ENVIROMENTAL TORQUES AND THE SOLAR ASPECT ANGLE

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Abstract: *The aim of this paper is to study the influence of the environmental torques in the angle between the spin axis and the Sun direction for spin stabilized satellite. The theory uses a cylindrical satellite which is in an illumined orbit. Mathematical model for the gravity gradient, aerodynamic, solar radiation, residual magnetic and eddy current torques are shown. The dynamic equations are represented in a satellite reference system and described by spin velocity, right ascension and declination angles of the spin axis. An analytical solution is gotten for the spin velocity and the attitude angles. The spin velocity has an exponential variation due to the eddy current and the gravity gradient torques. The combinations of the all torques contribute for a precession and drift on the spin axis, due to the temporal variation on the right ascension and declination angles respectively. These analytical solutions are used to study the behavior of the solar aspect angle. The theory is applied for the real data of the Brazilian Satellite of Data Collection - SCD1 and SCD2 and two approaches were presented. Results have shown the agreement between the analytical solution and the real satellite behavior for specific time simulation.*

Keywords: *spin stabilized satellite, aerodynamic torque, solar radiation torque, magnetic torques, gravity gradient torque.*

1. Introduction

The goal of this paper is to analyze the influence of the some external torques in the solar aspect angle, considering a cylindrical spin stabilized satellite in an illumined orbit. The Earth's shadow is not considered. Mathematical model for the gravity gradient, aerodynamic, solar radiation, residual magnetic and eddy current torques are shown.

The gravity gradient torque (GGT) is created by the difference of the Earth gravity force direction and intensity acting on each satellite mass element and is inversely proportional to the cube of the satellite geocentric distance. Therefore it decreases when the altitude increases.

The aerodynamic torque (AT) is created by the interactions of rarefied air particles with the satellite surface and is predominant in satellites with low altitude, because it depends on the quantity of air molecules in the Earth atmosphere. Calculation of aerodynamic torques for realistic spacecraft is not very accurate because of existing uncertainties in the atmospheric density and in drag coefficient. In this paper TD-88 model is used to describe the atmospheric density.

The solar radiation pressure (SRT) is created by the continuous photons collisions with the satellite surface, which can be able to absorb or reflect on this flow. The total change of the momentum of all the incident photons on the satellite surface originates from the solar radiation force and a torque can be produce.

Magnetic disturbance torques result from the interaction between the spacecraft's residual magnetic field and the Earth´s magnetic field. The residual magnetic torque (RMT) results from the interaction between the spacecraft's residual magnetic moment and the Earth magnetic field and its main effect are to produce a spin axis orientation drift. The torque induced by eddy currents (ECT) is caused by the spacecraft spinning motion and produces a reduction in the satellite spin rate with time.

A spherical coordinate system fixed in the satellite (*body system*) is used to locate the spin axis of the satellite in relation to the terrestrial equatorial system, with the axis z being along the direction of the spin velocity vector. The directions of the spin axis are specified by the right ascension (α) and the declination (δ) as represented in the Fig. 1.

The dynamic equations are represented in a body system and described by spin velocity and the right ascension and declination of the spin axis. These equations depend on the torques components in the body system. The averages of each torque's components are determined over on orbital period and are substituted in the equations of motion. Due to the cylindrical form of the satellite and others simplifications, only the gravity gradient and eddy currents torques have a non null z-components.

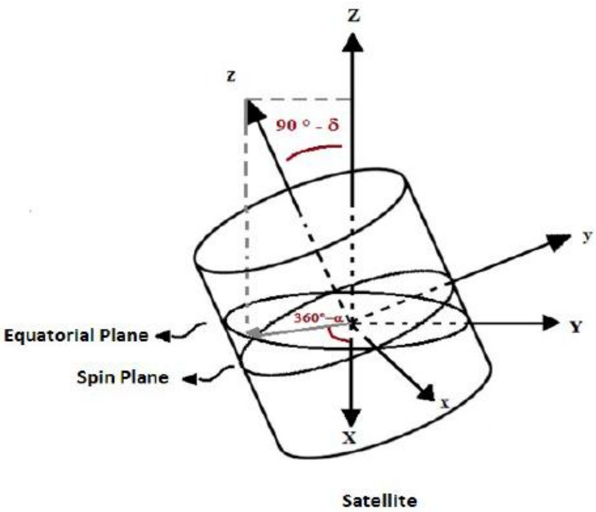

Figure 1 – Body System (xyz) and Equatorial System (XYZ): right ascension (α) and declination (δ) of the spin axis (z).

An analytical solution is gotten for the spin velocity and the attitude angles for one orbit period. These analytical solutions are used to study the behavior of the solar aspect angle, which is gotten by the dot product of the Sun direction and the spin axis, and depend on the declination and the right ascension of the spin axis and declination and the right ascension of the Sun direction.

