TO THE ADAPTIVE MULTIBODY GRAVITY ASSIST TOURS DESIGN IN JOVIAN SYSTEM FOR THE GANYMEDE LANDING

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Introduction

Modern space missions inside Jovian system are not possible without multiple gravity assists (as well as in Saturnian system, etc.). The orbit design for real upcoming very complicated missions by NASA, ESA, RSA should be adaptive to mission parameters, such as: the time of Jovian system arrival, incomplete information about ephemeris of Galilean Moons and their gravitational fields, errors of flybys implementation, instrumentation deflections. Limited dynamic capabilities, taking place in case of maneuvering around Jupiter's moons, demand multiple encounters (about 15-30 times) for these purposes. Flexible algorithm of current mission scenarios synthesis (specially selected) and their operative transformation is required.

Mission design is complicated due to requirements of the Ganymede Orbit Insertion (GOI) ("JUICE" ESA) and also Ganymede Landing implementation ("Laplas-P" RSA [1]) comparing to the ordinary early “velocity gain" missions since “Pioneers” and "Voyagers”. Thus such scenario splits in two parts.

Part 1(P1), as usually, would be used to reduce the spacecraft’s orbital energy with respect to Jupiter and set up the conditions for more frequent flybys. Part 2(P2) would be used to lower the spacecraft’s Ganymede relative velocity to set up the correct conditions for the GOI. The technique of P1 implementation is well known: it is the sequence of resonant same-body transfers “from Ganymede to Ganymede”(the solution based on the Lambert solution technique). But the P2 implementation couldn’t be the same (in requirements of quasi ballistic gravity assist with low cost propellant consumption). The cause is the invariance of Tisserand’s parameter in a circular restricted three-body system (CR3BP) [2-3]. Therefore it is impossible to change the magnitude of the $V\infty$velocity vector of the spacecraft with respect to the Ganymede. Furthermore, the same-body flybys sequence on the Tisserand-Poincare (T-P) graph [2-3] falls according the $V\infty$-isoline to the “Extra Radiation” zone (XR-zone) with spacecraft’s orbit pericenter height $r_p \leq 10R_J$, R_J - Jupiter’s radius (This height is critical for missions “JUICE” ESA and “Laplas-P” RSA. Approximately they shell have Total ionizing dose TID of 250-300 Krad behind a 10-ml Al spherical shell). So regular expensive corrections of pericenter’s height increasing are demanding in this case.

But the other implementation of P2 can be used. They are- “crossed” gravity assists from one small body G(“Ganymede”) to the second small body $\bar{G}$(“not Ganymede”, generally for a not big TID it is Callisto) and then – in the opposite direction. We can “to outwit” the Tisserand’s criterion like this. It allows us to build special effective “phase beam"(PB) methods of orbit design synthesis for "Laplas-P"-mission in classes of encounter sequences-disjunctions $G_1 \wedge \bar{G} \wedge G_2$ (or modified $G_1 \wedge \ldots \wedge G_k \wedge \bar{G}_m \wedge \ldots \wedge \bar{G}_{m+n} \wedge G_{k+1} \wedge \ldots$).

Tisserand-Poincare graph [2] allow to design mission dynamical maquette (but only in phase-free form). Algorithms with utilizing the real satellites configurations at the real time of close approach are required for upcoming missions implementation. NAIF JPL NASA Ephemeris have been employed in our simulations. Patched Conics can not be applied immediately in the trajectory design. The rigid fixation of scenario's conic points should be threat to the mission's implementation.

As the result, the idea of adaptive synthesis of mission design was constructed by authors. Then it was implemented by effective algorithm. The corresponding numerical scheme was developed with using Tisserand-Poincare graph and the simulation of hundreds of thousands of options formation of the calculation (Fig.1). The ΔV-low cost searching was utilized also for trajectory design with help of multiple rebounds of phase beams modeling. The techniques developed by the authors specifically to the needs of the mission "Laplas P" RSA. In the selection of hundreds of thousands of options for each date selected only those chains that form closed loops on Ganymede gravity assist after intermediate Callisto's flybys. The adaptive scheme, which retains the flexibility to clarify or an unforeseen change in external conditions, also in the case of errors during encounters implementations is presented, that provides the reliability of the mission and its multi-variant flexibility.

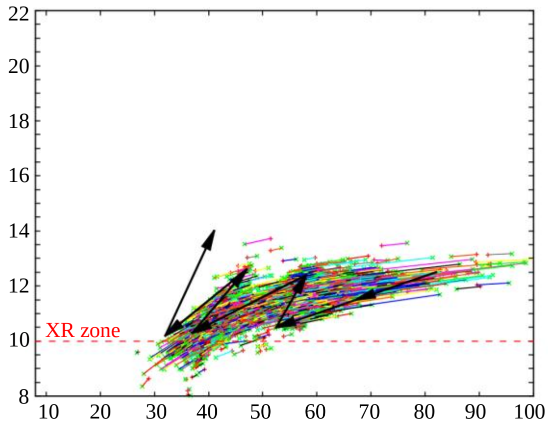

Fig. 1. T-P graph under a modified synthesis script with imposing requirements of multi rebounds. X-axis and Y-axis are spacecraft apocenter and pericenter height in R_J (Jupiter radii)

1. SPECIFIC FEATURES OF JUPITER MISSIONS TARGETING TO LAND ON ITS SATELLITE

The flight to the Jovian system can be performed in various ways, in particular, using gravity assist maneuvers [4- 8]. An efficient way is to use the VEEGA (Venus--Earth--Earth--Venus Gravity Assist) route. This route is proposed to be used as the reference trajectory in the ESA JUICE mission to Jupiter aimed at entering a Ganymede artificial satellite orbit [6-8]. The flight should take six years on the average.

In the first approximation, the heliocentric spacecraft trajectory in the Jovian frame of reference is a flyby hyperbola. In order to enter a Jupiter artificial satellite orbit, a Jovian Orbit Insertion (JOI) braking impulse is needed. After this maneuver, the spacecraft will stay on a highly elongated elliptical near Jupiter orbit. The JOI impulse can be applied in the pericenter of the flyby hyperbola both before and after the gravity assist maneuver near one of the Jovian moons. The execution of the gravity assist maneuver immediately after the completion of the interplanetary part of the flight is more energy efficient; however, it is strictly coupled with the time of arriving in the Jovian system.

To make the mission more flexible and stable, Jupiter tours that begin with a JOI are preferable. A specific feature of the JOI maneuver is that it must ensure the subsequent rendezvous with the Jupiter moon (Ganymede) and such a flyby of it that reduces the period and orbital velocity of the spacecraft.

Upon the spacecraft becomes a Jupiter artificial satellite on a highly elongated elliptical orbit, a series of gravity assist maneuvers using the gravitational fields of Galilean moons must be performed; this series consists of two phases [5,9-10]. In the first phase, the orbital velocity is reduced and the orbital period is decreased down to several Ganymede orbital periods (about 7.155 day). Thereby, the total mission duration is controlled and frequent flybys near natural Jupiter moons are performed to execute multiple gravity assist maneuvers.

