DETERMINATION AND PREDICTION OF ORBITAL PARAMETERS OF THE RADIOASTRON MISSION

Mikhail Zakhvatkin(1)**, Victor Stepanyants**(2)**, Yuri Ponomarev**(3)**, and Andrey Tuchin**(4)
(1)(2)(4)*Keldysh Institute of Applied Mathematics, 125047, Russia, Moscow, Miusskaya sq., 4, +7-499-250-79-96, zakhvatkin@kiam1.rssi.ru*
(3)*Astrospace Center of Lebedev Physical Institute, 117997, Russia, Moscow, Profsoyuznaya st., 84/32, yupon@asc.rssi.ru*

Abstract: *Direct solar radiation pressure (SRP) and operation of stabilization thrusters during unloadings of reaction wheels are the major uncertainties that affect the Radioastron. Both effects have significant impact on the orbit and cannot be ignored since accurate orbit is vital for correct processing of interferometric observations. This paper introduces developed direct SRP model, which allows to calculate both acceleration and torque due to impacting solar radiation. The spacecraft is not equipped with accelerometers, but a telemetry of the reaction wheels can be used to measure perturbing torque and to obtain additional information about unknown parameters of the SRP model. The paper shows how the SRP model along with consideration of unloadings significantly improves the accuracy of the orbit.*

Keywords: *Radioastron, orbit determination, solar radiation pressure, reaction wheels*

1. Introduction

The Radioastron [1] is a spacecraft carrying a 10-meters space radio telescope (SRT). The main purpose of the mission is to perform Very Long Baseline Interferometric (VLBI) observations in conjunction with the greatest ground-based VLBI networks. Highly elliptical orbit of the spacecraft provides baselines up to 350 000 km and corresponding resolution of the interferometer, which cannot be achieved by using only ground-based radio observatories. Moreover the orbit significantly evolves due to lunisolar perturbation, thereby providing opportunity to observe large variety of radio sources of interest.

Correlation of interferometric observations of such ground-space interferometer requires good enough knowledge of the parameters of its baseline, which in turn requires precise enough a posteriori knowledge of orbital parameters of the Radioastron. Meanwhile Radioastron orbit determination is complicated by significant nongravitational perturbations.

Due to size of the SRT direct solar radiation pressure (SRP) produces significant perturbing acceleration of the spacecraft's center of mass and a moment about the center of mass. Both SRP force and the moment change greatly depending on spacecraft's attitude relative to the Sun. Because of the fact that the spacecraft most of the time is moving far from the Earth and other gravitating bodies SRP becomes the major source of errors during calculation of spacecraft motion.

Another considerable nongravitational perturbation arises from operation of the spacecraft's attitude control system. During the flight and especially during observations spacecraft maintains stable orientation with respect to inertial frame by means of reaction wheels. While in constant orientation angular speed of reaction wheels increases monotonically and at some point the system requires an unloading — slowdown of reaction wheels and reduction of net angular momentum by means of jet thrusters. Because of specific position of the thrusters an unloading causes small change in velocity of the center of mass of the spacecraft. In case of the Radioastron unloadings occur one–two times a day changing spacecraft's velocity by 3–7 mm/s.

Both of emphasized nongravitational perturbations have significant impact on the motion of the Radioastron and should be properly reflected in its motion model. This paper describes such model and its applications to the problems of the orbit determination and prediction.

2. Motion model

Radioastron motion can be considered as passive motion, which is interrupted from time to time with unloadings of reaction wheels. An unloading consists of several dozen of thrust firings conducted over 2–3 minutes in almost constant orientation of the spacecraft. Since an unloading lasts much lesser than the orbital period it is convenient to represent its effect on the motion as instantaneous velocity increment applied at some average moment of time. Since parameters of every firing are reflected in the telemetry average time of application and measured velocity increment of i-th unloading can be calculated as follows

$$t_i = (\sum_{j=1}^{N} \Delta v_{i,j} t_{i,j}) / (\sum_{j=1}^{N} \Delta v_{i,j}), \quad \Delta \mathbf{v}_i^0 = \sum_{j=1}^{N} \Delta \mathbf{v}_{i,j}, \tag{1}$$