The theory is applied for the real data of the Brazilian Satellite of Data Collection - SCD1 and SCD2, which are quite appropriated for verification and comparison of the data generated and processed by the Satellite Control Center (CCS) of the Brazil National Research Institute

(INPE). Two approaches were presented. In the first one the attitude and orbital data are daily updated with real attitude data supplied by INPE. In the second approach the attitude and orbital data are not updated.
Others analysis of the solar aspect angles [1,2,4,13,15] have been developed but they haven't considered the all torques acting together. Then the present theory could be more realistic than the other ones.

2. Mathematical Model for the External Torques

Gravity Gradient Torque Model

The GGT [1,3,8,11] for a spin stabilized satellite in a body system ($\hat{i}, \hat{j}, \hat{k}$) can be expressed by:

$$\vec{N}_g = N_{gx}\,\hat{i} + N_{gy}\,\hat{j} + N_{gz}\,\hat{k}, \tag{1}$$

with

$$N_{gx} = 3\frac{\mu}{r'^3}\left[a_{21}\,a_{31}\left(I_z - I_y\right)\cos\theta - a_{11}\,a_{31}\left(I_x - I_z\right)sen\theta\right], \tag{2}$$

$$N_{gy} = 3\frac{\mu}{r'^{\,3}}\left[a_{21}\,a_{31}\left(I_z - I_y\right)sen\theta + a_{11}\,a_{31}\left(I_x - I_z\right)\cos\theta\right], \tag{3}$$

$$N_{gz} = 3\frac{\mu}{r^3}\left[a_{11}\,a_{21}\left(I_y - I_x\right)\right], \tag{4}$$

where μ (3.986 x 10^{14} m³/s²) is the Earth gravitational parameter, r' is the satellite geocentric distance, a_{11}, a_{21} and a_{31} are the direction cosines which relate the orbital system and the satellite fixed system, I_x, I_y, I_z are the Principal Moments of Inertia of the satellite and θ is the angle between the satellite principal axis of inertia x' and the body axis x, defined in each instance by the product of the spin velocity W and the time t. The elements a_{11}, a_{21} and a_{31} depend on the orbital elements (orbit inclination, true anomaly, longitude of the ascending node and argument of the perigee), the angle θ and the right ascension and declination of the spin axis [3,14,15]. Equation 2 shows that this torque decreases with the cube of the altitude and depends on the shape, dimension and mass distribution of the satellite.

To compute the mean components of each considered torque in the body system, an average time in the fast varying orbit element, the mean anomaly, is used. This approach involves several rotation matrices, which are dependent on the orbit elements, right ascension and declination of the satellite spin axis. The mean GGT was developed in [1,3] and it also presented in [15].

Aerodynamic Torque Model

For this paper will be adopt the following model to represent the AT [2,8,11]:

$$\vec{N}_A = \left[D_z m_{ey} - D_y m_{ez}\right]\hat{i} + \left[D_x m_{ez} - D_z m_{ex}\right]\hat{j} + \left[D_y\, m_{ex} - D_x\, m_{ey}\right]\hat{k} \ , \tag{5}$$

where m_{ex}, m_{ey}, m_{ez} and D_x, D_y, D_z are the components of the position vector between the center of pressure and the center of mass of the satellite and the drag force in a body system. In this paper the influence of the lift force in the AT is neglected and

$$D_x = -D\left[a_{11}\, cos(\gamma_S) + a_{21} sen(\gamma_S)\right], \tag{6}$$

$$D_y = -D\left[a_{12}\, cos(\gamma_S) + a_{22} sen(\gamma_S)\right], \tag{7}$$

$$D_z = -D\left[a_{13}\, cos(\gamma_S) + a_{32} sen(\gamma_S)\right], \tag{8}$$

$$D = \tfrac{1}{2}\rho v^2\, S\, C_D \ , \tag{9}$$

where ρ is the local density, v represents the magnitude of the satellite's velocity relative to the atmosphere, S is a reference section area of the satellite, C_D is the Drag Coefficient, γ_S is the angle between the position vector and the orbital velocity vector and a_{ij} , i=1,2,3, j=1,2, are the direction cosines which relate the orbital system and the body system [3,14,15].

In order to estimate the influence of the AT magnitude in the rotational motion, in this paper the thermosphere model TD-88 is used for the atmospheric density [9] and some simplifications are done. The velocity v is assumed equal to the orbit velocity, the drag coefficient is fixed (equal to 2.2). In the applications for the SDC1 and SCD2 and by the analyses development in [2], it also assumed that $m_{ex} = m_{ey} = 0$ and $m_{ez} = 0.1m$. Then in that case the z - component of AT is null.

By the same way of the GGT, the mean AT was gotten by [2, 15].

