND PLANETARY GEODESY USING LOW GAIN ANTENNAS: TEST RESULTS FROM CASSINI

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Abstract: *The lack of expensive scan platforms for remote sensing instruments in many interplanetary missions has a strong and detrimental influences on scientific investigations, like geodesy and radio-science, based on observables (range, range-rate, deltaDOR) generated by the radio link. Indeed, the communications to ground using high gain antennas are incompatible with simultaneous pointing of other instruments, like radar or camera. That affects the amount of data collectable and therefore the science return.*

This limitation could in principle be overcome by establishing the radio link through an on-board low gain antenna (LGA), allowing Doppler data to be taken while the spacecraft body is articulated for other science investigations. The LGA's broader power pattern permits communication but of course results in a reduced power of the uplink and downlink signals, and therefore a reduced SNR at the spacecraft and the ground station. Moreover, such a configuration must compensate the effect of spacecraft rotations on range rate measurements when the entire spacecraft articulates to maintain pointing for other remote sensing instruments.

On 15 August 2010, the Cassini mission did a dedicated test to assess the feasibility of gravity (i.e. precision Doppler) measurements with the LGA. During that pass, Cassini was turned repeatedly about its x- and y-axis to investigate the degradation of link stability and the ability to correct for the attitude motion. Open loop data acquired by the radio-science receiver (RSR) were processed to reconstruct sky frequency versus time as the spacecraft articulated. This Doppler frequency time series was then processed by means of an orbital and an attitude fit, the latter driven by the attitude motions model (angular frequencies and quaternions) provided by Attitude Control System (ACS). We report here the results of this test.

Keywords: *Cassini, Navigation, LGA, Radio-science*

1. Introduction

Articulating elements such as scan platforms and steerable high gain antennas (HGA) introduce significant complications to the spacecraft design and surely result in increased cost. Spacecraft designers tend therefore to avoid their use, even if this choice invariably leads to constraints on spacecraft operations and potential reduction in science return. For example, most planetary missions lack expensive scan platforms for remote sensing instruments and rather rely on attitude changes of the spacecraft as a whole for pointing to targets. This approach to spacecraft design, while significantly simplifying the overall system, makes communication to ground by means of high gain antennas (i.e. Earth pointing) incompatible with simultaneous pointing of other instruments for remote sensing observations. This limitation is especially detrimental to celestial mechanics, geodesy and radio science investigations, which rely on the radio link to generate observable quantities (range, range rate, and delta-DOR).

The lack of a scan platform severely affected science operations in Cassini's exploration of the Saturnian system. Because of the limited number of flybys of the moons, the impossibility of operating several instruments simultaneously on the same target forced the project to assign each flyby a single, prime instrument. The pointing to the desired target was obtained by turning the entire spacecraft, a rather slow maneuver for a large platform. Gravity measurements were substantially affected by the limited number of flybys assigned to this science goal. Indeed, the determination of a gravity field relies crucially on global coverage, while the Cassini radio science investigations could count only on a handful of flybys with sometimes suboptimal geometries.