In the second phase, the spacecraft's orbital velocity is reduced to a magnitude comparable with the Ganymede orbital velocity to be able to land on its surface.

Before each Jupiter moon flyby as a result of which a gravity assist maneuver is performed, the spacecraft's trajectory must be corrected to ensure the desired parameters of this moon flyby. The expenditure of the characteristic velocity for such corrections should not exceed 15 m/s for one correction. Such corrections can be combined with the maneuvers aimed at increasing its periapsis. Note that there are alternative methods for increasing the periapsis, which will be discussed below.

To summarize, we list the main phases of a Jovian space mission intended to land on Ganymede.

1. Spacecraft injection from the flyby hyperbola into a highly elongated Jupiter elliptical orbit using JOI.

2. Reducing the spacecraft's orbital period and decreasing its major semiaxis using an initial series of long but decreasing gravity assists.

3. Approaching the target Jupiter moon (e.g., Ganymede).

It is necessary to make the orbital velocities of the spacecraft and the target close to each other and, taking into account the fact that the parts of the spacecraft trajectory between reflections are elongated and the target moon's orbit is close to a circle, to reduce the eccentricity of the spacecraft's orbit using a series of shortened gravity assist maneuvers.

4. Phasing the spacecraft with Ganymede.

5. Execution of the Ganymede Orbit Insertion (GOI) braking maneuver that injects the spacecraft to a Ganymede artificial satellite orbit.

6. Execution of a series of maneuvers to form the pre-landing orbit.

7. Turn the autonomous landing system on.

2. MODEL OF THE GRAVITY ASSIST MANEUVER

The patched conics method [11] that is often used to design gravity assist maneuvers represents the trajectories of a celestial body with negligible small mass (in our case, this is a spacecraft) in piecewise Keplerian form with kinks at the points where the gravity assist maneuvers are executed. The domains where the maneuvers are performed (from the time t_1 when the spacecraft enters the sphere of influence of the second gravitating body up to the time t_2 when the spacecraft leaves this sphere) are assumed to be negligibly small compared with the Keplerian parts of the trajectory near the central body; these short parts of the trajectory are replaced with patched points (Fig. 2).

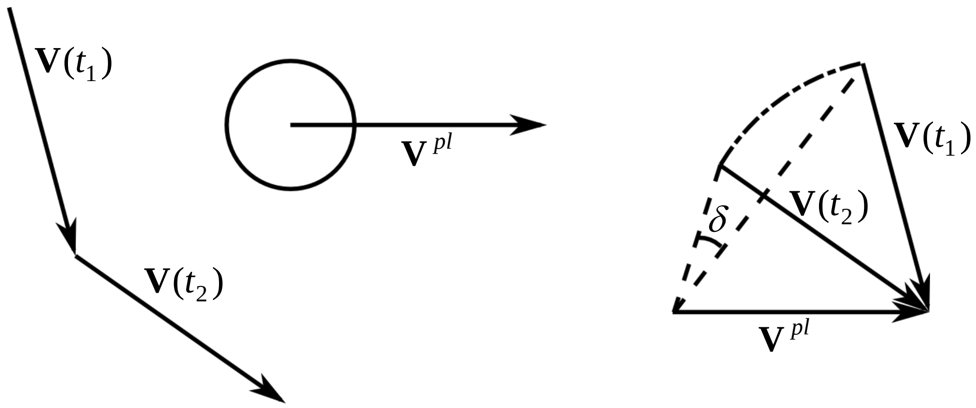

Fig. 2. Geometry of the gravity assist maneuver.

The pumping angle δ is determined by the "internal geometry" of the patched point based on the angle between the branches of the flyby hyperbola "packed in this point" relative to the flyby celestial body. The magnitude of the spacecraft velocity relative to the moon does not change.

The variation of the spacecraft velocity V relative to the central body is (see [11])

$$\Delta \mathbf{V} = \mathbf{V}(t_2) - \mathbf{V}(t_1),$$

$$\Delta V = \|\Delta \mathbf{V}\| = 2V_\infty \sin\delta = \frac{2V_\infty \mu}{\mu + r_p V_\infty^2},$$

where μ is the gravitational constant of the flyby body, V_∞ is the asymptotic spacecraft velocity relative to this body, and r_p is the pericentral distance of the flyby hyperbola.

Depending on how the spacecraft moves near the flyby body, the gravity assist (GA) maneuvers can be V_∞ increase GA or V_∞ decrease GA maneuvers.

It can be easily verified that the asymptotic velocity is related to the planet velocity V^{pl} and the spacecraft arrival velocity $V(t_1)$ as (see [11])

$$V_\infty = \left[V(t_1)^2 + (V^{pl})^2 - 2V(t_1)V^{pl}\cos\delta\right]^{1/2}$$

In the case of the cranking gravity assist maneuver, the maximum change in the inclination I of the spacecraft's orbit plane after a single flyby can be written as

$$\sin\Delta i = \frac{V_\infty \sin 2\delta}{V^{pl}} = \frac{V_\infty}{V^{pl}}, \quad V_\infty \le V^{pl}.$$

3. CAPABILITIES OF GRAVITY ASSIST MANEUVERING

It can be shown (see [11]) that the maximum variation in the magnitude of the velocity $\Delta V_{\max}$ is achieved at $V_\infty = \left(\frac{\mu}{r^{pl}}\right)^{1/2}$

and the magnitude of the increase in the hyperbolic velocity cannot exceed the planet's escape speed: $\Delta V_{\max} = \left(\frac{\mu}{r^{pl}}\right)^{1/2}$ Here, r^{pl} is the planet radius.

The table shows the capabilities of gravity assist maneuvers using flybys of influence spheres of planets and large dwarf planets of the Solar system; we completed and improved this table compared with the data provided in [11, 12] using the latest space observation data [13-15].

If we change to model dimensionless variables, the gravity assist maneuvering in the vicinity of any celestial body will be determined by the model normalized spacecraft speed coefficient $\chi = V/V^{pl}$ Therefore, the intensity of any gravity assist maneuver can be described by the dimensionless parameter $\chi_{\mathrm{mod}} = \Delta V_{\max}/V^{pl}$ the greater this parameter, the greater is the possible deformation of the beam of dynamically feasible spacecraft phase trajectories in the maneuver. The corresponding values of $\chi_{\mathrm{mod}} = \Delta V_{\max}/V^{pl}$ are also presented in the table.

4. BRIEF ANALYSIS OF POSSIBLE MANEUVER PARTICIPANTS IN THE JOVIAN SYSTEM

The four biggest Jupiter satellites-Galilean moons Io, Europa, Ganymede, and Callisto-care the first candidates for the partnership with the spacecraft in gravity assist maneuvering in the Jovian system.