Residual Magnetic Torque

The spacecraft's magnetic moment is usually the dominant source between the disturbances torques. If $\vec{m}$ is the magnetic moment of the spacecraft and $\vec{B}$ is the geocentric magnetic field, the RMT is given by [4, 14]:

$$\vec{N}_r = \vec{m} \times \vec{B}. \tag{10}$$

Here the magnetic torque is developed only for a spin-stabilized satellite. In this case, the spacecraft's spin velocity vector and the satellite magnetic moment are along z-axis and the RMT can be expressed in the satellite fixed system by [4,14]:

$$\vec{N}_r = -mB_y\,\hat{i} + mB_x\,\hat{j}\,, \tag{11}$$

where B_x, B_y, B_z are the components of the geomagnetic field in the satellite fixed system [14]. These components are obtained in terms of the geocentric inertial components of the geomagnetic field [11] and the attitude angles of the satellite. In order to describe the geomagnetic field the dipole vector model [14] is used.

The mean components of this torque were obtained in [14] and some details can be observed in [6].

Eddy Currents Torque

The torque induced by eddy currents is caused by the spacecraft spinning motion. It is known [8,11] that the eddy currents produce a torque which causes the precession in the spin axis and causes an exponential decay of the spin rate. If $\overrightarrow{W}$ is the spacecraft's spin velocity vector and p is a constant coefficient which depends on the spacecraft geometry and conductivity, this torque is given by:

$$\vec{N}_i = p\ \vec{B} \times (\ \vec{B} \times \vec{W}\ ). \tag{12}$$

For the spin stabilized satellite the spin velocity vector and is along z-axis and the ECT can the expressed in the body system by [13]:

$$\vec{N}_i = p\ W\ \left(\ B_x\, B_z\, \hat{i}\ +\ B_y\, B_z\ \hat{j} - (\ B_x{}^2 + B_y{}^2\ )\ \hat{k}\ \right)\ . \tag{13}$$

The mean components of this torque are developed in [13] and some details can also been observed in [6].

Solar Radiation Torque

A Solar Radiation Torque model was developed in [12] for the case which the illuminated surfaces of the satellite are a circular flat surface S_1, and a portion of the cylindrical surface S_2. It is given by:

$$\vec{N}_s = N_{Sx}\,\hat{\imath} + N_{Sy}\,\hat{\jmath}\ , \tag{14}$$

$$N_{Sx} = -\frac{\overline{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 u_z{}^*\left(u_y{}^*\cos(\theta) + u_x{}^* sen(\theta)\right), \tag{15}$$

$$N_{Sy} = -\frac{\overline{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 u_z{}^*\left(u_y{}^*\,\mathrm{sen}(\theta) - u_x{}^* cos(\theta)\right), \tag{16}$$

with

$$u_x{}^* = (a_s R_x + r' a_1)\ \ , \tag{17}$$

$$u_y{}^* = \left(a_s R_y + r' a_2\right), \tag{18}$$

$$u_z{}^* = (a_s R_z + r' a_3)\ , \tag{19}$$

where $\overline{K}$ is a solar parameter with assumed value given by 1,01x10[17] kgm/s^2, a_s is Sun-Earth distance and here assume the value 1,49597870x10[11] m, r' is the satellite geocentric distance, R is the Sun satellite distance, β_i, γ_i, $i = 1,2$, are specular and total reflection coefficients, respectively, for each satellite surface (which assume constant values), a_1, a_2 e a_3 are direction cosines which relate the Orbital System and the principal system, given in [6,7] and R_x, R_y e R_z give the Sun direction in the body and are obtained by:

$$R_x = b_1\, cos(\delta_s)\, cos(\alpha_s) + b_2\, cos(\delta_s)\, sen(\alpha_s) + b_3\, s\,en(\delta_s)\,, \tag{20}$$

$$R_y = b_4\, cos(\delta_s)\, cos(\alpha_s) + b_5\, cos(\delta_s)\, sen(\alpha_s) + b_6\, s\,en(\delta_s)\,, \tag{21}$$

$$R_z = b_7\, cos(\delta_s)\, cos(\alpha_s) + b_8\, cos(\delta_s)\, sen(\alpha_s) + b_9\, s\,en(\delta_s), \tag{22}$$

being b_1, b_2, …, b_9 the direction cosines which relate the Equatorial System and body system (given in terms of satellite's rotation angle, right ascension and declination of the spin axis [6,7]) .

The Sun-satellite distance is represented in Fig. 1 and can be obtained by:

$$R^2 = a_s{}^2 + r'^2 + 2r'\mathrm{a_s}\left(a_1R_x + a_2R_y + a_3R_z\right). \tag{23}$$

By the equation Eq.(14) it is possible to observe that the component of the satellite axis "Oz" is zero, due to the satellite geometric symmetry (cylindrical shape [12]).

The mean components of SRT are developed in [7] and some details can beobserved in [6].