Cassini has a 4m, fixed HGA and a radio system operating at X and Ka-bands. The narrow half- power beam width (HPBW), about 0.6 deg at X-band [1], 0.16 deg at Ka-band, provides excellent SNR during flybys devoted to gravity science, but unfortunately makes gravity measurements impossible during flybys assigned to other instruments. This limitation could in principle be mitigated if the radio link were enabled through one of the two onboard low gain antennas (LGA). LGA1 is located on the tripod of the HGA, therefore pointing in the same direction as the HGA (the –z axis in the body frame). LGA2 is mounted on a short boom close to the engines, pointing in the +z direction. Each has a HPBW of about 30 deg. The lower gain (about 38dB on axis [1]) caused by the broad antenna pattern results in reduced power of the received uplink signal such that 2-way transponder lock can be achieved only for pointing up to about 55 deg off the z-axis. This sub-optimal configuration has two drawbacks for precision Doppler measurements, namely the much smaller SNR of the up- and downlink signals and the need to compensate the effect of spacecraft rotation on range rate measurements as the entire spacecraft body articulates. To assess the feasibility of scientifically useful gravity measurements using the LGA a dedicated test was carried out on 15 August 2010, when Cassini was tracked in coherent 2-way mode from DSS-63. The main goals were (1) a quantitative assessment of the degradation of link Doppler stability, measured by rms velocity error, under low SNR conditions and spacecraft rotation and (2) verification of our ability to control systematic errors when correcting the Doppler data for the moving phase center of the LGA. To this end, the spacecraft was turned repeatedly about its x- and y-axes in order to assess capability to maintain lock at large off-boresight angles and determine the attainable accuracy in the compensation of the attitude motion. This latter step is carried out in post-processing by using quaternions and angular rates generated by the ACS. For comparison, range rate data were acquired on the LGA also before and after the period when the spacecraft turns took place.

The analysis, explained in detail in the following paragraphs, was based on sky frequencies generated by processing the electric field samples acquired by the open loop Radio Science Receivers (RSR). The reconstructed sky frequencies were then processed through JPL Orbit Determination Program (ODP) to separate the effects of orbital motion and attitude dynamics on Doppler data. Starting from the orbital solution provided by the Cassini Navigation (NAV) team using only HGA data, the LGA range rate observables were first fitted to generate residuals that include the spacecraft rotations. The Doppler shift induced by these rotations can be computed if the velocity of the LGA phase center (PC) in the inertial frame is known. While the ACS provides accurate attitude quaternions and angular rates, the PC position in the spacecraft frame is not accurately known *a priori* and therefore was estimated by means of a simple fit. After the orbital and then the attitude fits, the residuals are substantially free from orbital and attitude dynamics effects, and contain only noise. Analyzing and comparing them to those obtained under normal condition (HGA and no attitude motion), is possible to investigate the feasibility of LGA antenna use.

2. Sky frequencies reconstruction

The raw data used here were the electric field (i.e. pre-detection) measurements of the downlink. The data were taken with the Deep Space Network's Radio Science Receiver (RSR) which tunes the downlink signal (based on the predicted trajectory) into a narrow band (in this case: 1kHz), where it was digitized along with the tuning information. We used a non-coherent approach to estimate Doppler frequency, motivated by two *a priori* considerations: (1) the SNR on the LGA might drop to a level too low for reliable phase coherent estimates, and (2) the non-coherent method employed guaranteed that the data used in each frequency estimate was symmetrical with respect to the frequency estimate's time tag. We analyzed chunks of data 11 seconds long (i.e. 11 x 1024 = 11,264 complex samples). We zero-padded to a Fourier transform length of 2^{17} samples (i.e. a naïve resolution of about 8 mHz) and computed the Fourier-transform-squared (i.e. the sample RF spectrum for that time interval), estimating the Doppler frequency for each chunk using a Bayesian technique [2]. For the SNRs actually observed this technique gave remarkably small formal frequency errors (the method was also tested on synthetic data over the same range of SNRs to verify that the frequency estimates were unbiased and with errors consistent with the formal errors expected from the Bayesian technique.). The Doppler frequency estimate for each chunk was determined by a fit to the peak of the sample RF spectrum. These frequency estimates were associated with the midpoint time of the 11 second data chunk. The data window was then advanced by 1 second and the process repeated; thus nearby frequency estimates are correlated, since e.g. 10/11 of the data used was common in adjacent estimates. Sky frequencies versus time were then reconstructed by adding the known tuning function used by the RSR to these RSR-level frequency estimates.