Io. The Galileo mission used all four Jupiter moons for maneuvering; however, this mission was very unfavorable from the radiation point of view. In the Io flyby, a part of the research hardware was damaged. The total absorbed dose exceeded 650 krad by the year 2004. Taking into account the fact that the currently planned ESA and Russian Space Agency Roskosmos missions [7-10] should be performed under more moderate radiation conditions (the total absorbed dose should be about 100--260 krad), which requires lighter protection of the spacecraft body and excludes spacecraft flybys lower than the Europa orbit, Io could be eliminated from the initial list of candidates. However, NASA and Jet Propulsion Laboratory also examine other "impenetrable" approaches to the radiation safety problem when flying near the Jupiter close moons. In particular, the severe Europa tour [16] is designed so as to operate until the research hardware fails. This mission allows for lower periapses and, respectively, higher radiation doses from 80 krad for 1000 mm Al shield to 2.1 Mrad for 100 mm Al shield.

Table. Maximally possible variations of a spacecraft speed in the flyby of large celestial bodies of the Solar system

Planet	Speed variation $\Delta V_{\max}$, km/s	$\chi_{\mathrm{mod}} = \dfrac{\Delta V_{\max}}{V^{pl}}$	Central body
Mercury	3.005	0.063	Sun (S)
Venus	7.326	0.209	S
Earth	7.912	0.265	S
Mars	3.557	0.147	S
Ceres	0.101	0.056	S
Jupiter	***42.57***	***3.257***	S
Saturn	25.52	2.634	S
Uranus	15.12	2.22	S
Neptune	***16.67***	***3.07***	S
Pluto	0.85	0.18	S
Haumea	1.16	0.26	S
Makemake	1.11	0.25	S
Eris	1.09	0.3	S
Moon	1.680	1.6	Earth
Io	1.809	0.1	Jupiter
Europa	1.433	0.1	Jupiter
Ganymede	1.949	0.17	Jupiter
Callisto	1.725	0.21	Jupiter
Titan	1.867	0.3	Saturn

Europa. The importance of scenarios of Europa flybys, rendezvous with it up to orbiting it [5,9-10] (and, possibly, landing on it) is high. Europa is one of the most attractive research targets because it has an ocean covered with ice, which is heated by Jovian tides. Even though Europa is smaller than the Moon, it is still the sixths-size natural satellite in the Solar system. Therefore, gravity assist maneuvers near Europa are theoretically effective (see table). However, as in the case of Io, the high radiation level near Europa significantly complicates the implementation of such scenarios. Note that the ESA JUICE mission assumes two Europa flybys, and the dose absorbed in this phase is about 200 krad if the Al body protection shield is 10 mm thick. Landing on Europa gives the dose many times higher; therefore, it cannot be planned for near future. When designing missions with the total absorbed dose below 150--250 krad with protection not thicker than 10 mm Al, Europa should also be excluded from the list of candidates, which significantly cuts down the list of feasible scenarios and complicates their design.

Ganymede. Ganymede is the largest satellite in the Solar system. The interest in the exploration of this giant is very high, as in the case of Europa.

The design of space missions that plan Ganymede rendezvous and landing on its surface is a priority line of research in ESA and Roskosmos. On the one hand, the radiation conditions of Ganymede landing are fairly benign. On the other hand, the Ganymede gravitational constant is 9887.8 $км^3/с^2$, which makes the landing on it from the orbit more difficult than in the case of Europa.

Callisto. The gravity assist maneuvering near Callisto is very efficient in itself and as an alternative in the case when maneuvering near other moons is impossible (due to the phase configuration of the system, radiation safety, etc.).

Landing on Callisto is also promising. Moderate radiation conditions near it make it possible to lighten the spacecraft by decreasing the radiation protection of its body (the limiting total dose is 150 krad).

5. STRATEGY OF DESIGNING TOURS IN THE JOVIAN SYSTEM: THE FIRST PHASE (DEBUT)

Each feasible chain of gravity assist maneuvers requires that the tempo be preserved, the spacecraft flight from one rendezvous to the next one must be quasi-ballistic and almost inertial, the fuel consumption must be minimized, and the characteristics of the escape from the preceding maneuver must be used to aim at the next target. This recurrent energy saving of the designed chain can cut off if the spacecraft caught in a deadlock when there is no attainable goal at the exit from the current gravity assist maneuver. The well-designed scenario must ensure that the goals for the spacecraft are timely provided.

The technology of next goal provision in the initial phase of the planetary mission is fairly clear. This technology consists of the solution of the initial Lambert problem [11] based on the condition that the spacecraft reaches Ganymede after performing JOI and implementing the subsequent series of solutions of the Lambert problems such that the spacecraft's orbital period at the end of each gravity assist maneuver is a multiple of the orbital period of the satellite chosen as the maneuvering partner. The resonance of periods thus formed ensures a new rendezvous with this partner in a certain amount of time in a vicinity of the true anomaly of the last rendezvous. For this purpose, it is sufficient to perform a low cost correction immediately before the planned gravity assist maneuver; this correction should control the distance from the target moon, and it is typically performed in the vicinity of one of the last orbit periapses before rendezvous.

Improving Correction of the Solution to the Lambert Problem by Parameterization of the Distance to the Target Moon

To improve the correction impulse based on the desired distance from the target moon, the spacecraft equations of motion in the Jovian reference frame is integrated taking into account the gravitation of its natural satellites (see [17, 18]). The correction impulse is calculated in three steps.

At the first step, the impulse is found from the condition of minimizing the difference between the prescribed altitude of the periapsis of the flyby hyperbola and its realized value due to the impulse. The minimum is sought under the condition that at the time when the spacecraft is at the minimum distance from the moon on the flyby hyperbola, its distance to the baricenter of the Jovian system is greater than the distance from the target moon to the baricenter. This condition ensures that flyby is on the outer side of the moon orbit, which corresponds to the decrease gravity assist maneuver. If the flyby were on the inner side of the orbit, the velocity would increase. The minimum is sought using the combined gradient and coordinate descent method [17, 18]. To this end, along with the gradient, feasible directions are determined. A direction is feasible if baricenter condition is fulfilled when the component of the velocity varies in this direction. If all the directions are feasible, the gradient descent method with step control is used. If there are prohibited directions, then the descent follows the most effective coordinate.

At the second step, the multiplicity coefficient is determined as the minimum integer greater than the ratio of the spacecraft's orbital period to the Ganymede orbital period. The target value of the spacecraft's orbital period after the gravity assist maneuver is equal to the product of the multiplicity coefficient and the Ganymede orbital period.

At the third step, the impulse is calculated more precisely based on the minimization of the functional equal to the difference between the target value of the period and its actual magnitude. As in the case of designing the flyby at the prescribed altitude, the minimum is sought under the condition that the flyby is on the outer side of the moon orbit.

In the case of the cranking gravity assist maneuvers, a parameterization with respect to the flyby cross range is needed. The two-dimensional parameterization of the flyby altitude makes it possible to design both pumping gravity assist (PGA) and cranking gravity assist (CGA) maneuvers, where the latter intend to change the spacecraft's orbit inclination.