3. Analytical Solution for the Equations of Rotational Motion

The variations of the spin velocity, the declination and the right ascension of the spin axis for spin stabilized artificial satellites are given by the Euler equations in spherical coordinates [10], when the mean components of the external torque are included:

$$\frac{dW}{dt} = \frac{N_{gzm}}{I_z} + \frac{N_{izm}W}{I_z}, \tag{24}$$

$$\frac{d\delta}{dt} = \frac{N_{tym}}{I_z W} + \frac{N_{iym}}{I_z}, \tag{25}$$

$$\frac{d\alpha}{dt} = \frac{N_{txm}}{I_z W \cos\delta} + \frac{N_{ixm}}{I_z \cos\delta}, \tag{26}$$

where I_z is the moment of inertia along the spin axis, N_{ixm}, N_{iym}, N_{izm} are the mean components of eddy currents torques and $N_{tym} = N_{Ay} + N_{sy} + N_{gym} + N_{rym}$ and $N_{txm} = N_{Ax} + N_{sxm} + N_{gxm} + N_{rxm}$, being N_{Axm}, N_{Aym}, N_{Azm}, N_{sxm}, N_{sym}, N_{szm}, N_{gxm}, N_{gym}, N_{gzm}, N_{rxm}, N_{rym}, N_{rzm} the mean components of the aerodynamic, solar radiation, gravity gradient and residual magnetic torques respectively. The details of each mean external torque can be observed in [6].

The differential equations of Eqs. (24) - (26) can be integrated assuming that mean components of each torque is constant over one orbital period. Then for one orbit period the solution of the differential equation of spin velocity is given by:

$$W = \left(W_0 + \frac{N_{gzm}}{N_{izm}}\right) e^{\frac{N_{izm}}{I_z}t} - \frac{N_{gzm}}{N_{izm}} . \tag{27}$$

In order to get the solution of the others equations, it is necessary to substitute the solution given by Eq. (27) in the Eqs (25) and (26) for declination and right ascension of spin axis, respectively, and after some algebraic manipulations the analytical solution can simply be expressed as:

$$\delta = \frac{t}{I_z}\left(N_{iym} - \frac{N_{tym}\,N_{izm}}{N_{gzm}}\right) + \frac{N_{tym}}{N_{gzm}}\ln\left(\frac{W}{W_0}\right) + \delta_0 , \tag{28}$$

$$\alpha = \frac{t}{I_z\cos\bar{\delta}}\left(N_{ixm} - \frac{N_{txm}\,N_{izm}}{N_{gzm}}\right) + \frac{N_{txm}}{N_{gzm}\,\cos\bar{\delta}}\ln\left(\frac{W}{W_0}\right) + \alpha_0 , \tag{29}$$

with: $\bar{\delta} = \frac{\delta-\delta_O}{2}$, δ is the computed declination, W_0, δ_0 and α_0 are the initial values for spin velocity, declination and right ascension of the spin axis.

These solutions are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth oblateness. With this approach, the analytical theory will be close to the real attitude behavior of the satellite.

4. The Solar Aspect Angle

The goal of this paper is to analyze the solar aspect angle η, which is the angle between the spin axis $\hat{k}$ and the Sun direction $\hat{u}$, which can be computed by the dot product of the Sun direction $\hat{u}$ and the spin axis $\hat{k}$, and depend on the declination (δ) and the right ascension (α) angles of the satellite spin direction and the declination (δ_S) and the right ascension (α_S) angles of the Sun direction. They are represented in the Fig. 2.

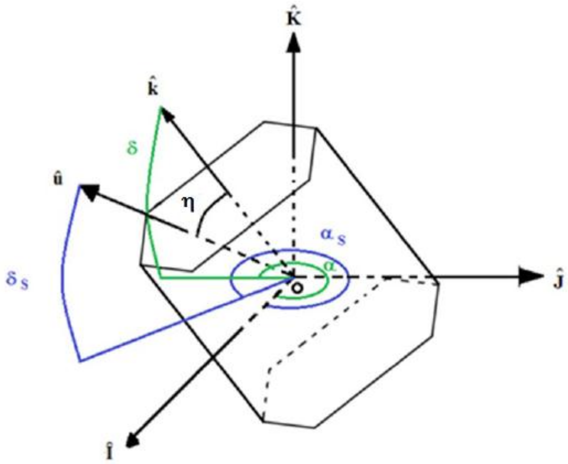

Figure 2 – Solar aspect angle η.

The solar aspect angle can be computed by:

$$\cos\eta = \hat{u} \bullet \hat{k}. \tag{30}$$

The vectors $\hat{u}$ and $\hat{k}$ can be represented in the equatorial system using the angles α_S, α, δ_S and δ, it means:

$$\hat{u} = cos(\alpha_S)cos\,(\delta_S)\hat{I} + sin(\alpha_S)cos\,(\delta_S)\,\hat{J} + \; sin(\delta_S)\hat{K}, \tag{31}$$

$$\hat{k} \;=\; cos(\delta)cos(\alpha)\hat{I} \;+ cos(\delta)sin(\alpha)\hat{J} \;+\; sin(\delta)\hat{K}. \tag{32}$$

Then the angle η is given by:

$$cos\,\eta = cos(\delta)cos(\alpha)cos(\delta_S)cos(\alpha_S) + cos(\delta)sin(\alpha)cos(\delta_S)sin(\alpha_S) + sin(\delta)sin(\delta_S) = M, \tag{33}$$

$$\eta = acos\, M \tag{34}$$

with $0^o < \eta < 180^o$.