3. Orbital fit

The first step of the analysis consists in separating the effect on the Doppler data (sky frequencies) due to the orbital motion from that due to the attitude motion. That was carried out comparing the open loop observables (open loop sky frequencies) with those computed on a reference trajectory obtained propagating the state vector and the covariance matrix computed by the Cassini NAV team from an orbital solution that does not include the LGA data. These residuals (Fig. 1) contain only the Doppler shift induced by the motion of the LGA about the spacecraft center of mass (CoM), while the orbital motion has been effectively removed. The residuals are therefore “observables” in the following fit for the estimation of the LGA PC position in the body frame (centered in the spacecraft CoM).

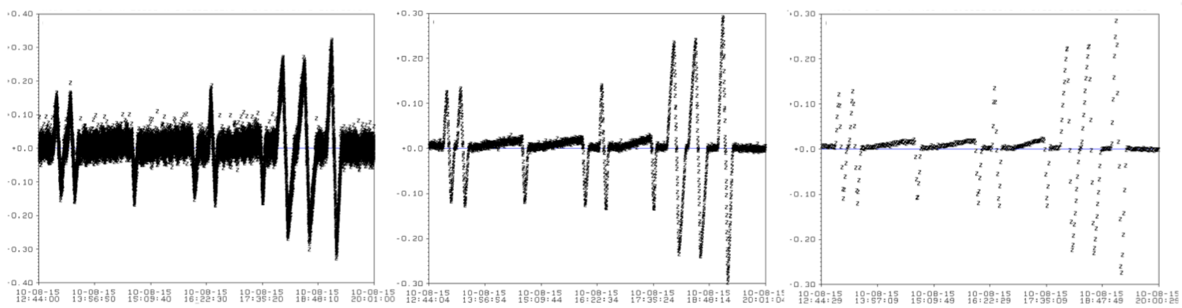

Figure 1. Doppler residuals (Hz) after orbital fit (from left to right: data with time count of 1s, 10s, 60s). The signatures reflect the attitude dynamic effects.

4. Attitude fit

Since ACS provides accurate attitude quaternions and angular rates, the LGA PC position in the spacecraft frame can be estimated by means of a simple fit. Is then necessary to write an attitude model in order to provide the partial derivatives necessary for the fit.

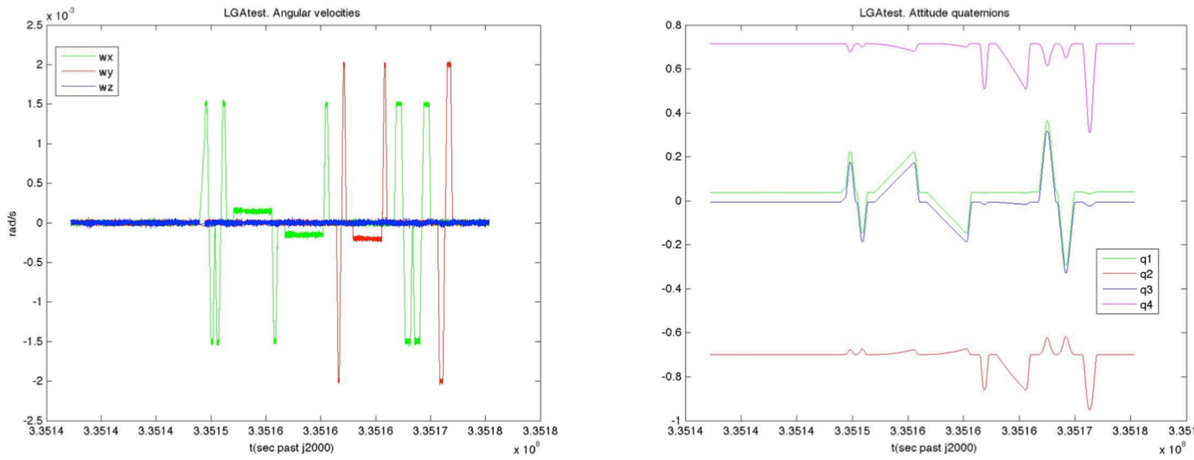

Figure 2. Cassini angular velocities (left) and quaternions evolution (right) during LGA test. Provided by ACS team.