The design of the first phase (debut) was performed under flexibly specified conditions of arrival in the Jovian system using improved ephemeris of the Solar system and Jovian system NAIF JPL NASA and the program package ESTK of BC (Ballistic Center) of the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences (BC of KIAM RAS).

Figure 4 shows a typical trajectory of the spacecraft in the first phase.

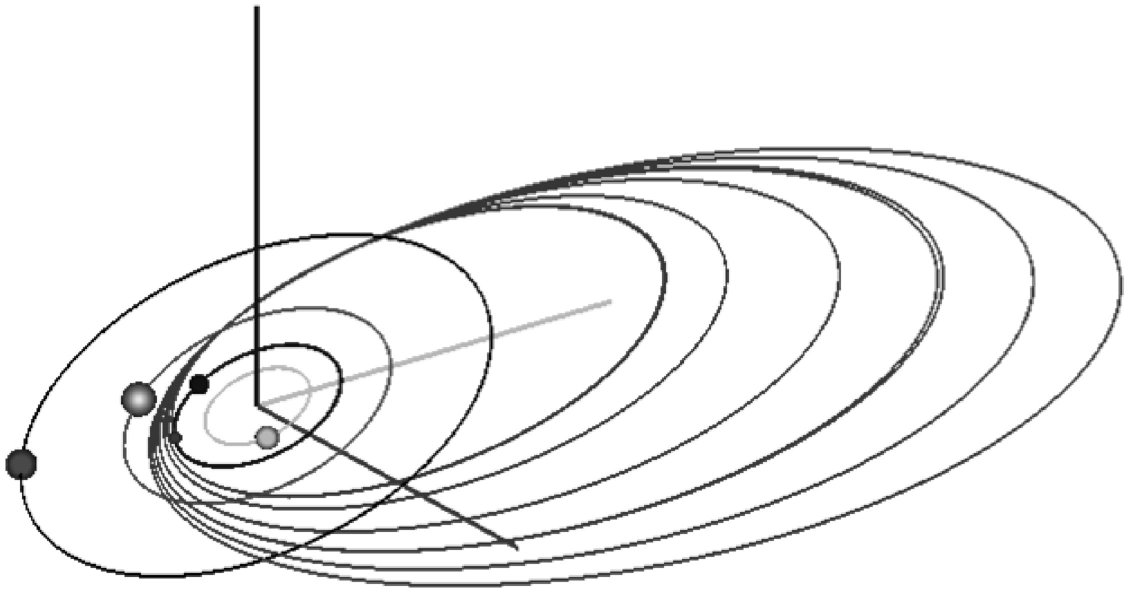

Fig. 3. The first phase of the mission in the Jovian system (debut).

The implemented sequence of decreasing periods of the spacecraft can be written in terms of the Ganymede periods as $6 \to 5 \to 5 \to 4 \to 3 \to 2.5 \to 2$.

Even though the orbit periapses are localized with respect to the true anomaly, they nevertheless drift from one maneuver to another. The a priori fixing of the line of apsides of the spacecraft's orbit, which is used for designing mission scenarios in some studies, results in losing the actual spacecraft motion when the Keplerian trajectories are replaced with actually observed ephemeris of the Jovian celestial bodies. The trajectories are destroyed under small variations of the parameters. A similar pattern is observed when the patched conics method [11, 19-21] is used, which replaces the gravity assist maneuver domain with a reflection point; this is a representative example in this case.

6. STRATEGY OF DESIGNING TOURS IN THE JOVIAN SYSTEM: PRECONDITIONS OF THE SECOND PHASE IMPLEMENTATION

The gravity assist maneuvering procedure described above cannot be repeated infinitely for a number of reasons.

The first reason is that the gravity assist maneuvers lose their efficiency as the asymptotic velocity of the spacecraft relative to the moon decreases while the expenditure of the characteristic velocity for these maneuvers increases [19].

The endgame problem in the case of a single partner is analyzed in a number of modern NASA and ESA studies, For example, a thorough survey can be found in [19].

The second, more fundamental cause is due to the properties of trajectories in the restricted three body problem. In the case of low cost corrections in the course of gravity assist maneuvering in the vicinity of the target moon, they have the Jacobi integral (and the Tisserand parameter) as their invariant; correspondingly, the magnitude of the asymptotic velocity $\mathbf{V}_\infty$ [2-3,19] is also preserved in the series of any resonance gravity assist maneuvers.

In the case of cranking gravity assist maneuvers, the possible values of $\mathbf{V}_\infty$ cannot be arbitrary either-they lie on an invariant surface - the pseudo-sphere «$\mathbf{V}_\infty$-Globe» [21].

As a result, the spacecraft cannot approach the target moon without significant additional expenditure of the characteristic velocity; furthermore, it is destined to slide into the “death zone” (the zone with low periapses and extra radiation, XR radiation) on an asymptotic velocity isoline.

Let us discuss these reasons in more detail.

7. JACOBY INTEGRAL AND THE TISSERAND CRITERION

Let us write out the dimensionless Jacobi integral for the restricted three body problem (the central body is Jupiter with the gravitational constant μ_1, the small body is its natural satellite (moon) with the gravitational constant $\mu_2 \ll \mu_1$ and the third body is the spacecraft with an infinitesimal mass). We introduce the rotating barycentric reference frame BXYZ, where the axis BX passes through the central and the small bodies, the axis BY is perpendicular to BX in the plane of the small body orbit, and the axis BZ completes the reference frame to the positively oriented one. Let a_{sat} be the major semiaxis of the Jupiter satellite orbit. Change to the dimensionless $\tau = \sqrt{a_{\mathrm{sat}}^3/(\mu_1+\mu_2)}$ the dimensionless spacecraft coordinates BX, BY, BZ and distances R_1, R_2 from the spacecraft to Jupiter and its satellite normalized to a_{sat}. The spacecraft velocity V_{Bsc} in the reference frame BXYZ is written in the dimensionless form as $V = V_{\mathrm{Bsc}}/V_{\mathrm{sat}}$, where $V_{\mathrm{sat}} = \sqrt{(\mu_1+\mu_2)/a_{\mathrm{sat}}}$. Then, the dimensionless Jacobi integral can be written as the double difference of the potential and kinetic energy of the mechanical system divided by the spacecraft mass (see [19, 2-3]):

$$J = 2U - V^2 = (X^2+Y^2) + 2\frac{1-\mu}{R_1} + 2\frac{\mu}{R_2} + (1-\mu)\mu - V^2\,, \tag{1}$$

$$\mu = \frac{\mu_2}{\mu_1+\mu_2} \ll 1\,. \tag{2}$$

With regard to eq. (1), (2), we have in the inertial reference frame Oxyz fixed to the central body that

$$V^2 = \dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 = v^2 + (x^2+y^2) - 2h\cos i\,,$$

$$J = ((x-\mu)^2 + y^2) + 2\frac{1-\mu}{R_1} + 2\frac{\mu}{R_2} + (1-\mu)\mu - v^2 - R_1^2 + 2h\cos i\,, \tag{3}$$

where $v = V_{\mathrm{sc}}/V_{\mathrm{sat}}$, V_{sc} is the spacecraft velocity in the reference frame Oxyz, x and y are the spacecraft coordinates normalized to a_{sat}, i is the inclination of the spacecraft's orbit, and h is the dimensionless area integral constant. Let e be the eccentricity of this orbit and $a = a_{\mathrm{sc}}/a_{\mathrm{sat}}$, where a_{sc} is its major semiaxis. Then, using the relation for energy and the equation for the angular momentum about the axis Oz

$$v^2 = \frac{2(1-\mu)}{R_1} - \frac{1-\mu}{a}, \; h = \sqrt{(1-\mu)a(1-e^2)},$$

we derive from (3) the Jacobi integral in the form

$$J = \frac{1-\mu}{a} + 2\sqrt{a(1-e^2)(1-\mu)}\cos i + 2\frac{\mu}{R_2} - 2x\mu + \mu^2 + (1-\mu)\mu. \qquad (4)$$

