ORBIT RECONSTRUCTION FOR THE GAIA MISSION

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Abstract: *The upcoming ESA cornerstone mission Gaia which is due to be launched to the second Sun-Earth Lagrange point in 2013 has the scientific objective to create a star catalogue with unprecedented precision. In order to do so a very good knowledge of the spacecraft position and velocity is required a posteriori. It is shown in this paper that the required orbit reconstruction accuracy cannot be guaranteed with conventional tracking data, range and Doppler, which are usually used to determine the orbits of Lagrange point missions for the purpose of spacecraft operations. Therefore very precise - down to 10 mas - ground-based astrometric measurements of Gaia are foreseen to augment the conventional tracking data in order to reconstruct the Gaia orbit to sufficient accuracy. This paper describes the usage of the astrometric measurements within the orbit determination system of Gaia, outlines the challenges to acquire them on a regular basis, provides results from an analysis that demonstrates the feasibility of the approach and shows test results that have been achieved with ground-based astrometric measurements of the flying ESA spacecraft Planck.*

Keywords: *Gaia, orbit determination, astrometric measurements, Lagrange point*

1. Introduction

Gaia is one of the European Space Agency's (ESA) science cornerstone missions with the scientific objective to create a precise three-dimensional map of about one billion stars aiming at visual magnitudes as faint as V = 20. The envisaged astrometric accuracy for stellar objects is 25 μas at V = 15, or 300 μas at V = 20 which can be compared with values achieved by ESA's Hipparcos mission of 1 mas. Moreover, Gaia will make - to lower but still unprecedented precision - astrometric measurements of solar system objects, mainly minor planets, for the purpose of improving the knowledge of their physical and orbital properties. Besides astrometry, Gaia will perform radial velocity determination at a 2–10 km/s level for stars with V < 17, low resolution spectroscopy in the same brightness range, and spectrophotometry in 25 colours for V < 20.

Gaia is due to be launched in November 2013 on a Soyuz-Fregat from Kourou, French Guyana. The launcher injects the spacecraft directly into a fast transfer orbit to L_2, the second Lagrange point in the Sun-Earth system. After about one month transfer time a large manoeuvre (> 100 m/s) inserts the spacecraft into a Lissajous orbit around L_2 where it will spend its 5 years nominal mission lifetime with an optional mission extension of 1 year.

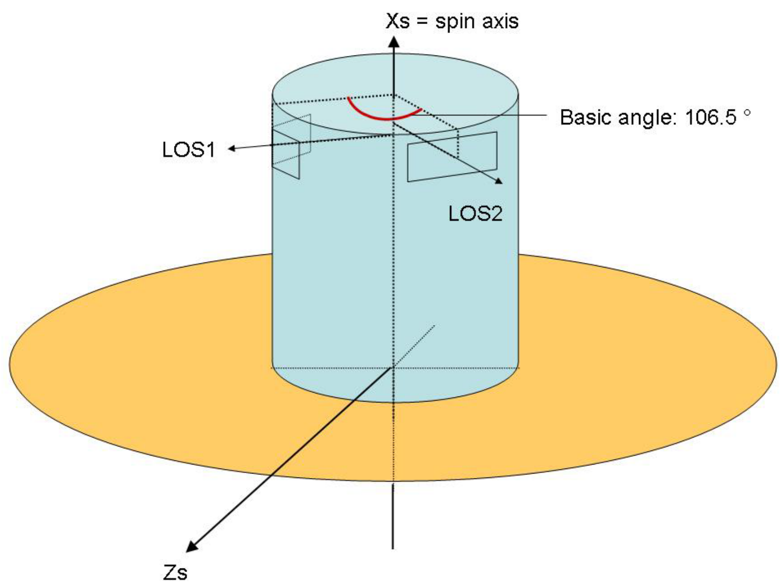

Figure 1. Gaia spacecraft schematic view

A schematic view of Gaia is displayed in Fig. 1. It is hat-shaped with a large brim serving as Sun shield to protect the telescopes from sunlight. The diameter of the Sun shield is 10.1 m yielding a total area of more than 80 m^2. Two telescopes with viewing directions separated by the so-called basic angle of 106.5° share the same focal plane in which the detectors are located. With this arrangement the positions of stars can be measured and compared between the two lines of sight, which are arranged in a plane perpendicular to the spacecraft spin axis around which the spacecraft performs a continuous scanning motion that allows the telescope's field of view to sweep across the sky with a nominal speed of 1°/min i.e. once every 6 hours. The spacecraft spin axis is tilted by 45° with respect to the Sun-Earth direction around which it precesses with a period of 63 days. This precessing motion is referred to as Gaia's nominal scanning law with which full coverage of the sky is achieved within a few months. During periods in which Gaia scans through the galactic plane, which will happen approximately for about 20 days every 100 days, the precession rate will be reduced and the so-called modified scanning law is executed to account for the increased star density.

During its routine mission Gaia will be tracked in X-band daily from one of ESA's 35 metre deep space antennae with an approximate station pass duration of 8 hours. The majority of the station time is required to dump the large amount of scientific data. In order to do so the downlink is modulated with a Gaussian Minimum Shift Keying (GMSK) digital modulation scheme that allows to downlink information rates of up to 8.7 Mbits/s. However, this modulation scheme is mutual exclusive with performing ranging. Since 2-way range measurements are required for the orbit determination dedicated range sessions of approximately 10 minutes duration are performed at the start and end of each tracking pass. In contrast, Doppler measurements are not (significantly) affected by the GMSK modulation and are recorded in 2-way mode for the entire duration of the pass. During periods of the modified scanning law the ground station coverage must be increased to daily pass durations of up to 16 hours in order to be able to downlink the increased amount of scientific data. This will be accomplished with the daily usage of two deep space antennae.