At any time, the position **r** of the PC in the inertial frame EMEJ2000 with origin in the spacecraft CoM is obtained from the transformation:

$$\mathbf{r} = q\mathbf{R}\overline{q} \tag{1}$$

where **R** is the PC position in the body frame and q is the quaternion associated to the transformation. The corresponding velocity is

$$\mathbf{V} = \dot{\mathbf{r}} = \dot{q}\mathbf{R}\overline{q} + q\mathbf{R}\dot{\overline{q}} = \dot{q}\mathbf{R}\overline{q} + \overline{\dot{q}\overline{\mathbf{R}}\overline{q}} = \dot{q}\mathbf{R}\overline{q} - \overline{\dot{q}\mathbf{R}\overline{q}} = 2\dot{q}\mathbf{R}\overline{q} \tag{2}$$

The latter equality follows because the scalar component is null. From the attitude evolution equation

$$\dot{q} = \frac{1}{2}\Omega(\omega)q(t)$$
$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{3}$$

we get

$$\mathbf{V} = 2\dot{q}\mathbf{R}\overline{q} = \Omega q\mathbf{R}\overline{q} \tag{4}$$

The projection of this vector along the line of sight yields the Doppler shift in a two-way radio link:

$$\Delta f = 2\frac{f}{c}\left[\mathbf{E}\cdot(\Omega q\mathbf{R}\overline{q})\right] \tag{5}$$

where, f is the downlink frequency, c the speed of light and **E** the earth unit vector (from the spacecraft CoM to Earth).

To estimate the phase center position **R** we need the partial derivatives of Δf with respect to X_{pc},Y_{pc} and Z_{pc}, the components of **R** in the body frame. They are:

$$\begin{aligned}
\frac{d(\Delta f)}{dX_{PC}} &= 2\frac{f}{c}\{E_x[\omega_y(-2q_1q_3-2q_2q_s)+\omega_z(2q_1q_2-2q_3q_s)]+E_y[\omega_y(2q_1q_s-2q_3q_2)+\omega_z(q_2q_2+q_sq_s-q_1q_1-q_3q_3)]+E_z[\omega_y(q_2q_2+q_1q_1-q_sq_s-q_3q_3)+\omega_z(2q_2q_3+2q_1q_s)]\} \\
\frac{d(\Delta f)}{dY_{PC}} &= 2\frac{f}{c}\{E_x[\omega_x(2q_1q_3+2q_2q_s)+\omega_z(q_3q_3+q_2q_2-q_1q_1-q_sq_s)]+E_y[\omega_x(2q_2q_3-2q_1q_s)+\omega_z(-2q_2q_1-2q_sq_3)]+E_z[\omega_x(q_3q_3+q_sq_s-q_2q_2-q_1q_1)+\omega_z(2q_sq_2-2q_1q_3)]\} \\
\frac{d(\Delta f)}{dZ_{PC}} &= 2\frac{f}{c}\{E_x[\omega_x(2q_sq_3-2q_2q_1)+\omega_y(q_1q_1+q_sq_s-q_3q_3-q_2q_2)]+E_y[\omega_x(q_1q_1+q_3q_3-q_sq_s-q_2q_2)+\omega_y(2q_2q_1+2q_sq_3)]+E_z[\omega_x(-2q_2q_3-2q_1q_s)+\omega_y(2q_1q_3-2q_2q_s)]\}
\end{aligned} \tag{6}$$

Since quaternions and angular rates are time tagged in SCET, the Doppler shift is actually recorded a one-way light-time (OWLT) later by the ground station. The conversion from SCET to UTC was performed using a simple linear interpolation of the OWLT at the beginning and the end of the observations (the OWLT changed by more than 1s between the first and the last data). Moreover, the attitude information are provided at intervals of 4s.