where N — total amount of firings during the unloading, $t_{i,j}$ — known time of j-th firing of the unloading, $\mathbf{v}_{i,j}$ — measured spacecraft's velocity increment due to j-th firing of the unloading, its direction is determined from spacecraft's orientation and its value — from telemetry data of the firing such as duration and mass of burned fuel. On the time span of interest, e.g. orbit determination time interval, the effect of the unloadings on the motion is described with the set of velocity increments $\{\Delta \mathbf{v}_i\}_{i=1}^{m}$, which may not exactly match to measured values $\{\Delta \mathbf{v}_i^0\}_{i=1}^{m}$, and the set of corresponding application times $\{t_i\}_{i=1}^{m}$.

Spacecraft's motion between two adjacent unloadings is passive. Numerical solution of the equations of motion accounts for gravitational perturbations caused by Earth's spherical harmonics, third body attraction, tidal potential and effects of general relativity. The model also accounts for atmospheric drag and Earth radiation pressure. Earth albedo coefficients were assumed constant, since Earth radiation has rather small impact on the motion due to characteristics of the Radioastron orbit.

Direct SRP strongly affects Radioastron motion. The acceleration itself is about $1.7 \cdot 10^{-7}$ m/s^2, which is higher than average due to construction of the spacecraft. Besides, the SRP is primary cause of perturbing torque and therefore of unloadings. There are several approaches to modeling SRP acceleration. Empirical models have been rejected because of not dense enough tracking data, lack of accelerometers and complicated spacecraft's motion due to often unloadings. The desired model should be adjustable and described with as less parameters as possible. We decided to use an analog of "box-wing" model [2, 3, 4], which accounts for optical properties of the spacecraft's surface and can be used with almost arbitrary attitude.

Impacting solar flux is decomposed into three parts: absorbed, specularly reflected and diffusely reflected [5, 6]. Each part of the flux has known impact on the surface. Quantitatively these parts are described by coefficients of reflectivity α and specularity μ. Net SRP force impacting a flat element is described as follows

$$\mathbf{F}_{sp} = (1-\alpha)\mathbf{F}_a + \alpha\mu\mathbf{F}_s + \alpha(1-\mu)\mathbf{F}_d. \tag{2}$$

$$\mathbf{F}_a = \Phi_0 A\cos\theta\cdot\mathbf{s},\quad \mathbf{F}_s = 2\Phi_0 A\cos^2\theta\cdot(-\mathbf{n}),\quad \mathbf{F}_d = \Phi_0 A\cos\theta\cdot\left(\mathbf{s}-\frac{2}{3}\mathbf{n}\right) \tag{3}$$

where Φ_0 — is solar irradiance divided by speed of light, A — area of the element, $\mathbf{s}$ — unit vector of incident rays, $\mathbf{n}$ — normal unit vector of the element, θ — angle of incidence of solar rays.

The model include approximation of spacecraft's surface containing antenna of the SRT, the spacecraft bus and solar panels (Fig. 1). The surface of the bus and solar panels consist of rectangles, the surface of the antenna consists of a number of triangles. The element of the antenna is considered to be shadowed by the spacecraft bus or by solar panels if its center is shadowed. Illuminated surface of the antenna and the bus is covered with multi-layer insulation, so optical properties of their surfaces are considered to be equal and described by coefficients α_1 and μ_1. Reflectivity of solar panels is considered to be equal to α_2, and reflection is assumed to be specular, i.e. $\mu_2 = 1$. Coefficient μ_2 is fixed because panels are always rotated almost normally to the Sun and both coefficients are strongly correlated. Thereby Radioastron SRP model depends on three unknown parameters α_1, μ_1 and α_2 and known spacecraft's attitude with respect to the Sun.