Since Gaia aims for very precise scientific measurements no movable parts such as momentum wheels or an articulated antenna are included in the spacecraft design that could give rise to attitude disturbances and spacecraft vibrations. During the science phase the attitude of the spacecraft is therefore controlled by cold gas micro-propulsion system (MPS), which provides a maximum thrust of 500 μN per thruster with a resolution down to 1 μN. Moreover, a Phased-Array Antenna (PAA) is used for the communication that allows steering the effective antenna beam into a desired direction by varying the relative phases of the individual antenna signals. The PAA is installed below the Sun shield that is visible in Fig. 1. The phase centre of the PAA is supposed to be located in the centre of the array that in turn lies on the spin axis of the spacecraft.

For orbit and attitude control during transfer orbit and for the regularly required orbit maintenance manoeuvres in the naturally unstable L2-Lissajous orbit the Gaia spacecraft facilitates a chemical propulsion system (CPS) of 8 redundant 10 N thrusters. The spacecraft is equipped with fine Sun sensors, gyros and star trackers as attitude sensors. In addition, the control loop is fed with instrument data to achieve the stringent pointing requirements of the nominal science mode.

2. Gaia Orbit Reconstruction Requirements

To fulfill the scientific objectives of the mission the Gaia orbit has to be known *a posteriori* to a precision that is unprecedented for ESA's space observatories. The requirement on the reconstruction precision of the spacecraft position is 150 m (1σ) in each component. The main driver for this requirement is the measurement of solar system object parallaxes for which the observer baseline needs to be known to high precision. The requirement on the reconstruction precision of the spacecraft velocity is 2.5 mm/s (1σ) in each velocity component. Moreover, there should be no systematic bias on the velocity of more than 1 mm/s in each velocity component. The main driver for this requirement is the need to correct astronomical observations for stellar aberration for which the observer velocity needs to be known to high precision.

For ESA missions to the Lagrange points the orbit is determined by using ESOC's interplanetary orbit determination system [1]. Interplanetary orbit determination is mainly relying on 2-way range and Doppler tracking measurements. With the existing tracking systems that are installed in ESA's deep space stations and by using the X-band communication link an end-to-end measurement precision of better than 5 m and 0.1 mm/s for 2-way range and range-rate (derived from Doppler observables integrated over a 1 minute count time) respectively is achieved. With these measurements the Gaia orbit reconstruction requirements would seem to be fulfilled easily. Yet, this is only true for one of the components in the direction of the line-of-sight. The so-called plane-of-sky components perpendicular to the line-of-sight have to be derived indirectly. For Earth or planetary obiters these components are derived due to a strong coupling to the spacecraft dynamics. In contrast, for deep space spacecraft this coupling is rather weak and the plane-of-sky components can only be derived via the sinusoidal modulation on the range and Doppler observables over one tracking pass that is induced by the rotational motion of the tracking station on Earth. Analysis results that are presented in section 4 show that with reasonable assumptions on the range and Doppler measurement precision the Gaia orbit reconstruction requirements can be only partly fulfilled, more precisely only in periods when the spacecraft is away from zero declination (i.e. away from the Earth equatorial plane). This behaviour, which is well known in the field of deep space orbit determination, is due to a singularity in the sensitivity function of the range and Doppler measurements on the declination position component of the spacecraft that occurs at zero declination.

The solution is to augment the line-of-sight measurement with direct measurements of the plane-of-sky components, i.e. with angular measurements of the spacecraft position. The tracking antenna being in auto-track with the spacecraft could supply these kinds of measurements but only with an accuracy of about 180 arcsec (corresponds to more than 1000 km at L_2), which makes them unusable for Gaia. Delta Differenced One-Way Range (Delta-DOR) measurements, which are a VLBI type measurement that requires two widely separated receiving stations on Earth, would be another option. These measurements are very precise (better than 3 mas which corresponds to 22 m at L_2) and often used for deep space spacecraft navigation. But they are also very resource demanding by needing several deep space stations for about 2 hours every night over the full duration of the Gaia mission. This is considered not to be affordable and would lead unavoidably to major conflicts with other missions relying on ESA's deep space network.

The solution of the problem came finally from the Gaia science team: they proposed the use of astrometric measurements of Gaia by ground-based optical telescopes. The precision of these measurements depends on the quality of the star catalogue that is used to reduce the optical images to angular measurements. It is reckoned that with the best star catalogues that are available today the angular measurements have a precision of about 50 mas (corresponds to 360 m at L_2). Unfortunately, this is not precise enough to meet the Gaia requirements. More precise angular data can only be derived from optical images once better star catalogues are available. The only improved star catalogue that will be available in the near future will be the Gaia star catalogue itself and it is reckoned that with this an angular precision of better than 10 mas (corresponds to 70 m at L_2) can be achieved. These measurements could therefore be used to meet the Gaia orbit reconstruction requirements. But now we have the dilemma that the generation of the Gaia star catalogue depends on a precise Gaia orbit reconstruction and the precise Gaia orbit reconstruction depends on the availability of the Gaia star catalogue. This interdependency calls for an iterative process, which is now the baseline for the Gaia mission:

1. The Gaia orbit will be reconstructed using range and Doppler measurements only with a somehow degraded reconstruction precision.
2. Based on this orbit a preliminary Gaia star catalogue will be generated.
3. The preliminary catalogue will be used to reduce the ground-based optical images of Gaia to angular measurements.
4. These angular measurements will be fed into a second orbit determination process, which will result in the finally reconstructed Gaia orbit with the required precision.
5. Based on this orbit the final Gaia star catalogue will be generated.