Consequently we integrated eq. 3 (de facto a simple linear interpolation) and generated a new attitude file with quaternions and angular velocities at the epochs of Doppler observations. For the goal of the attitude fit, the Earth unit vector **E** could be safely considered a constant throughout the time span of the test.

The simple, linear estimator provides the coordinates of the phase center, the associated covariance matrix and residuals. All data have been equally weighted using the post-fit variance of the residuals and the points closer to the peak angular accelerations have been edited out in the estimate at 60 s, as the model (eq. 5) provides the instantaneous Doppler shift and loses accuracy in the presence of large angular accelerations, especially at long integration times. The estimate of the phase center position and the associated statistical error are reported in Tab. 1.

Table 1. Estimate of the LGA phase center position and associated errors.

	Time count 1s	Time count 10s	Time count 60s
X	-13.35 cm (σ = 0.62 cm)	-13.34 cm (σ = 0.35 cm)	-13.01 cm (σ = 0.58 cm)
Y	-4.50 cm (σ = 0.43 cm)	-4.48 cm (σ = 0.24 cm)	-4.25 cm (σ = 0.39 cm)
Z	-383.37 cm (σ = 0.85 cm)	-383.20 cm (σ = 0.48 cm)	-383.74 cm (σ = 0.83 cm)

The dependence of the estimation error on the integration time is an indication that the noise is not white. Indeed, at 1 s and 10 s the contribution from numerical noise is large. Unlike white frequency noise, numerical noise, decreases as the inverse of the integration time, with a transition between the two regimes occurring approximately at integration times of 10 s. The residuals obtained by means of the attitude fit contain only noise and are shown in Fig. 3, Fig. 4 and Fig. 5.

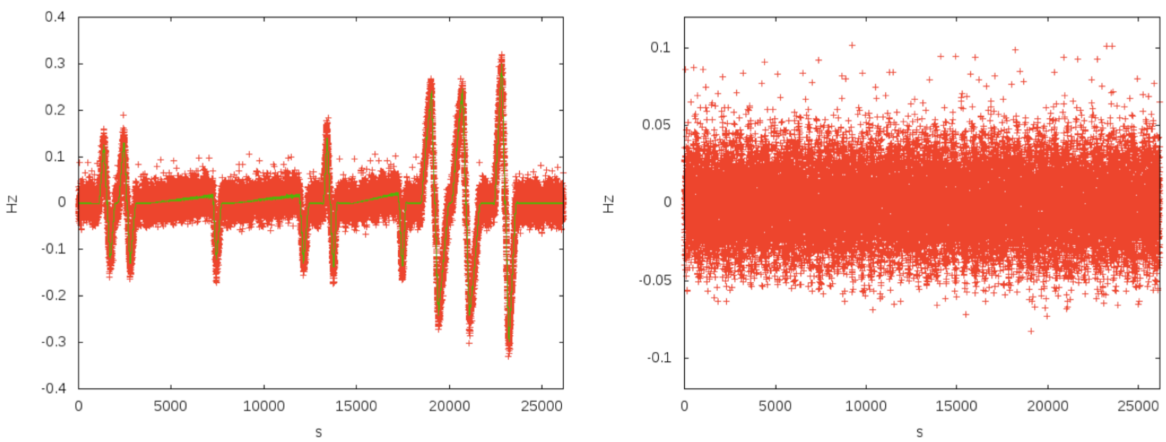

Figure 3: Attitude fit of open loop LGA data with time count equal to 1s. The red marks are the residual prefit (left) and postfit (right). The green line is the Δf prediction after the estimate of the phase center position.