Formula (4) is a correction of formula (A17) presented in the detailed paper [3], which (as in formula (A9) in the same paper) contains errors (which, however, do not affect the further calculations).

In papers [22-23], which cite [3], these errors are reproduced (formulas (7) and (3), respectively) degree “two” of the velocity in the formula for the Jacobi integral is omitted. The situation with errors in formulas of the restricted three body problem is not new - these formulas have been corrected beginning from Poincare's works [24,25].

Passing to the limit as $\mu \to 0$ and assuming that R_2 is not small, we obtain the local equivalence of the Jacobi integral ***J*** and the Tisserand parameter ***T*** [26]:

$$J = \frac{1}{a} + 2\sqrt{a(1-e^2)}\cos i = T$$

In particular, for an orbit with zero inclination, we have

$$J = T = \frac{1}{a} + 2\sqrt{a(1-e^2)} = \frac{2}{r_a + r_p} + 2\sqrt{\frac{2r_a r_p}{r_a + r_p}}$$

where r_a, r_p are the apocenter and pericenter of the spacecraft orbit normalized to a_{sat}.

Consider the configuration of the spacecraft flyby of a Jupiter moon. In the framework of the conics method, we have in the case of the gravity assist maneuver

$$V_\infty^2 = V_{\text{sc}}^2 + V_{\text{sat}}^2 - 2V_{\text{sc}}V_{\text{sat}}\cos\gamma\cos i \qquad (5)$$

where γ is the angle between the branches of the flyby hyperbola.

In dimensionless form normalized to V_{sat}, $v_\infty = V_\infty / V_{\text{sat}}$, (5) is written as

$$v_\infty^2 = v^2 + 1 - 2v\cos\gamma\cos i$$

Then, using the relation $-1/(2a) = v^2/2 - 1$ we obtain

$$\frac{1}{a} = 2 - v_\infty^2 + 1 - 2h\cos i, \quad h = v\cos\gamma,$$

hence, we immediately obtain the following relation between the Jacobi constant and the Tisserand parameter with the asymptotic velocity:

$$J = T = 3(1-\mu) - v_\infty^2 = 3 - v_\infty^2 \qquad (6)$$

In dimensional units, the Jacobi constant J_{dim} and the Tisserand parameter T_{dim} are written as

$$J_{\text{dim}} = T_{\text{dim}} = 3V_{\text{sat}}^2 - V_{\infty}^2$$

Thus, the Tisserand parameter provides a link between the models of motion in the two body problem and the restricted three body problem [19]: since the Jacobi constant of the three body problem is preserved in the gravity assist maneuver, the asymptotic velocity on the flyby hyperbolic orbit in the Kepler problem for the same maneuver is also preserved.

8. TISSERAND CRITERION AND "COMET INVARIANTS" OF THE JOVIAN SYSTEM

It is known that the Tisserand condition for the system Sun--Jupiter--small body is written in terms of the astronomical unit au (1 au = r_{0E} = 149 597 870.7 km, where r_{0E} is the average radius of the Earth orbit) as the ***comet invariant*** [27]

$$T_{H-J}^{au} = \Gamma = \frac{1}{(a_{\text{sc}} / r_{0E})} + 0.16860\sqrt{(a_{\text{sc}} / r_{0E})(1-e^2)}\cos i \qquad (7)$$

For the Jovian system, the local model "comet invariants" in the case of the spacecraft flyby through the influence spheres of large moons with the average orbit radii $r_{0-\text{sat}}$ can be written in terms of the dimensional value of the spacecraft's major semiaxis a_{dim} as

$$T = \frac{1}{(a_{\text{dim}} / r_{0-\text{sat}})} + 2\sqrt{(a_{\text{dim}} / r_{0-\text{sat}})(1-e^2)}\cos i \qquad (8)$$

Also, they can be conveniently written as analogs of the classical version of the comet invariant in terms of the local "Jupiter astronomical units" ***jau***, where 1 jau is the average radius of the orbit of the third Jupiter moon Ganymede $r_{0\gamma}$ (1 jau = 1 070 400 km, 1 jau=14.97 R_J in terms of the average Jupiter radius R_J):

$$a_{\text{sc}-\gamma} = a_{\text{dim}} / r_{0\gamma} \,,$$

$$T_{J-i}^{\text{dim}} = \Gamma_i = \frac{1}{a_{\text{sc}-\gamma}} + 8.09092\sqrt{a_{\text{sc}-\gamma}(1-e^2)}\cos i \quad \text{– spacecraft invariant for the Io flyby} \qquad (9)$$

$$T_{J-\varepsilon}^{\text{dim}} = \Gamma_\varepsilon = \frac{1}{a_{\text{sc}-\gamma}} + 4.03053\sqrt{a_{\text{sc}-\gamma}(1-e^2)}\cos i \quad \text{– for Europa,} \qquad (10)$$

$$T_{J-\gamma}^{\text{dim}} = \Gamma_\gamma = \frac{1}{a_{\text{sc}-\gamma}} + 2\sqrt{a_{\text{sc}-\gamma}(1-e^2)}\cos i \quad \text{– for Ganymede,} \qquad (11)$$

$$T_{J-c}^{\text{dim}} = \Gamma_c = \frac{1}{a_{\text{sc}-\gamma}} + 0.85739\sqrt{a_{\text{sc}-\gamma}(1-e^2)}\cos i \quad \text{– for Callisto .} \qquad (12)$$

Thus, when the spacecraft performs a gravity assist maneuver with a Jupiter moon, the "Jupiter comet invariant" of this very moon is fixed.

9. STRATEGY OF DESIGNING TOURS IN THE JOVIAN SYSTEM: THE SECOND PHASE

Strategies for designing chains of gravity assist maneuvers are actively developed in NASA and ESA. However, the technology of designing asymptotic velocity reducing series (relative to the target moon) is not formulated as an individual problem. Typically, only the results are described. We believe that the formalization of the asymptotic velocity reduction algorithm is a necessary element for the construction of a flexible mathematical model of a moon orbit insertion and landing on its surface.