The first iteration orbit reconstruction using only range and Doppler measurements corresponds to the routine orbit determination that is performed once per week for the purpose of spacecraft operations. For verification purposes astrometric measurements using standard star catalogues for data reduction will be delivered during the first 4-6 weeks after the Lissajous orbit insertion manoeuvre. It is foreseen that the first set of precise astrometric measurements that have been reduced using the preliminary Gaia star catalogue will be delivered 2 years into the mission and subsequently with a biannual frequency. The second iteration orbit reconstruction will hence be performed with the same frequency.

3. Astrometric Data

3.1. Ground Based Optical Tracking of Gaia (GBOT)

The acquisition and reduction of ground based optical images of Gaia is the responsibility of a group within the Gaia Data Processing and Analysis Consortium (DPAC), which is called GBOT (Ground Based Optical Tracking of Gaia) [2]. They have the duty to organise the observation campaign starting from identification, placement and testing of the required infrastructure during the development phase up to carrying out, analysing and providing the observations in collaboration with the participating telescopes during the 5 years operational phase. In order to do so they are searching for participating observatories that own telescopes with an aperture size between 1 and 2 m and a field of view of 5 x 5 arcmin or larger. The GBOT group is faced with some challenges for the practical implementation:

1. Measurements are required almost every night over the entire Gaia mission lifetime, i.e. more than one telescope has to be involved to mitigate missing observations due to e.g. cloudy nights.
2. They have to develop semi-automatic, robust observing and processing methods for a faint and fast moving object (apparent velocity of Gaia will be about 40 mas/s).
3. Suitable telescopes have to be found that commit to a reliable quasi-operational service.
4. During periods around full Moon the observations will be difficult if not impossible; certainly their precision will be degraded.
5. The brightness of Gaia is *a priori* unknown. Theoretical estimates exist but they are under debate. Its definite brightness will only be known once Gaia is in orbit around L_2 (see also section 3.4).
6. For the anticipated precision of the measurements and their proper usage in the orbit determination the following has to be known more precisely than is usually available:
    a. The location of the telescopes on Earth (more precisely the crossing of its two movable axes) has to be known to better than 7 m, i.e. a geo-survey at the telescope will be required.
    b. The time tag of the observations has to be accurate to at least 0.1 sec.

At the time of writing the only suitable telescope that has committed to the GBOT program is the 2.0 m Liverpool Telescope on La Palma. Other telescopes have been technically evaluated, the ESO VLT Survey 2.6 m telescope in Chile, the Pic du Midi 1.06 m telescope and the Faulkes South and North 2.0 m telescopes in Australia and USA respectively, with positive results. However, a firm commitment of those to the GBOT program is not yet in place.

3.2. Interface GBOT-ESOC

As outlined in the previous section the data reduction of the CCD images of Gaia in front of the stellar background is the full responsibility of the GBOT group. This includes as well calibration and corrections to the measurements like e.g. differential refraction. The processed observations that are transferred to the Gaia Flight Dynamics system in ESOC are pairs of topocentric right ascension and declination with respect to the International Celestial Reference Frame (ICRF). In addition, measurement uncertainties and ancillary information concerning the measurement process are made available via two ASCII interface files whose data contents and fixed format are defined in [3]. They have been agreed by all participants; ESOC, GBOT and the Gaia Science Operations Centre (SOC) and are in the following referred to as GBOT-ESOC interface files.

3.3. Processing of Astrometric Data in ESOC's Orbit Determination Program

In order to include the astrometric observations of Gaia into the orbit determination their normalised residuals need to be computed that are subsequently fed into the estimation filter [1]. The normalised residuals are the differences between the observed and computed observations factored by the inverse of the assumed measurement precision, the latter usually being the measurement standard deviation. In order to do so the astrometric observables have to be modelled within the orbit determination based on given *a priori* information. The overall error in the modelling has to be at least a factor 10 smaller than the expected measurement precision, i.e. in the case here the modelling has to be accurate to 1 mas.

The GBOT-ESOC interface files give the terrestrial (Earth-fixed) coordinates of the participating telescope. These are transformed to the corresponding Solar system barycentric coordinates w.r.t ICRF at the observation time t_3 which is converted to coordinate time. The space coordinate transformation encompasses a series of rotations and a translation. The former is performed using algorithms and parameters describing the Earth orientation and rotation that are provided by the International Earth Rotation and Reference Systems Service (IERS). The latter translation between the centre Earth and the Solar system barycentre is performed by a Lorentz transformation [4]. The time transformation between proper time at the telescope on Earth (which is assumed to be International Atomic Time, TAI) and coordinate time (for which Barycentric Dynamic Time TDB [5] is used) is applied by the following equation:

$$\left(\mathrm{TDB} - \mathrm{TAI}\right)_{\mathrm{telescope}} = \left(\mathrm{TDB} - \mathrm{TT}\right)_{\mathrm{geocentre}} + \frac{1}{\mathrm{c}^2}\left(\dot{\vec{\boldsymbol{r}}}_{\boldsymbol{E}}^{\boldsymbol{B}} \cdot \vec{\boldsymbol{r}}_{\mathrm{telescope}}^{\boldsymbol{E}}\right) + 32.184\ \mathrm{s}\ . \qquad (1)$$

The first term is the difference between coordinate time in the barycentric celestial reference system, TDB, and coordinate time in the geocentric celestial reference system (terrestrial time TT) at the centre of the Earth. This term is pre-computed and provided as a time ephemeris together with the planetary ephemerides. For Gaia INPOP (Integrateur Numerique Planetaire de l'Observatoire de Paris) time and planetary ephemerides [6] are used by both the Flight Dynamics and the DPAC system to ensure consistency between reconstructed orbital data and exploitation of scientific data. The second term accounts for the location of the telescope on the surface of the Earth and the third term is per definition the difference between TT and TAI. Applying these transformations gives $\vec{r}_3^{B}(t_3[TDB])$, the barycentric position of the telescope w.r.t. ICRF at the observation time t_3 in TDB.