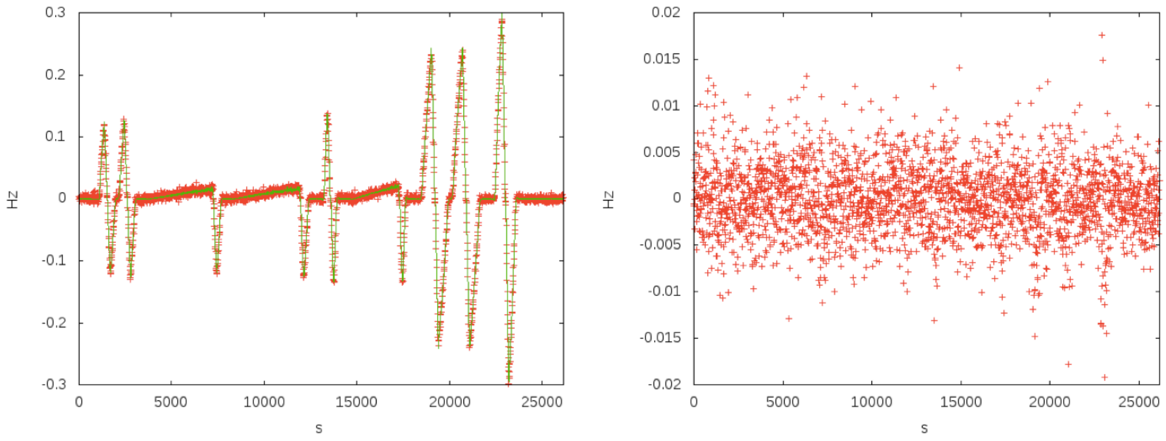

Figure 4: Attitude fit of open loop LGA data with time count equal to 10s. The red marks are the residual prefit (left) and postfit (right). The green line is the Δf prediction after the estimate of the phase center position.

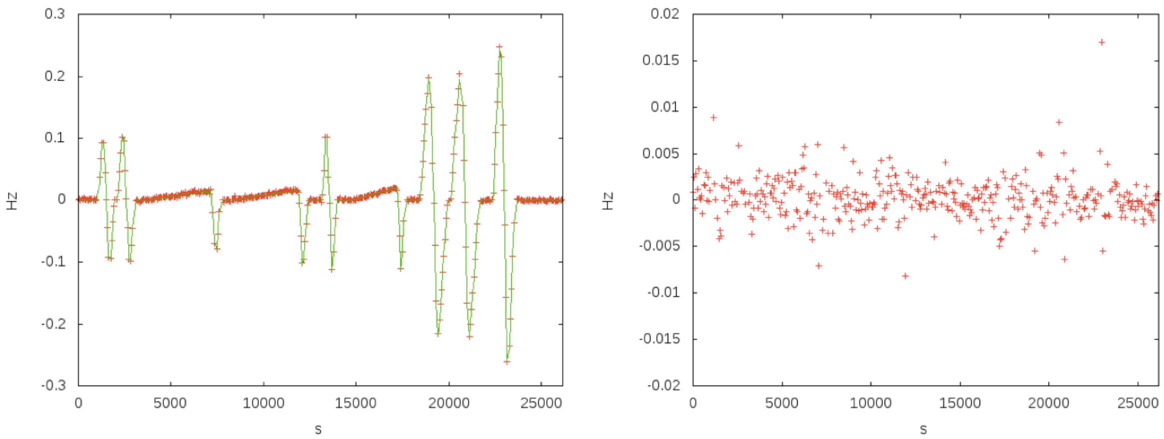

Figure 5: Attitude fit of open loop LGA data with time count equal to 60s. The red marks are the residual prefit (left) and postfit (right). The green line is the Δf prediction after the estimate of the phase center position.

The 60 s residuals have an rms value of 2.2 mHz (Fig. 5, Tab. 2). This value is nearly identical to the one obtained from two HGA passes immediately preceding and following the test, respectively 2.3 and 2.7 mHz.

Table 2: Mean and rms of the postfit LGA Doppler residuals at different count time.

LGA data	Mean (mHz)	Doppler residuals rms (mHz)
Tc=1s	-0.06	21.35
Tc=10s	-0.08	3.81
Tc=60s	0.19	2.20

By comparison, the rms values of the frequency residuals of the five Titan's gravity flybys (X-band, standard media calibration) T11, T22, T33, T45 and T68 are reported in Tab. 3.