Upon the spacecraft's orbital period is decreased, the phase of reducing its asymptotic velocity must be performed (middlegame). This phase is impossible to implement using gravity assist maneuvers near the same celestial body. This phase can be designed using various approaches that differ in the complex multicriteria "cost"-expenditure of the characteristic velocity, maneuvering time, and absorbed radiation dose. In any case, to move away from the extra radiation zone, the spacecraft's orbit periapsis must be increased. The standard method for this operation---corrections in the apocenter (which costs 50--100 m/s)---requires such a correction to be performed almost before every gravity assist maneuver, which consumes much energy.

However, there is a more energy efficient solution. It is based on the idea of leaving for the time being the concept of Lambert construction of period resonance moon flybys that ensure the new rendezvous with it. One can try to use low cost techniques to involve other participants of the Jovian system by "changing the line of the comet invariant." This naturally implies the necessity of developing techniques for performing criss-cross maneuvers in which, upon executing a gravity assist maneuver with one partner, the spacecraft goes to a trajectory on which it can perform a gravity assist maneuver with another partner. However, such a condition cannot be ensured using direct methods similar to the construction of completion of gravity assist maneuvers that are resonance with respect to orbital period.

The analysis of the event table (Jovian Tour Overview) of the JUICE Ganymede mission designed by ESA [7] shows that its developers follow a similar path.

The inclusion of velocity decreasing criss-cross gravity assist maneuvers was also used in the NASA Galileo mission [28]. The calculation procedure of the spacecraft trajectory assumed the involvement of a lot of experts to look through possible situations and draw the so-called porkchop plots containing asymptotic velocity isolines for all potential participants of the gravity assist maneuver.

10. TISSERAND—POINCARE GRAPHS

Recently, the design of such missions has been using different approaches. First of all, this is related to the introduction of the so-called Tisserand—Poincare diagrams called “T-P graphs” [29, 30]. In the initial modification, the orbital altitude of the spacecraft's pericenter was plotted on the vertical axis, and the spacecraft's orbital period was plotted on the horizontal axis [29,30]. Due to the energy invariance described above, the position of the spacecraft after executing a gravity assist maneuver with the moon lies on a curve of the

Tisserand-Poincare graphs. In particular, this curve is an isoline of the spacecraft's asymptotic velocity as it flybys the moon (this curve will be called *isoinfine* from the term V-infinity). The characteristics of the spacecraft orbit are depicted on the T-P graph by a point (note that the orbit inclination is close to zero, the longitude of the ascending node is an indetermined parameter, and the argument of the pericenter is determined by the flyby hyperbola). One can design the chain of gravity assist maneuvers with the moon as a sequence of points on the isoinfine. This sequence is constructed given the initial conditions of entering the Jovian system. The motion along the isoinfine makes it possible to decrease the spacecraft's orbit eccentricity and its orbital period, but it does not allow one to "reach" the moon itself.

In the case of several partners, the situation becomes more flexible. Several crossing isoinfines can pass through each point representing the spacecraft state. This opens opportunities for modifying the asymptotic velocity of the spacecraft relative to the target moon by “temporarily” changing the partner.

Recently, more illustrative modifications of the Tisserand-Poincare diagram have started to be used. The relation of the orbital period with the major semiaxis makes it possible to transform it to the following form: the apocenter altitude is plotted on the horizontal axis, and the pericenter altitude is plotted on the vertical axis (see [7, 21-23]). This form of the T-P graph makes it possible to quickly estimate the flyby radiation safety because the major radiation dose absorbed in an orbital revolution is absorbed in the pericenter [10].

Figure 5 shows the T-P graphs for Ganymede and Callisto; different colors highlight the domains bounded by isoinfines with a fixed asymptotic velocity step.

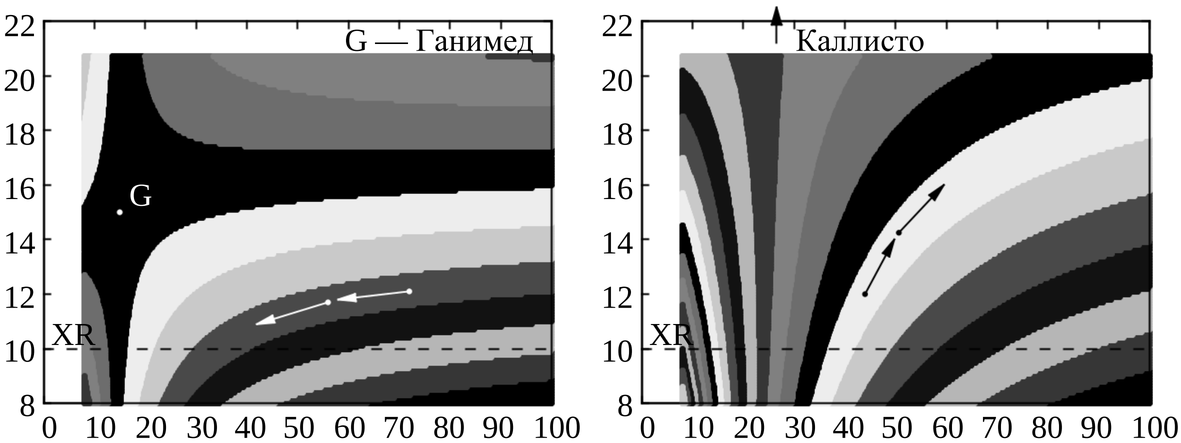

Fig. 4. Isoinfines on Tisserand—Poincare diagrams for Ganymede and Callisto; the apocenter and pericenter altitudes are given in terms of the Jupiter radius R_J.

11. PHASING PROBLEM

Note that the Tisserand—Poincare diagram makes it possible to design only the dynamic pattern of the strategy. This graph does not reflect the mutual positions of the spacecraft and the moon in the planetary system (phase-free model). Here, we face the need for developing new phase search methods; a prototype of such methods is the search for start windows and mapping in the manner of NASA porkchop plots. We adopted the use of beams of phase trajectories in the design.

Taking into account the fact that the geometry of the gravity assist maneuver is in essence the same as the billiard geometry [10-11, 31-35], we can generalize the optic-mechanical analogy, which is valid for billiard trajectories, for the case of investigating the spatial localization of the spacecraft on various trajectories emanating from the gravity assist maneuver domain.

With this purpose, the structure of the propagation of the variative beam of phase trajectories was simulated under the condition of imposing the requirements of "reflections" as the spacecraft flybys the moon and “rereflections” when the spacecraft trajectories that return to the moon after the subsequent reflection from another moon are sought.