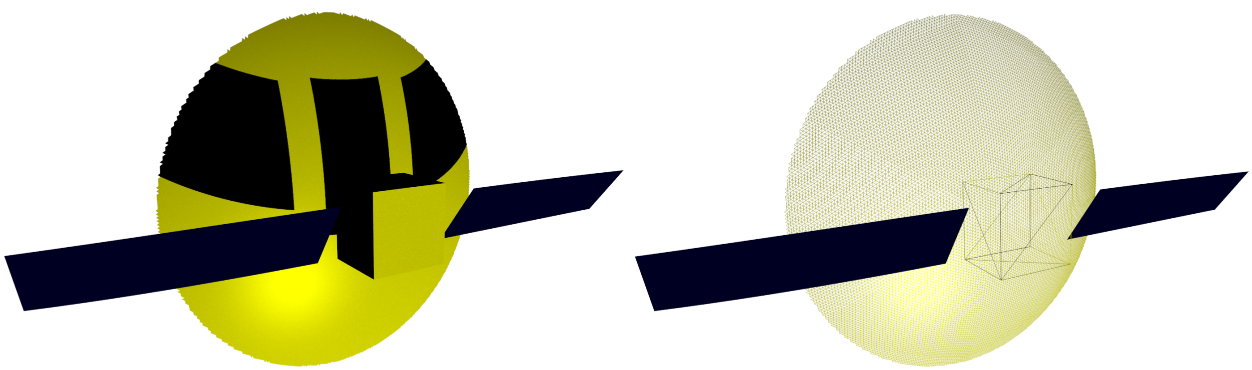

Figure 1. Approximation of the Radioastron surface

3. Orbit determination

On the interval of interest (t_b, t_e) we will solve for the spacecraft's initial state vector, three coefficients of the SRP model and m impulses of unloadings occurred on the interval: $\mathbf{Q} = \{\mathbf{X}_0(t_0), \alpha_1, \mu_1, \alpha_2, \Delta\mathbf{v}_1, \ldots, \Delta\mathbf{v}_m\}$. As mentioned above recorded spacecraft's telemetry can be used as additional source of information about solve-for parameters. Besides measured values $\Delta\mathbf{v}_i^0$ the telemetry contains measured rotation speed of reaction wheels. Since most of the time Radioastron maintains its attitude while observing radio sources the whole change of the angular momentum of reaction wheels is due to impacting SRP if the spacecraft is far from the Earth

$$\frac{\mathbf{K}(t_2)-\mathbf{K}(t_1)}{t_2-t_1} = \mathbf{M}_{SRP}(\Lambda, \alpha_1, \mu_1, \alpha_2), \tag{4}$$

Table 1. Orbit determination with several models

	SRP model	Unloadings	σ_1	σ_2	Δr, km	Δv, mm/s
1	Simple, 1 coeff.	Not considered	12.43677	9.18588	71.71	288.1
2	Simple, 1 coeff.	Nominal	4.72914	6.78832	36.76	113.3
3	Proposed, 3 coeff.	Nominal	1.20896	0.63767	7.57	8.9
4	Proposed, 3 coeff.	Solved for	0.28198	0.24907	0.21	2.3

where $\mathbf{K}(t)$ — is angular momentum of reaction wheels in body-fixed frame, t_1, t_2 — bounds of the interval of constant attitude, $\mathbf{M}_{SRP}$ — SRP torque in body-fixed frame, Λ — attitude quaternion. Rate of change of angular momentum on the left side of Eq. (4) is observed value and the SRP torque in the right side is computed one. We seek such realization of parameters $\mathbf{Q}$ that delivers minimum to the functional

$$\Phi = (\mathbf{\Psi}_o - \mathbf{\Psi}_c)^\top \mathbf{P} (\mathbf{\Psi}_o - \mathbf{\Psi}_c) + \sum_{j=1}^{N} \boldsymbol{\xi}_j^\top \mathbf{P}_j^{SRP} \boldsymbol{\xi}_j + \sum_{i=1}^{m} (\Delta \mathbf{v}_i^0 - \Delta \mathbf{v}_i)^\top \mathbf{P}_i (\Delta \mathbf{v}_i^0 - \Delta \mathbf{v}_i), \tag{5}$$

where $\mathbf{\Psi}_o, \mathbf{\Psi}_c$ — are respectively vectors of observed and computed values of tracking data, $\boldsymbol{\xi}_j$ — difference between observed and computed values of SRP torque (see Eq. (4)) during j-th interval of constant attitude, $\mathbf{P}$, $\mathbf{P}_j^{SRP}$, $\mathbf{P}_i$ — given weight matrices. First term describes tracking data fit, second term — how coefficients of the SRP model correspond to the measured SRP torque, third term — how actual impulses of unloadings differ from measured ones.