Given the spacecraft orbital data w.r.t. the ICRF as a function of TDB, the light time equation between the Gaia spacecraft and the telescope on Earth can be solved yielding the barycentric position of the spacecraft at light transmission time t_2 in TDB, $\vec{r}_2^{B}(t_2[TDB])$.

Let $\vec{r}_{32}^{B} = \vec{r}_2^{B}(t_2[\mathrm{TDB}]) - \vec{r}_3^{B}(t_3[\mathrm{TDB}])$ denote the topocentric position vector between spacecraft and telescope with components *(x,y,z)* and $d = \left|\vec{r}_{32}^{B}\right| = \sqrt{x^2 + y^2 + z^2}$ denote the topocentric distance to the spacecraft. The computed values of right ascension α and declination δ are then simply given by

$$\alpha_c = \tan^{-1}\left(\frac{y}{x}\right) + \Delta\alpha \tag{2}$$

$$\delta_c = \sin^{-1}\left(\frac{z}{d}\right) + \Delta\delta \tag{3}$$

where Δα and Δδ are observable specific biases that can be treated as uncertain within the orbit determination.

The GBOT-ESOC interface files give the standard deviation for each observation as well as a measurement error for the time tag. This timing error can be absorbed into the actual observation error as follows. Let an error with subscript T denote the total error including the effect of a time tag error and Δ denote a random error that has zero mean. Then:

$$\Delta\alpha_T = \Delta\alpha + \dot{\alpha}\Delta t \tag{4}$$

$$\Delta\delta_T = \Delta\delta + \dot{\delta}\Delta t\ . \tag{5}$$

From the definition of variance, denoted by *V*, and the fact that the time tag error is independent from the actual measurement error:

$$V(\alpha_T) = V(\alpha) + \dot{\alpha}^2 V(t) \tag{6}$$

$$V(\delta_T) = V(\delta) + \dot{\delta}^2 V(t). \tag{7}$$

With the notation of Eq. (2) and (3) it is straightforward to show that

$$\dot{\alpha} = \frac{x\dot{y} - \dot{x}y}{x^2 + y^2} \tag{8}$$

$$\dot{\delta} = \frac{\dot{z}\left(x^2 + y^2\right) - z(x\dot{x} + y\dot{y})}{d^2\sqrt{(x^2 + y^2)}}. \tag{9}$$

The astrometric observations in the case here are not uncorrelated, since they are derived from the same measurement and by using the same data reduction process. The GBOT-ESOC interface files give therefore a correlation coefficient between right ascension and declination. Similarly to Eq. 6 and 7, from the definition of covariance, denoted by *Cov*,

$$Cov(\alpha_T, \delta_T) = Cov(\alpha, \delta) + \dot{\alpha}\dot{\delta}V(t) \tag{10}$$

a total correlation coefficient ρ_T that takes the time tag error into account can be derived:

$$\rho_T = \frac{Cov(\alpha_T,\delta_T)}{\sqrt{V(\alpha_T)V(\delta_T)}}\,. \tag{11}$$

The normalisation of the residuals is then achieved by multiplying the observation equation in the orbit determination filter by the inverse of the matrix square root of the 2x2 observation covariance matrix which in the case of correlated observations is non-diagonal. There is more than one solution and the one chosen here leaves the normalised right ascension residuals unaltered, i.e.

$$\widetilde{RES}(\alpha) = \frac{RES(\alpha)}{\sigma_{\alpha_T}} \tag{12}$$

$$\widetilde{RES}(\delta) = \frac{1}{\sqrt{1-\rho_T}}\left(\frac{RES(\delta)}{\sigma_{\delta_T}} - \frac{\rho_T\, RES(\alpha)}{\sigma_{\alpha_T}}\right) \tag{13}$$

where $\widetilde{RES}$ and RES stand for normalised and unormalised residuals respectively and the σ's are the (total) standard deviations of the corresponding observation that are the square root of the variances as defined in Eq. 6 and 7.

The normalised residuals defined in Eq. 12 and 13 are fed into the orbit determination filter. Moreover, there are defined parameters that affect the modelling of the astrometric observations directly and that can be treated as uncertain – solve-for or consider (see Sec. 4.1) - within the orbit determination process. These are the following:

- Observation bias (see Eq. 2 and 3);
- Earth orientation parameters;
- Telescope terrestrial (Earth-fixed) coordinates;
- Telescope clock off-set and drift.

In order to treat them as uncertain the partial derivatives of the observation w.r.t. the parameter is computed in the orbit determination software. The derivation of these partial derivatives is outside the scope of this paper.

3.4. Tests with Flying Spacecraft

In 2008 the NASA WMAP spacecraft was observed from the European Southern Observatory La-Silla 2.2 m telescope and Pic du Midi 1.06 m telescope. In April, August and September 2010 the ESA Planck spacecraft was observed from the Liverpool telescope on La Palma. A further observation campaign of Planck has been performed from the Pic du Midi telescope in April 2011. Both spacecraft are in orbit around the L_2 point as Gaia will be. However, size and relative geometry of the Sun shield orientations to the Earth direction (Sun aspect angle) are significantly different from those of Gaia. The diameter of the WMAP sunshield is 5 m and its Sun aspect angle 22.5 deg. The diameter of the Planck sunshield is 4 m and its Sun aspect angle is smaller than 10 deg. Gaia's sunshield will have a diameter of 10.1 m and a Sun aspect angle of 45 deg. The observed visible magnitude of WMAP was between 18.5 and 19.2 and those of Planck between 17 and 19. Mainly because of its larger size and despite its unfavourably large Sun aspect angle the brightness of Gaia is expected to be similar to that of Planck.