For the mission of the satellite SCD1 and SCD2 there is a restriction for the solar aspect angle: $60^o < \eta < 90^o$ for SCD1 and $80^o < \eta < 100^o$ for SCD2 [4,6].

5. Applications

The theory developed has been applied to the spin stabilized Brazilian Satellite SCD1 and SCD2 for verification and comparison of the theory against data generated by the SCC of INPE. Operationally, SCC attitude determination comprises: sensors data pre-processing, preliminary attitude determination and fine attitude determination. The pre-processing is applied to each set of data of the attitude sensors collected from every satellite that pass over the ground station. Afterwards, from the whole pre-processed data, the preliminary attitude determination produces estimative to the spin velocity vector from every satellite that pass over a given ground station. The fine attitude determination takes (one week) a set of spin velocity vector and estimates dynamical parameters (spin velocity vector, residual magnetic moment and Foucault parameter). Those parameters are further used in the attitude propagation to predict the need of attitude corrections.

Two approaches are assumed to examine the influence of the considered external torques actuating during the evolution of rotational motion of the satellite. In the first approach the attitude and orbit data are updated every 24 hours with the data generated by SCC of INPE. In the second approach the computed attitude and orbit data aren't updated in order to determine the validate period of the analytical solution. In all numerical simulation the orbital elements are updated taking in account the main influence of the Earth oblateness.

Applications for SCD1 satellite

The initial conditions of attitude had been taken from July, 24th, 1993 at 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC) for 40 days and are presented in the Appendix.

Some initial data for SCD1:

Semi-major axis = 7139615.83m
Eccentricity = 0.00454
Inclination= 25^o
Longitude of ascending node = 260.43^o
Argument of perigeu = 260.23^o
Mean anomaly = 102.89^o
Foucault parameter = p = 311.35N m s/T^2 rad
Magnetic moment = m = 0. 809Am^2
I_x=11.06kgm^2
I_y=10.67kgm^2
I_z=13.00kgm^2
γ_1=0.7
β_1=0.1
γ_2=0.5
β_2 =0.1

Results for the first approach: daily updated data

The results for the first approach are shown in Fig. 3 and Fig 4. Figure 3 presents the results for temporal behavior of the solar aspect angle. Figure 4 represents the deviation between the computed values and real values of this angle and it is important to observe that they are smaller than 0.5^o for the time simulation, which is within the dispersion range of the attitude determination system performance by SCC (0.5^o). Then the mean error deviation for the solar aspect angle was 0.23127^o for the time simulation.

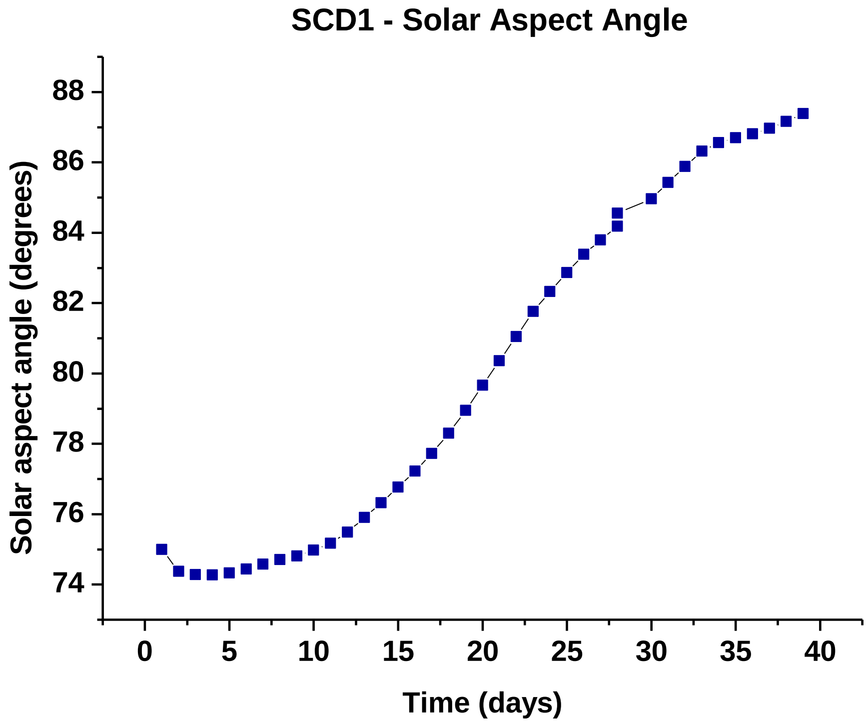

Figure 3 – Temporal behavior of the solar aspect angle *η*, with daily updated data, for SCD1.