Table 3: Doppler residuals rms of the five Titan's gravity flybys (X-band, standard media calibration)

Titan flybys	SEP (deg)	Doppler residuals rms (mHz)
T11	147	0.7
T22	133	0.8
T33	45	1.3
T45	29	2.8
T65	119	1.1

As the Sun-Earth-Probe angle (SEP) during the LGA test was 40 deg, the comparison with T33 and T45 is the most appropriate. The values of the rms residuals are similar in the three cases (within a factor of 2). The strong compatibility, in terms of rms, of the LGA data with those of HGA should be explained considering the low SEP at which the LGA test occurs. The plasma noise indeed, seems, to be in that case, dominant with respect to the thermal noise, making the residuals insensitive to the degradation of the SNR in case of an antenna lower gain.

The phase center position estimate has been also translated in the Cassini structural reference frame (Tab. 4), using the CoM position in that reference frame. The PC estimated position results to be aligned (Fig. 6) on the spacecraft z-axis and located nearly the antenna feed position, as expected.

Table 4: CoM and PC position in the structural reference frame.

Structural r.f.	X (m)	Y (m)	Z (m)
CoM	0.1298	0.0479	1.0843
PC (1s)	-0.004 (σ=0.006)	0.003 (σ=0.004)	-2.749 (σ=0.009)
PC (10s)	-0.004 (σ=0.004)	0.003 (σ=0.002)	-2.749 (σ=0.005)
PC (60s)	0.000 (σ=0.006)	0.005 (σ=0.004)	-2.749 (σ=0.008)

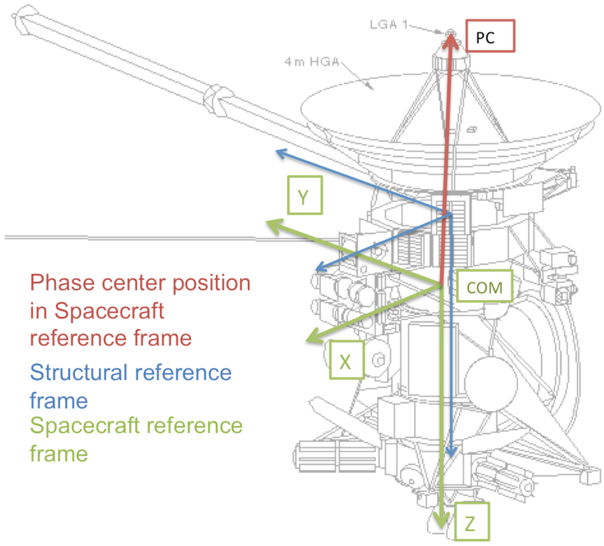

Figure 6: Qualitative representation (based on Tab. 4 values) of the estimated PC position with respect to the structural and spacecraft reference frame.

5. Conclusions

We showed that, for angles 55 degrees of less off boresight, Cassini LGA can give scientifically-useful Doppler data for gravity. Besides demonstrating the feasibility of gravity measurements with the LGA, the test provided the determination of the phase center (PC) location in the body frame. This result is important because the knowledge of the vector from the spacecraft CoM to the PC allows to process Doppler data without the need to fit for additional parameters: the PC coordinates, the attitude quaternions and angular rates are sufficient to remove the effect of spacecraft rotations in any future flyby. A small change of this vector is expected from propellant consumption, which changes the CoM location with respect to the spacecraft structure, but its variation could be computed with sufficient accuracy from the known mass variation.

6. Acknowledgements

The successful completion of the LGA test required the effort of many people. We thank in particular the Cassini Radio Science Support Group (A. Anabtawi, E. Barbinis, D. Fleischman, D. Kahan) for the support to operations and data analysis, D. Roth and R. Ionasescu for providing the orbital reconstruction and the closed-loop data files, and T. Burk for the attitude data.

7. References

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