The study of the Radioastron orbit has been performed on two adjacent time intervals about 50 days each. Used tracking data includes two-way radio range and Doppler data, one-way Doppler data and optical observation of right ascension and declination. Besides proposed model, which includes complex SRP model and adjustable impulses of unloadings, we tried several less complex models to compare the impact of accounting for unloadings and complex SRP. The results describing how tracking data fits particular orbit are shown in Tab. 1. The table contains dimensionless standard deviation of tracking data after orbit determination on two intervals 2013-02-20–2013-04-10 (σ_1) and 2013-04-10–2013-05-30 (σ_1) and also difference in position (Δr) and velocity (Δv) between two solutions in the mid point 2013-04-10 00:00UT. Tracking data weights: radio range, 10 m.; one-way Doppler, 2 mm/s; two-way Doppler 5 mm/s, angles, 1 arcsec. The difference between observed and computed values of the Radioastron tracking obtained with the most complex model (No. 4 in Tab. 1) is shown in Fig. 2.

In the simple SRP model the force directed from the Sun to the spacecraft and depends only on one solve-for parameter and distance to the Sun. Nominal unloadings imply application of measured impulses $\Delta \mathbf{v}_i^0$ to the spacecraft's motion. The table also contains comparison of two orbits obtained with the same model at the boundary point.

As shown in the table use of only complex SRP model without introduction of large number ($3m$) of parameters of unloading impulses significantly improves quality of the orbit. Solving for parameters of unloadings is necessary measure. From the one side complexity of the motion and not dense enough tracking data do not allow to use small time intervals for orbit determination, from the other side even small uncertainties in measured values $\Delta \mathbf{v}_j^0$ lead to significant errors when using large