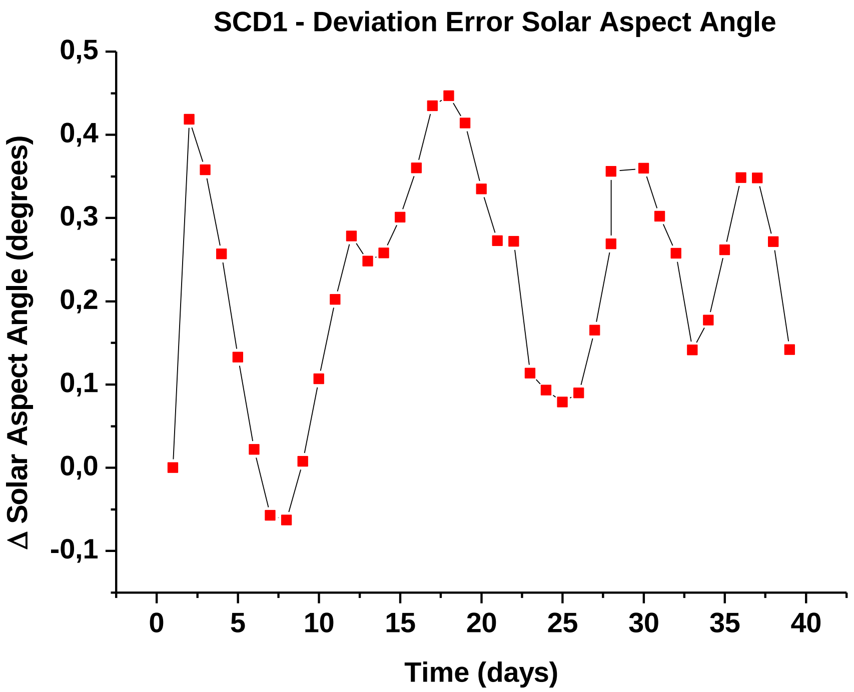

Figure 4 – Difference of computed and real values of the solar aspect angle *η*, with daily updated data, for SCD1.

Results for the second approach: without daily updated data

Table 3 presents the results for this approach for 2 days of two different periods. The simulations were interrupted in the 3rd day because the mean deviations errors between the computed values and real values for the spin velocity, right ascension and declination were bigger than INPE's required precision. Other simulations were done for different initial data but in all of them the results were similar, which means that the computed values have a good agreement with the real data only for the 2 days simulations.

Table 3– Deviation between computed and real values without the daily updated data for SCD1

Day	**$\Delta\eta$ (°)**	**Day**	**$\Delta\eta$ (°)**
07/27/1993	0	08/25/1993	0
07/ 28/1993	0.13293	08/26/1993	0.18526
07/29/1993	0.13945	08/27/1993	0.48049

Applications for SCD2 satellite

The initial conditions of attitude had been taken from February, 1st, 2002 at 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC) for 40 days and are presented in the Appendix. It is important to observe that when the attitude control is acting, the data are equal to the data supplied by SCC, because the theory doesn't include control torques.

Some initial data for SCD2:
Semi-major axis = 7133679.70m
Eccentricity = 0.00175
Inclination = 25.01^o
Longitude of ascending node = 88.30^o
Argument of perigeu = □192.51^o
Mean anomaly =300.03^o
Foucault parameter = p =311.35N m s/T^2 rad
magnetic moment = m =0.188A m2
I_x=12.33kg m^2
I_y=12.35kg m^2
I_z=14.50kg m^2
$\square_{\square}$=0.7
β_1=0.1
$\square_{\square}$=0.5
β_2 =0.1

Results for the first approach: daily updated data

Figures 5 and 6 present the results for temporal behavior of the solar aspect angle and the deviation between the computed values and real values of this angle, respectively. It can also be observed that they are smaller than 0.25^o, with the mean error deviation for the solar aspect angle is 0.10614^o for the time simulation. Note that the differences are zero when the attitude control is acting. Then they are within the dispersion range of the attitude determination system performance by SCC.

Results for the second approach: without daily updated data

Table 4 presents the results for this approach for 2 days of two different periods. The same way of the SCD1, the simulations were interrupted in the 3rd day because the mean deviations errors between the computed values and real values for the others parameters were bigger than INPE's required precision.

Table 4– Deviation between computed and real values without the daily updated data for SCD2

Day	Δη (°)	Day	Δη (°)
02/09/2002	0	**03/05/2002**	0
02/ 10/2002	-0.24611	**03/ 06/2002**	0.01108
02/11/2002	-0.24029	**03/07/2002**	-0.04099