From these observations campaigns, the Planck reduced astrometric data have been provided to ESOC for testing purposes. As an example, results of the observation campaign using the Liverpool telescope in August and September 2010 are shown in this paper. Between six and ten pairs of reduced astrometric observations taken on 7 nights in the period between 05 August and 01 September 2010 were made available. These were processed with the ESOC software and the *a priori* residuals using the Planck reconstructed orbit were computed (a so-called pass-through). The Planck orbit had been reconstructed using radiometric range and Doppler data. Fig. 2 displays the results and Tab. 1 gives the corresponding residuals statistics. The given right ascension residuals are weighted with the cosine of the declination. The deviation from zero is mostly due to an error in the Planck reconstructed orbit which should be in the order of 500 to 1000 m. A difference of 100 mas corresponds at the time to an error of 700 m in the Planck ephemeris. Any systematic error in the observation cannot be detected with a pass-through without any independent plane-of-sky measurement. In contrast, the residuals spread of observations that have been collected during one night give an indication of the random error. From Tab. 1 one can read the worst case standard deviation being 110 mas in right ascension on 05 August. This appears to be a one-off. The standard deviations of the residuals from the other nights are between 20 and 70 mas. It can be concluded that astrometric measurements of a spacecraft at L2 with brightness magnitude between 17 and 19 that have been reduced using nowadays available star catalogues are possible with a precision of about 50 mas.

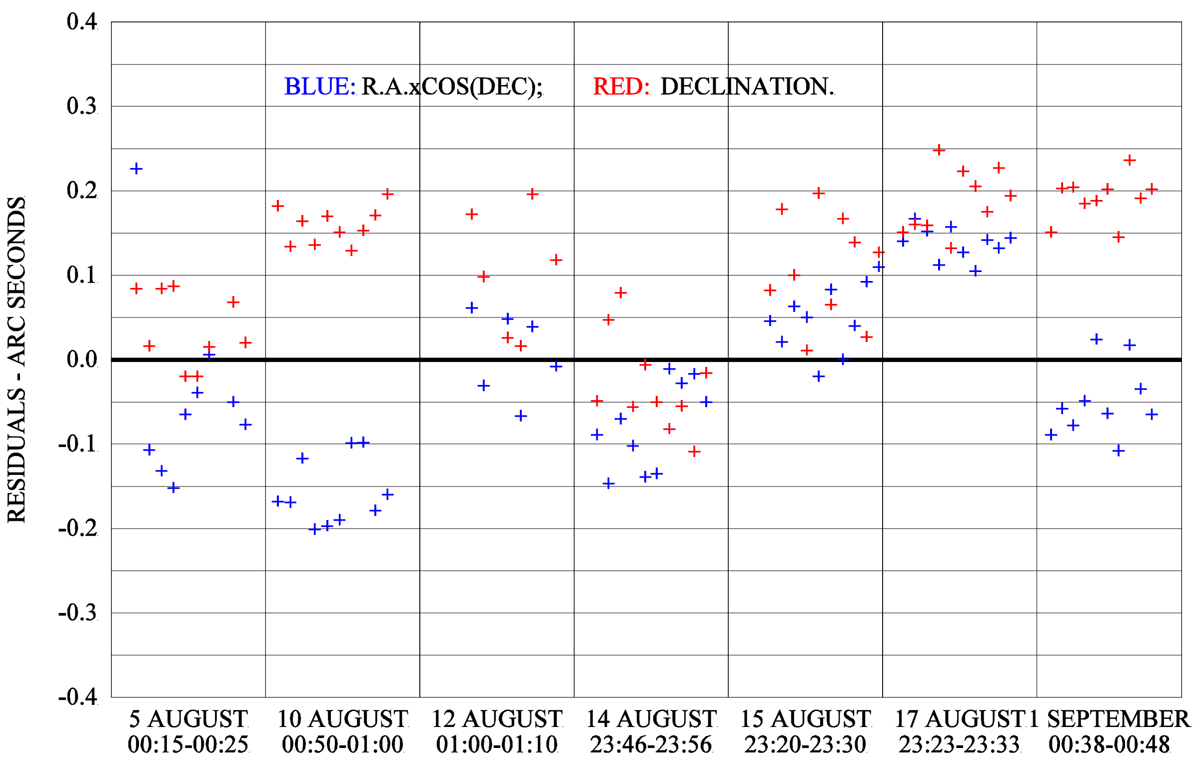

Figure 2. Residuals of Planck astrometric data from La Palma in 2010.

Table 1. Residuals statistics of Planck astrometric data from La Palma in 2010. All values are given in arcsec.

Date	No.	Mean		RMS		Stand. Dev.	
		RACD	DEC	RACD	DEC	RACD	DEC
05 August	9	-0.04	0.04	0.11	0.06	0.11	0.04
10 August	10	-0.16	0.16	0.16	0.16	0.04	0.02
12 August	6	0.01	0.10	0.05	0.12	0.05	0.07
14 August	10	-0.08	-0.03	0.09	0.06	0.05	0.05
15 August	10	0.05	0.11	0.06	0.12	0.04	0.06
17 August	10	0.14	0.19	0.14	0.19	0.02	0.04
01 September	10	-0.05	0.19	0.06	0.19	0.04	0.03
All	65	-0.02	0.11	0.11	0.14	0.11	0.09

4. Orbit Determination Performance Verification Study

This section summarises the results of a study into the accuracy and precision to be expected from the updated operational orbit determination system. The scope of the study is to derive the operational configuration of the orbit determination system and assess the performance of that system by means of processing sets of simulated data. Multiple scenarios reflecting different assumptions on observations quality and quantity, spatial geometry and dynamical environment are analysed. These scenarios have been subject to a covariance analysis (section 4.2) and to a tracking data simulation (section 4.3). The full details of the study are given in [7].