This fact makes it possible to focus virtual beams and "sow" them on selected promising reflection segments in the refinement mode in order to form a more effective maneuver. Moreover, it becomes possible to form gravity assist maneuvers that follow after the planned one using the *redirection of reflected beams*. The procedure proposed in [5], which yields good uniformity of the distribution density of trajectories in the beams, turned out to be very effective. Note that the automatic use of search Monte Carlo methods without paying attention to specific features of variative localization of beam "segment-points of growth" cannot detect (using today's computational facilities) criss-cross maneuvers under the real-life conditions of the Jovian system; therefore, "pilot charts" are needed. Furthermore, fixing these points of growth and associating them with a specific domain of the planetary system results in losing details of the actual spacecraft motion when the Keplerian trajectories are replaced with actually observed ephemeris of celestial bodies. The trajectories are destroyed under small variation of the parameters. The patched conics method is a representative example in this sense because the gravity assist maneuver domain is replaced with a reflection point. The drift of the points of growth appearing when the complete model of the motion is used on the one hand makes the spacecraft's trajectory robust; but on the other hand, it complicates the design of multicomponent gravity assist maneuvering.

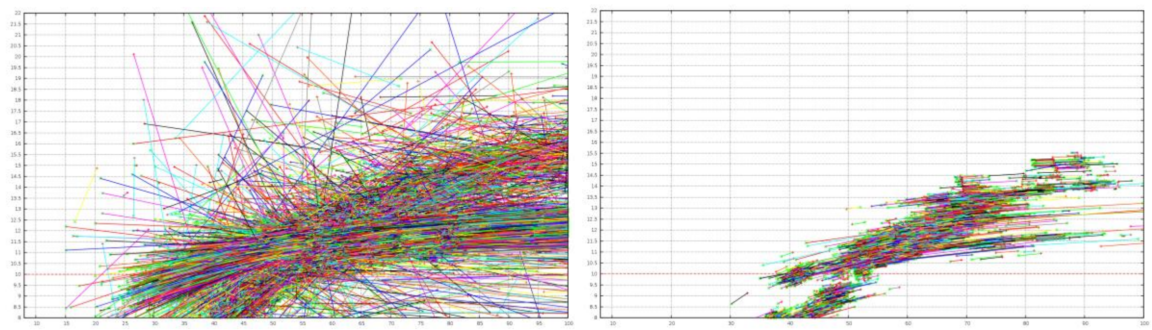

Figure 5. Tisserand—Poincare diagrams. The apocenter and pericenter altitudes are given in terms of the Jupiter radius R_J.

12. COMPUTATIONAL RESULTS

Radiation Conditions in the Jovian System

The level of radiation in the Jovian system is very high (Fig. 6), this presents a severe difficulty, and it must be taken into account in designing flight scenarios to Jupiter satellites.

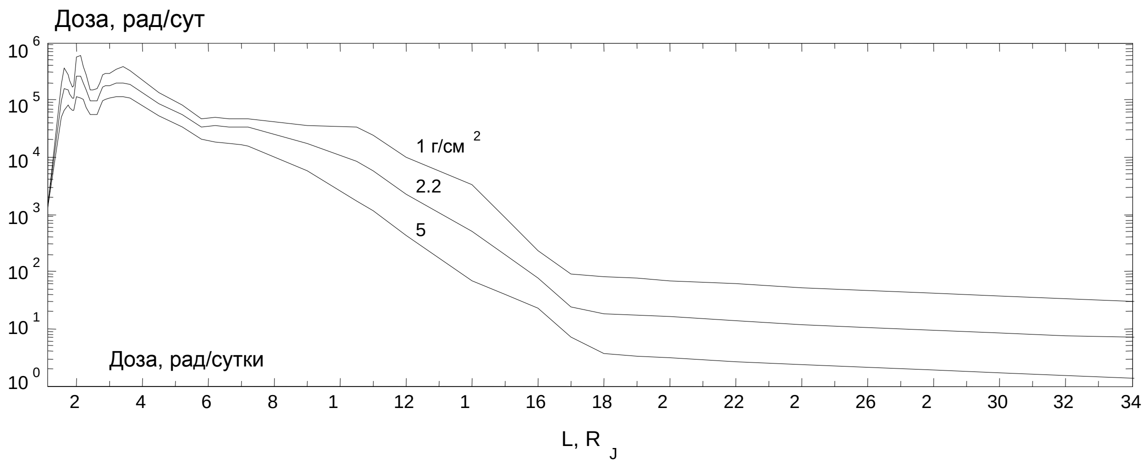

Figure 6. Radiation level in the vicinity of Jupiter [40]; the distance from the center of Jupiter in terms of its radius is plotted on the horizontal axis.

The dependence of the radiation level obtained from the measurements performed by NASA missions was taken into account in the software designed by the authors of this paper for seeking chains of gravity assist maneuvers. As a result, one can interactively estimate the total dose absorbed by the spacecraft "on the fly."

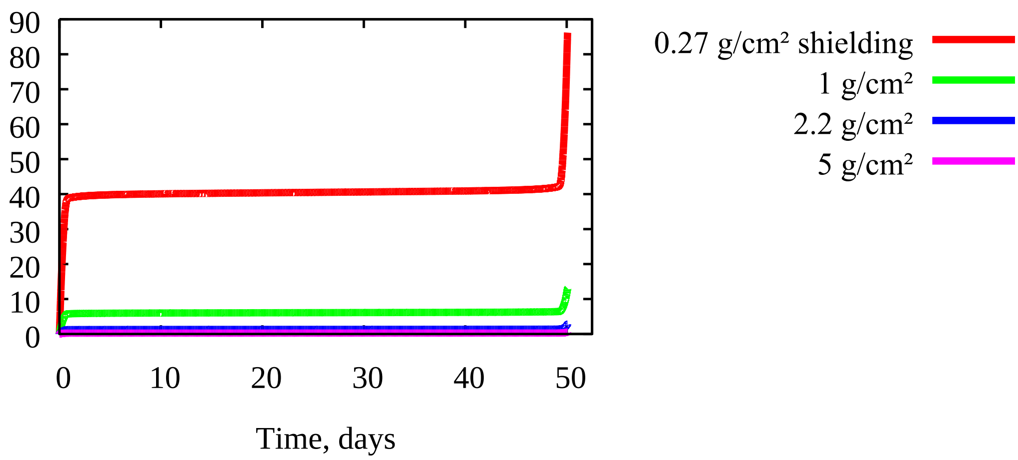

Figure 7. Dynamics of radiation dose accumulation (TILD) in the vicinity of Jupiter in one orbital revolution for various protection levels (in g/см²).

Figure 7 shows a typical plot of the calculated dose absorbed by the spacecraft (TILD) in one orbital revolution depending on the distance to the center of the system. The calculation was performed using the model described in [36].

Figure 7 implies that the spacecraft absorbs the major part of the dose in the pericenter almost quasi-singularly because the major part of the orbital revolution lies outside this zone according to the property of the area integral.

Figure 8 shows the numerically integrated function of the accumulated dose for the standard "moderate radiation level" chain of gravity assist maneuvers in the first phase of the tour (the computations were performed by the authors).