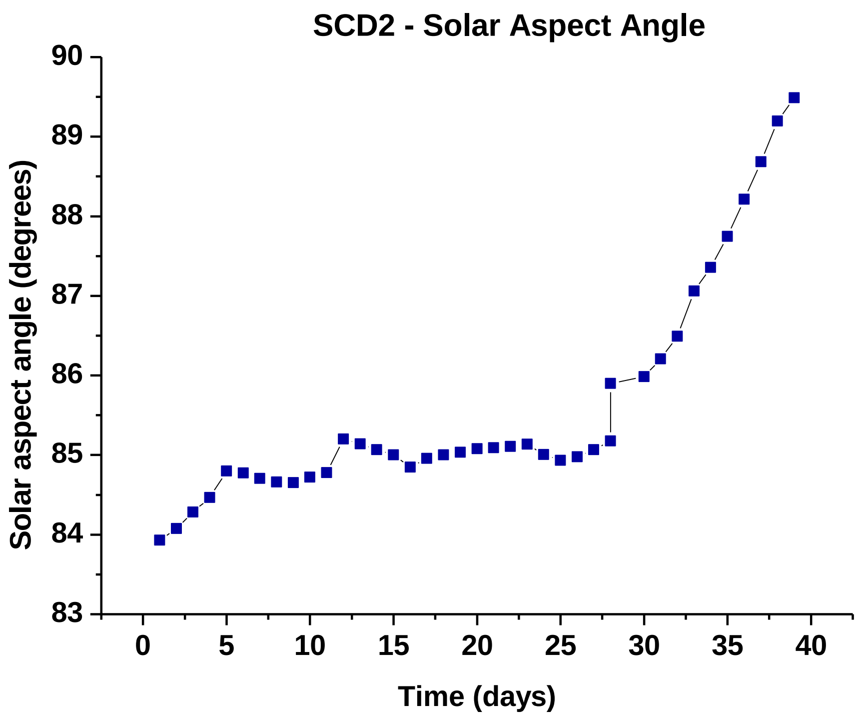

Figure 5 – Temporal behavior of the solar aspect angle *η*, with daily updated data, for SCD2.

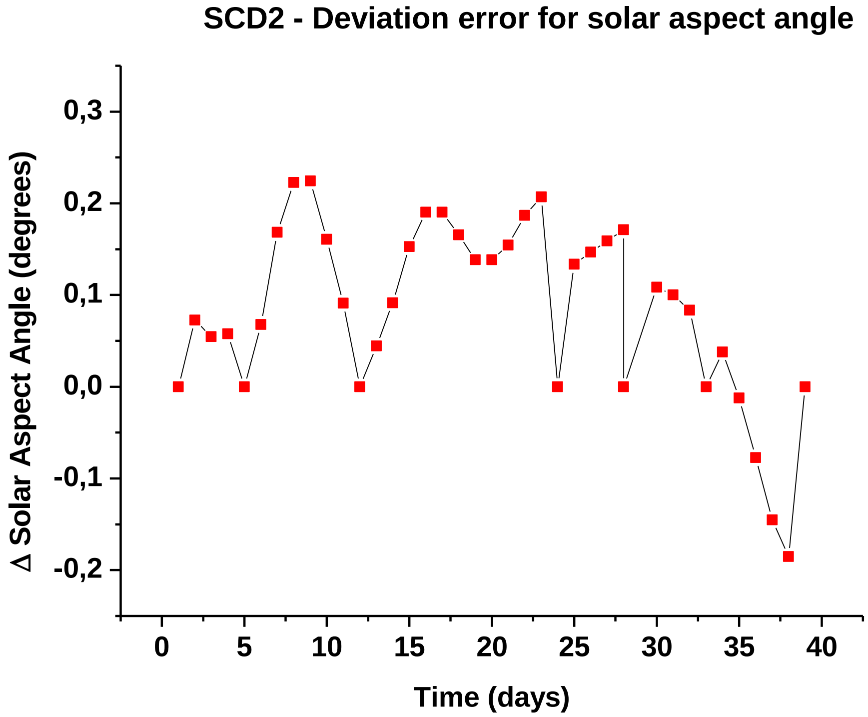

Figure 6 – Difference of computed values and real values of the solar aspect angle *η*, with daily updated data, for SCD2.

7. Conclusions

In this paper an analytical approach for the spin-stabilized satellite rotational motion was presented taking into account the influence of torques: aerodynamic, gravity gradient, solar radiation, residual magnetic and eddy currents. The goal was to analyze the influence of these torques in the solar aspect angle.

The analytical solution shows that the spin velocity has an exponential variation due to the eddy current torque and a linear variation due to the gravity gradient torque. The combinations of the others torques contribute for a precession and drift on the spin axis, due to the temporal variation on the right ascension and declination angles respectively. The theory was applied to the spin stabilized Brazilian's satellites SCD1 and SCD2.

In the first one the attitude and orbital data are daily updated with real attitude data supplied by INPE. The results showed a good agreement between the computed and real data during 40 days. The mean error deviation for the solar aspect angle was 0.23127^{o} for SCD1 and 0.10614^{o} for SCD2, which are within the dispersion range of the attitude determination system used for this satellite.

In the second approach the attitude and orbital data are not updated. The results presented a good agreement between the analytical solution and the actual satellite behavior only for two days simulation. For more than 2 days the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required for SCC (0.5^{o}).

For both approaches it is possible to note the influence of the declination of the spin axis in the calculation of the solar aspect angles. Results have shown the agreement between the analytical solution and the real satellite behavior for specific time simulation and two approaches were presented. Then the theory has consistency and can be applied to predict the behavior of the solar aspect angle.

Others analysis of the solar aspect angles [1,2,4,13,15] have been developed but they haven't considered the all torques acting together. Then the present theory could be more realistic than the other ones.