4.1 Orbit Determination Scenarios

Several radiometric tracking scenarios differing in station schedules and total contact times are analysed. In all cases, the data collection rate is 1 measurement per minute, and the minimum elevation is 15 degrees. This applies to both range and Doppler. The noise levels of the radiometric tracking data types are taken as 1 m for 2-way range and 0.1 mm/s for 2-way Doppler. The latter refers to the standard compression time of 60 s. For orbit determination purposes, the weight of both range and Doppler observables is decreased by a factor of 3.36, as suggested in [8], to obtain a more realistic *a posteriori* covariance. Two different noise levels of the astrometric measurements are analysed: the anticipated 10 mas and a more conservative value of 20 mas. The study assumes that there is one astrometric measurement available per telescope every day, with the exception of data gaps centred on the middle of the observations arc of a minimum duration of three days (to simulate data outage around the time of a full Moon). Astrometric data source considered in this study is the Liverpool telescope located at La Palma. Usage of more than one telescope simultaneously that are widely spread in location is also analysed and no significant change in results is observed.

The orbit of Gaia is numerically integrated with the Earth as centre of integration. The dynamic model includes the central potential of all major solar system bodies (whose states are taken from the INPOP planetary ephemerides [6]), the acceleration due to solar radiation pressure (SRP), and the acceleration due to the MPS (for fine attitude control) and the CPS (for orbit control manoeuvres). The noise of the MPS is believed to assume a level of 10^{-12} km/s^2, which is more than one order of magnitude larger than residual accelerations due to the spherical harmonic expansion of the gravity fields of the Earth and the Moon as well as their relativistic contributions. Therefore, a higher fidelity model of the dynamical environment is deemed unnecessary.

Because of the 45 degree tilt of the bottom plate relative to the Sun-spacecraft line, the SRP force exerted on Gaia has a significant component perpendicular to the principal direction. The SRP force is therefore conveniently represented in an orbital frame with axes along the Sun-spacecraft line (x), perpendicular to x and Gaia's spin axis (z), and perpendicular to x and z in a right-handed sense. In this frame the SRP acceleration is along the x and y axis whereas along z it is nominally zero. Due to the dimension of Gaia's Sun shield the acceleration due to SRP is comparatively large. In its operational orbit the magnitude is 1.4 x 10^{-10} km/s^2 and 0.8 x 10^{-10} km/s^2 along x and y respectively.

Within the orbit determination two types of uncertain parameters are distinguished: solve-for parameters and consider parameters. Associated with parameters of either type are uncertainties, given either as standard deviations or covariance matrices. The values of solve-for variables are adjusted in the orbit determination process to yield the best fit solution. In parallel, the *a priori* covariance is reduced to yield the *a posteriori* covariance. Consider parameters, on the other hand, are not subject to estimation and retain both their *a priori* value and uncertainty. Table 2 lists the configuration employed for both covariance analysis and orbit determination purposes.

Table 2. Orbit determination filter configuration

Parameter	**Type**	***A priori* standard deviation**
Spacecraft position component	Solve-for	1000 km in each component
Spacecraft velocity component	Solve-for	100 m/s in each component
SRP acceleration	Solve-for	5 % of nominal in x and y, 0.5 % of nominal total in z (see text for details)
CPS acceleration (OCM)	Solve-for	5% in magnitude and 3 deg in direction
MPS acceleration	Solve-for	1.7x10^{-12} km/s^2 in each component
Station range bias per pass	Solve-for	1 m
Station terrestrial position	Consider	10 cm in each component
Telescope terrestrial position	Consider	5 m in each component
Spacecraft transponder delay	Consider	10 ns
Earth pole position error	Consider	30 nrad
Earth rotation error	Consider	0.75 ms
Troposphere zenith delay (wet part)	Consider	4 cm
Troposphere zenith delay (dry part)	Consider	1 cm
Ionosphere delay	Consider	25% of nominal

Two different station schedules for the acquisition of radiometric tracking data, in the following referred to as case 1 and case 2, are investigated (see Tab. 3). Case 1 is the reference case during the nominal scanning law featuring daily passes from only one ESA deep-space antenna either in Cebreros (CEB) or New Norcia (NNO) yielding 6 hours' worth of Doppler data and 2×5 minutes worth of range data. Case 2 corresponds to the time of the modified scanning law during which the daily contact time is significantly increased, with both CEB and NNO tracking daily most of the time. Ranging is however performed from one station only and again only in two ranging session each giving range data over a period of 5 minutes.

All analyses conducted in the study focus on either of two observations arcs. The first covers the time period when the geocentric declination of Gaia is low. In the following this case is referred to as the low declination scenario. The second observations arc looked into covers the time period when the (absolute) declination of Gaia is high (> 20°). This high declination scenario represents the best case in terms of orbit determination performance using line-of-sight observables.

The orbit determination for Gaia is performed in a batch mode, i.e. a batch of observations is processed and the orbit over the length of the batch time is reconstructed. The orbit reconstruction in the first iteration is conducted on a weekly basis and the length of the observations batch is about four weeks. Hence, two successive orbit determination solutions share a common three-week interval of the data arc. From the weekly solution the reconstructed orbit is updated not from the beginning of the observations arc but only from about day 12 into the arc. The orbit start time is chosen to be well inside the "trust region" of the orbit solution in order to avoid artefacts at the observations arc's early boundary. Given the weekly operations schedule, the reconstituted orbit eventually consists of patches of orbit solutions, each patch covering a time span of seven days and coinciding with the central week of the respective observations arc. Therefore, the orbit determination performance figures of interest are given by the maximum values of the formal standard deviations of the individual spacecraft state components during the central week of the observations arc. Note that in the study the OCM (see Tab. 2) is scheduled such that it occurs in the middle of the central week of the observations arc.