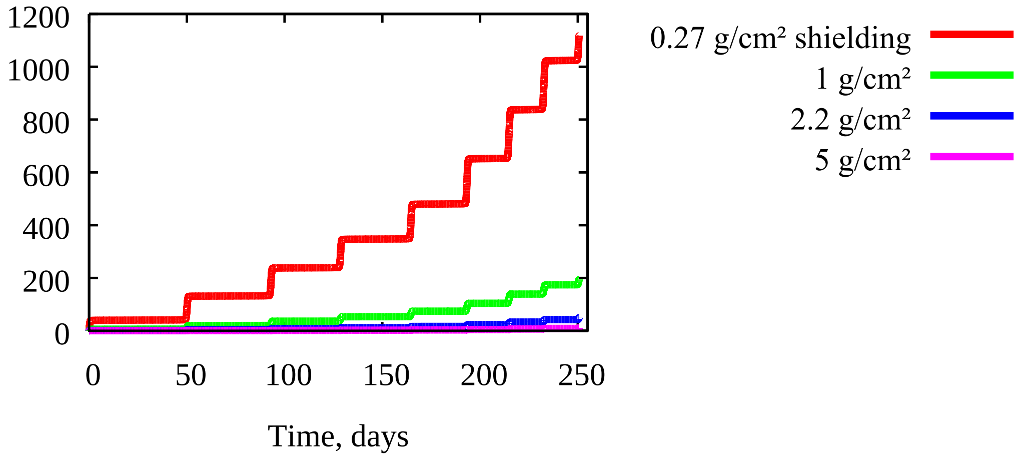

Figure 8. Dynamics of radiation dose accumulation in the debut of the Jupiter moons mission.

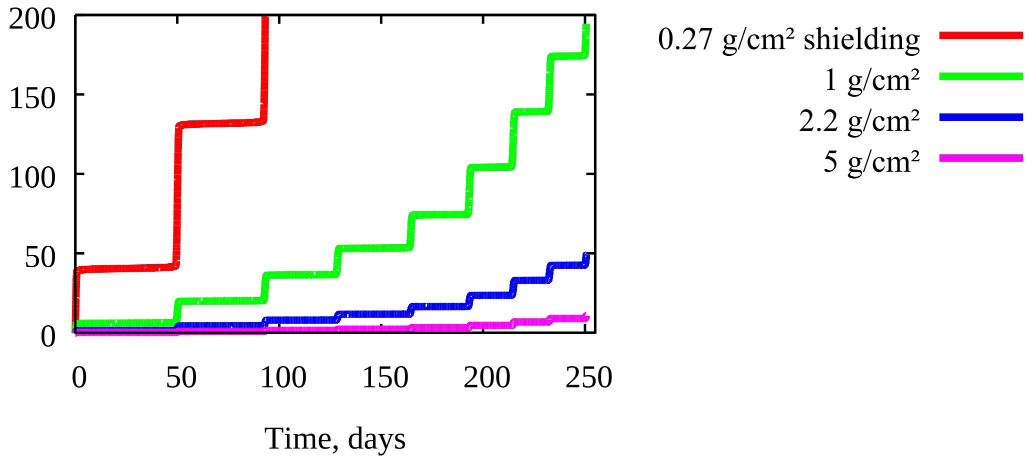

Figure 9. Dynamics of radiation dose accumulation in the debut of the Jupiter moons mission (zoomed in).

Figure 9 illustrates the dynamics of the absorbed dose accumulation on a larger scale.

In summary, we conclude that the total dose of radiation absorbed in the course of the entire mission in the vicinity of Jupiter can be very high; the high level is caused not so much by the total mission duration as by the number and altitude of the lowest periapses of the tour.

Taking into account the high radiation level in the Jovian system, we conclude that if the total dose must not exceed 150-250 krad under the Al protection 10 mm thick, only Ganymede and Callisto are available as partners for performing gravity assist maneuvers.

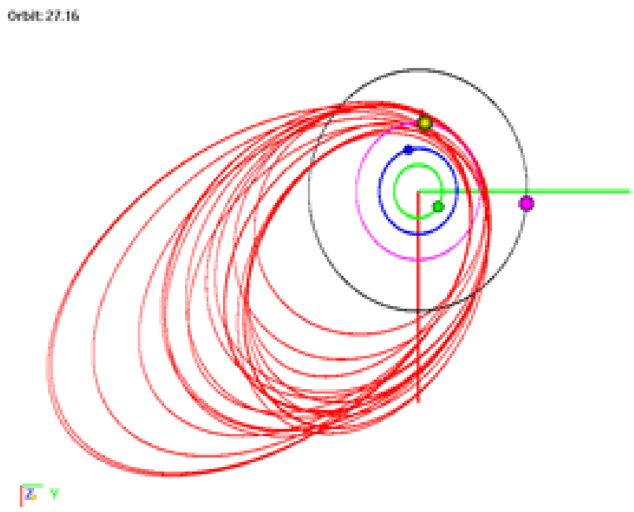

Figure 10. The synthesis of typical scenario of Ganymede Landing mission

13. THE THIRD PHASE ("ENDGAME")

There are various scenarios of a moon orbit insertion and landing on this moon. No gravity assist maneuvers with J4 Callisto are possible below its orbit. Simultaneously, gravity assist maneuvers with Ganymede become inefficient. The ESA JUICE mission assumes the use of high altitude gravity assist maneuvers at this stage; these maneuvers employ fine effects in the behavior of solutions of the restricted three body problem in this phase. In fact, here we are dealing with maneuvering in the vicinity of the Lagrangian libration points [3,7]. Formally, this is reflected in flybys of the partner moon at high altitudes (of an order 10-50 thousands of km) above its sphere of influence.

The gravity assist maneuvers of this type can also be found using the visual design software; they also can be used to design a low cost near Jupiter endgame. However, in the current version developed by the authors of this paper, which takes into account the features of the Laplace-P mission of the Russian Space Agency Roskosmos (in distinction from JUICE mission, Laplace-P plans to land on a Jupiter moon). In the endgame phase, the expenditures of the characteristic velocity are exchanged for the endgame duration. An example of a long but low cost endgame is provided by a version of JUICE. However, in order to reduce the total duration of the mission and improve the accuracy of guiding to the moon landing point, a version including the Ganymede orbit insertion maneuver (with a break impulse of about 0.6-0.85 km/s) after a series of gravity assist maneuvers can be used. Then, the spacecraft is brought in phase with Ganymede, and a deorbiting break impulse is given. The mission goes to the phase of landing on the Jupiter moon.

CONCLUSIONS

The design of missions to the Jovian system meets a number of difficulties that increase when constraints on resources, total absorbed radiation dose, and the tour duration are imposed. However, the variety of feasible gravitational interactions in the Jovian system on the one hand complicates the mission ballistic analysis and on the other hand makes it possible to adapt the scenario design algorithms to the interests of specific space missions. In particular, landing on a Jupiter moon is possible under reasonable expenditure resources and time (the near planet tour can last about two years with the total absorbed radiation dose being about 250 krad under 10 mm Al protection). On elliptic Jupiter orbits of the spacecraft, the major radiation dose in one orbit revolution is absorbed quasi-singularly in the vicinity of periapsis.

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