ACKOWLEDGMENTS

This present work was supported by FAPESP (Process No. 2012/21023-6).

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Appendix

Data supplied from INPE's CSS for SCD1

SCD1				SCD2			
day	**α_{INPE}(°)**	**δ_{INPE}(°)**	**W_{INPE}(rpm)**	**day**	**α_{INPE}(°)**	**δ_{INPE}(°)**	**W_{INPE}(rpm)**
07/24//93	234.1000	77.3000	90.8100	**02/01/2002***	281.7200	62.7400	34.5700
07/25/93	233.7400	77.6900	90.7100	**02/02/2002**	281.5300	62.9499	34.5900
07/26/93	233.5400	78.0900	90.6200	**02/03/2002**	281.3800	63.2019	34.6100
07/27/93	233.5300	78.5000	90.5200	**02/04/2002**	281.2800	63.4429	34.6300
07/28/93	233.7300	78.9300	90.4200	**02/05/2002***	280.0500	63.3900	34.6300
07/29/93	234.1400	79.3500	90.3300	**02/06/2002**	280.0600	63.4747	34.6200
07/30/93	234.8300	79.7800	90.2300	**02/07/2002**	280.0900	63.5517	34.6200
07/31/93	235.8000	80.2000	90.1200	**02/08/2002**	280.1300	63.6142	34.6100
08/01/93	237.1200	80.6000	90.0200	**02/09/2002**	280.1800	63.6780	34.6100
08/02/93	238.8200	80.9900	89.9100	**02/10/2002**	280.2500	63.7348	34.6000
08/03/93	240.8900	81.3400	89.8100	**02/11/2002**	280.3100	63.7863	34.6000
08/04/93	244.0400	81.8600	89.5400	**02/12/2002***	278.7100	63.4700	34.4800
0805/93	246.6200	82.1200	89.3500	**02/13/2002**	278.7300	63.5146	34.4200
08/06/93	249.5300	82.3300	89.1600	**02/14/2002**	278.7400	63.4636	34.3700
08/07/93	252.7400	82.4800	88.9700	**02/15/2002**	278.7400	63.4090	34.3100
08/08/93	256.1500	82.5800	88.7900	**01/16/2002**	278.7200	63.3570	34.2600
08/09/93	259.7000	82.6000	88.5900	**02/17/2002**	278.6800	63.3160	34.2000
08/10/93	263.2000	82.5600	88.4100	**02/18/2002**	278.6300	63.2964	34.1400
08/11/93	266.5500	82.4400	88.2200	**02/19/2002**	278.5700	63.2926	34.0800
08/12/93	269.7000	82.2800	88.0300	**02/20/2002**	278.5000	63.3014	34.0200
08/13/93	272.5400	82.0600	87.8500	**02/21/2002**	278.4200	63.3170	33.9600
08/14/93	275.7500	81.8500	87.6100	**02/22/2002**	278.3300	63.3421	33.9000
08/15/93	277.4500	81.6200	87.4200	**02/23/2002**	278.2300	63.3590	33.8300
08/16/93	278.9000	81.3700	87.2400	**02/24/2002***	276.6000	61.2200	33.6900
08/17/93	280.0900	81.1000	87.0600	**02/25/2002**	276.4200	61.1443	33.6900
08/18/93	281.0100	80.8200	86.8800	**02/26/2002**	276.2000	60.9304	33.5500
08/19/93	281.7400	80.5300	86.7100	**02/27/2002**	275.9400	60.7028	33.4800
08/20/93	282.2400	80.2300	86.5400	**02/28/2002**	275.6400	60.4678	33.4000
08/21/93	282.5700	79.9300	86.3700	**03/01/2002***	273.7500	59.4002	33.4300
08/22/93	282.7000	79.6400	86.2100	**03/02/2002**	273.3900	59.1207	33.4100
08/23/93	282.6700	79.3500	86.0400	**03/03/2002**	272.9700	58.8507	33.3800
08/24/93	283.5000	79.2200	85.8800	**03/04/2002**	272.5200	58.5730	33.3500
08/25/93	283.0100	78.9500	85.8000	**03/05/2002***	271.6300	58.2500	33.3400
08/26/93	282.4300	78.7000	85.7300	**03/06/2002**	271.1400	57.9950	33.3600
08/27/93	281.7600	78.4800	85.6600	**03/07/2002**	270.6300	57.7446	33.3800
08/28/93	281.0100	78.2700	85.5800	**03/08/2002**	270.0700	57.5159	33.4000
08/29/93	280.1800	78.0800	85.5100	**03/09/2002**	269.4900	57.3094	33.4200
08/30/93	279.2900	77.9100	85.4400	**03/10/2002**	268.8700	57.1157	33.4400
08/31/93	278.3400	77.7800	85.3700	**3/11/2002**	268.2400	56.9538	33.4600
09/01/93	277.3600	77.6700	85.3100	**3/122002**	267.8400	56.7966	33.5100

***Days with the acted attitude control .**