The second iteration orbit reconstruction could be performed in a different mode with a much longer time batch of observations and hence a longer time period for the reconstructed orbit, in particular since the precise astrometric measurements are foreseen to be delivered on a biannual basis. However, for the purpose of the study the first and second orbit reconstructions are run in the same mode and no significant differences in the results are expected if later on a different mode is chosen for the real second iteration orbit reconstruction.

Table 3 defines the numeration of the scenarios that are analysed. All scenarios are run with observations arcs at high and low geocentric declination.

Table 3. Definition of scenarios.

ID	Scenario
1	Nominal scanning law, daily tracking passes of either NNO or CEB of 6 hours duration
2	Modified scanning law, daily CEB and NNO passes with 8 hours duration each
3	Scenario 1 + one daily pair of astrometric observation with 10 mas precision with the exception of a 3 day data gap about the centre of the observations arc.
4	Scenario 1 + one daily pair of astrometric observation with 20 mas precision with the exception of a 3 day data gap about the centre of the observations arc.
5	Scenario 2 + one daily pair of astrometric observation with 10 mas precision with the exception of a 3 day data gap about the centre of the observations arc.
6	Scenario 2 + one daily pair of astrometric observation with 20 mas precision with the exception of a 3 day data gap about the centre of the observations arc.
7	Scenario 4 + astrometric observations have a systematic error of 10 mas
8	Scenario 6 + astrometric observations have a systematic error of 10 mas

4.2. Covariance Analysis

Figure 3 shows the results of covariance analyses of the aforementioned scenarios. Shown are the maximum *a posteriori* formal standard deviations of each component of the reconstructed spacecraft position and velocity within the one week reconstructed orbit. They are given with respect to the plane-of-sky reference system with red and black denoting the components in the plane-of-sky in the directions North-South (N-S) and East-West (E-W) respectively and green denoting the component orthogonal to the plane of sky in the direction Earth to spacecraft (radial). The requirements on the final Gaia orbit reconstruction are also shown as dotted horizontal lines. From Fig. 3 the following conclusions can be drawn:

- The radial spacecraft position and velocity component is the best determined component since it is directly measured by range and Doppler. The *a posteriori* covariance mainly reflects the assumed noise and bias on those observables.
- As expected, at low declination the least well determined component when only line-of-sight measurements are available is the N-S component in the plane-of-sky.
- The increase in tracking data quantity between scenario 1 and 2 does not improve the orbital knowledge any further. The reason is that in this case the contribution of the consider covariance is dominating the covariance of the spacecraft state and hence the latter does not decrease with increasing amount of tracking data.
- The addition of astrometric observations reduces the covariance in the N-S component. The effect is not seen in the E-W direction because the dominating contributor to the uncertainty is the assumed consider error in the Earth rotation (Tab. 2) which the radiometric data are very sensitive to.
- The Gaia reconstructed position requirement can formally be fulfilled either at high declination or at low declination with the availability of daily precise (< 20 mas) astrometric data.
- The Gaia reconstructed velocity requirement can formally be fulfilled with conventional radiometric data alone at both high and low declination.

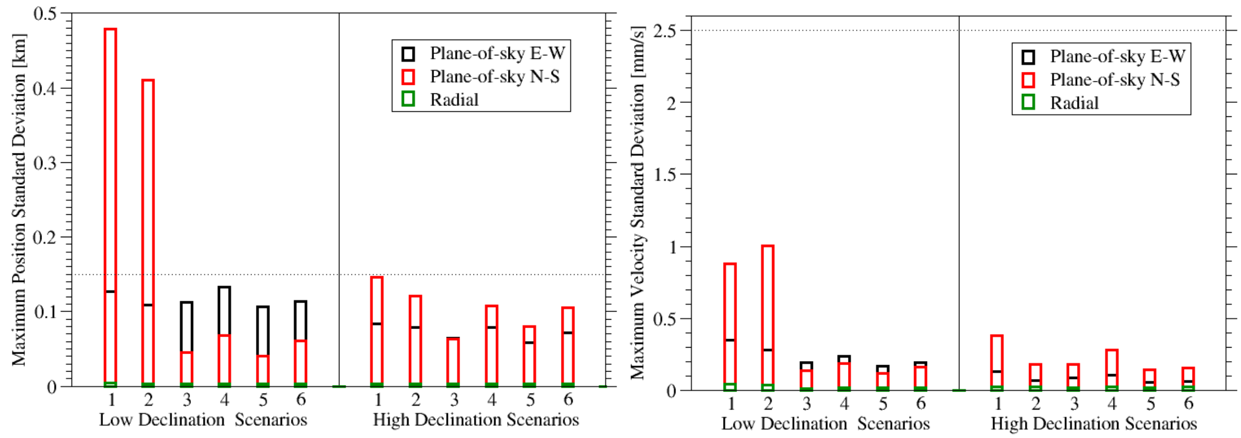

Figure 3. Covariance analysis results (see text for details)

4.3. Simulation

A simulation is performed in addition to the covariance analysis with the purpose to assess the differences between the true orbit and the determined orbit and if those differences are commensurate with the formal covariance figures presented in the previous section. The true orbit emerges from the propagation of an initial state which is taken from the study reference orbit and then perturbed. The dynamical model employed by the orbit determination system is chosen deliberately different from the model that is used for the propagation of the true orbit. Based on the true orbit radiometric tracking and ground based astrometric data are simulated and superimposed with random and systematic errors that are by and large commensurate with the assumptions made in the orbit determination system (see Sec. 4.1 and Tab. 2). The results are shown in Fig. 4 in the same way as the covariance results have been shown in Fig. 3. The difference is that Fig. 4 shows the maximum difference between the determined and the true orbit within the central week of the observations batch and that two more scenarios are included (7 and 8 from Tab. 5) in which the astrometric measurements are subject to systematic biases (which is difficult to treat in a covariance analysis). From Fig. 4 the following conclusions can be drawn:

- For the radiometric data only case 1 at low declination the N-S position difference is 923 m and exceeds the upper limit of the plot. The difference corresponds to 2σ of the corresponding covariance displayed in Fig. 3. Surprisingly, the maximum position differences in the radiometric-only scenario 2 at low declination are comparatively small compared to their corresponding covariance in Fig. 3. This is likely to be a one-off.
- The maximum position deviations between the true orbit and the determined orbit are by and large consistent with the covariance analysis results. The maximum velocity deviations between the true orbit and the determined orbit turn out to be larger than the formal errors would suggest, especially at low declination. This means that the covariance analysis results tend to be too optimistic.
- A systematic error of 10 mas or larger has a detrimental effect on the orbit determination especially in the ability to reconstruct the spacecraft position at low declination.
- The position reconstruction requirement can be only marginally fulfilled at low declination and only if highest precision and unbiased astrometric data are available.

- The velocity reconstruction requirement in terms of both – precision and accuracy - is confirmed to be fulfillable at low and high declination and apparently with radiometric tracking alone. However, in order to verify the true accuracy of the reconstructed orbit using only radiometric tracking data an independent plane-of-sky measurement is still required.

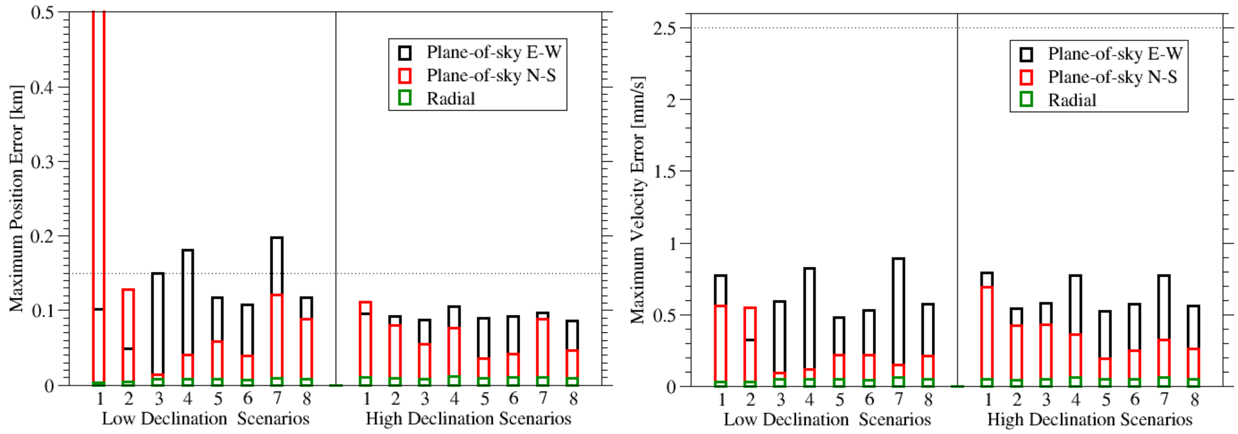

Figure 4. Simulation results (see text for details)

6. Conclusion

ESOC's interplanetary orbit determination system has been enhanced to include precise ground based astrometric observations of a spacecraft in order to fulfil the demanding orbit reconstruction requirements of the Gaia mission. In a study that has been performed using the new software it could be shown that the availability of astrometric plane-of-sky information is of paramount importance for the sake of orbit determination, especially for two main reasons:

1. to improve the knowledge on both the spacecraft's geocentric declination (at times when the spacecraft is at low geocentric declination) and right ascension (which is significantly impaired by any error in Earth rotation model if only radiometric data are used);

2. to provide independent means to verify the correctness of the orbit solution that has been reconstructed using radiometric line-of-sight measurements alone.

The velocity reconstruction requirement appears to be met in all analysed scenarios. In contrast, the position reconstruction requirement can only be fulfilled (but without any margin) if the spacecraft is either at high geocentric declination or the highest precision astrometric data are available on an almost daily basis. As a result the chosen baseline for the Gaia mission is to aim for the availability of precise astrometric data on an almost daily basis throughout the entire mission.

There are nevertheless some risks involved in the chosen orbit reconstruction approach for Gaia:

1. The true brightness of Gaia is only known after launch once placed in orbit around L_2. It is assumed to have a visible magnitude between 17 and 19 and the measurement process for the

astrometric data is prepared and optimised for this range. Deviations in either direction will cause degradation in the quality of the measurements.

2. The achievable accuracy and precision of astrometric data that will be reduced using the Gaia preliminary star catalogue will only be known late into the science phase of the mission when the measurements have already been taken. Any deficiency in the measurement process that is detected at this stage cannot be changed anymore.

3. For the first time at ESA, the provision of observables for the purpose of orbit determination is partly the responsibility of a mission science consortium, i.e. the orbit reconstruction process relies on non-proven operational data with yet unknown accuracy. One concern along that line is the risk of having systematic errors in the astrometric data that are difficult if not impossible to detect if no other independent plane-of-sky measurements are available. To mitigate this, occasional Delta-DOR measurements of Gaia are foreseen that will be taken from the ESA deep space antenna network. Delta-DOR is an ESA operationally validated data type whose reliability has been proven with past - mainly deep space - missions. Tests with the Gaia transponder have confirmed recently its ability to support this type of measurement although it was originally not designed for it.

7. References

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