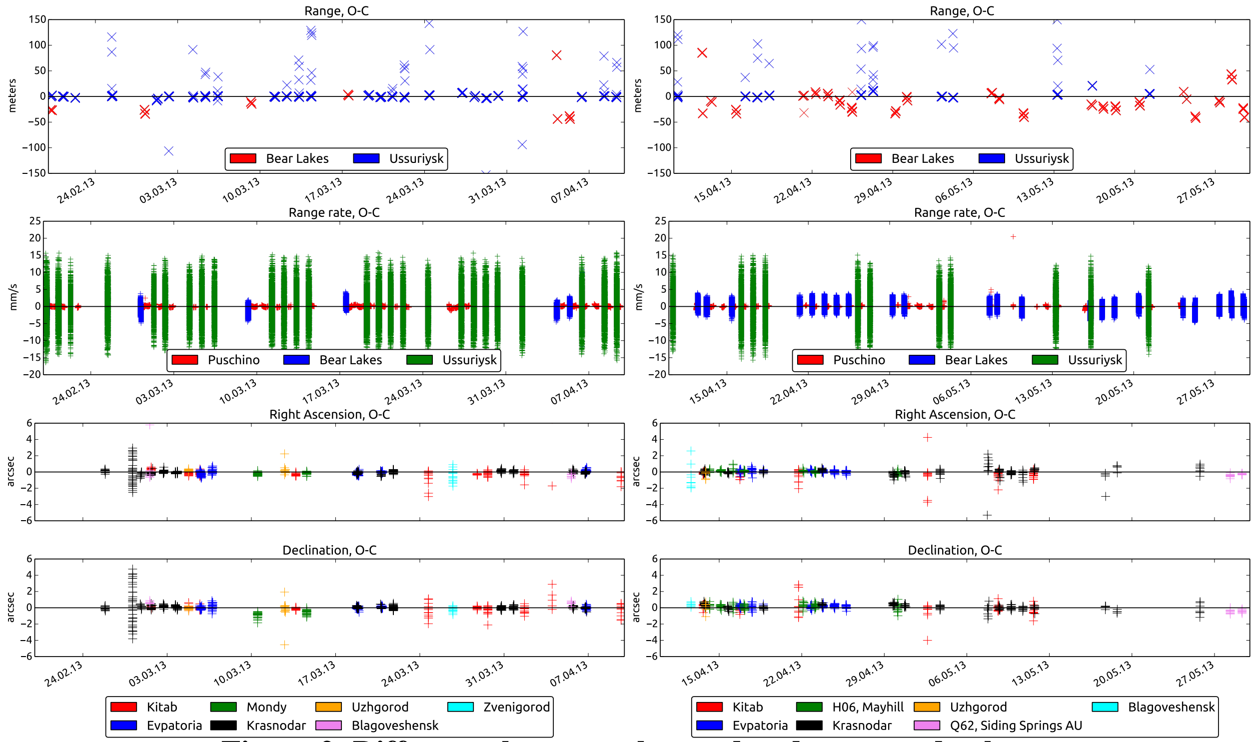

Figure 2. Difference between observed and computed values

orbit determination intervals. Worth noting that standard deviation of values of unloading impulses is 0.1–0.2 mm/s, which corresponds to expected level of uncertainty. Estimated directions of the impulses remain relatively close to observed analogs, the maximum deviation was less than 0.7 degrees.

Estimated coefficients of the SRP model are shown in Tab. 2. Their values are qualitatively similar to expected optical properties of the spacecraft's surface. The MLI, which covers the SRT and the spacecraft bus, has high reflectivity, but less than expected specularity. The reflectivity of the solar panels is low as expected. Estimated SRP parameters have minor difference depending on interval of orbit determination. The difference may be caused by different set of spacecraft's attitudes on the intervals. In general, the obtained results indicate that proposed model adequately accounts for the impacting solar radiation pressure.

Table 2. Estimated coefficients of the solar radiation pressure model

Parameter	20-Feb – 10-Apr	10-Apr – 30-May
α_1	0.754	0.791
μ_1	0.087	0.089
α_2	0.063	0.102

4. Orbit prediction

While quality of the predicted orbital parameters is not so important for scientific purposes of the Radioastron than a posteriori knowledge of the orbit, not accurate predictions may cause some difficulties during Radioastron observations scheduling. The quality of the predicted orbit as well suffers from two major perturbations: solar radiation pressure and impulses of unloadings. As shown above proposed SRP model well enough describes the perturbation if the spacecraft's attitude is known. Because the set of radio sources observed by the Radioastron is defined for several months ahead, future spacecraft's orientation can be considered as known. The situation with unloadings is less certain, they can be conducted in different time and therefore in different attitude. Moreover the value of an impulse depends on several factors such as current spacecraft's attitude and the angular moment stored by the reaction wheels.

Because the SRP model allows to calculate also the torque, one can predict angular momentum of the reaction wheels in case if there were no unloadings and no limits on the stored angular momentum. Based on predicted angular moment of the reaction wheels we will calculate times of upcoming unloadings and values of corresponding impulses. Consider two rules of conducting the unloadings:

- Stored angular momentum since previous unloading went outside some set U, which is determined by allowed rotation speeds of reaction wheels.
- Unloading are conducted every day in the same time. Based on the analysis of unloadings of the Radioastron the time was set to 05:30 UT.

We will refer to the unloadings calculated according to the first rule as "forced", because in that case the system has to perform an unloading in order to operate properly in the future. The unloadings performed on daily basis in the same time we will call "regular".

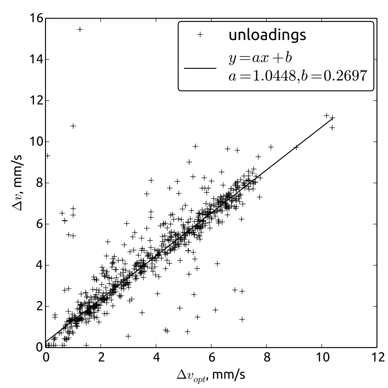

Figure 3. Relation between calculated optimal impulses and measured actual ones

In order to determine how the value of an impulse depends on stored angular momentum consider an unloading performed while the spacecraft maintains constant attitude. Let p_j be the momentum of exhausted gases of j-th thruster during the unloading, $\mathbf{r}_j$ — position of the thruster's nozzle with respect to the center of mass in the body-fixed frame, $\mathbf{e}_j$ — thrust direction in the body-fixed frame, $\mathbf{K}$ — stored angular momentum of the reaction wheels in the body-fixed frame. If the unloading is conducted relatively fast the following equation holds

$$\mathbf{K} = \sum_{j=1}^{q} \mathbf{r}_j \times \mathbf{e}_j p_j, \tag{6}$$

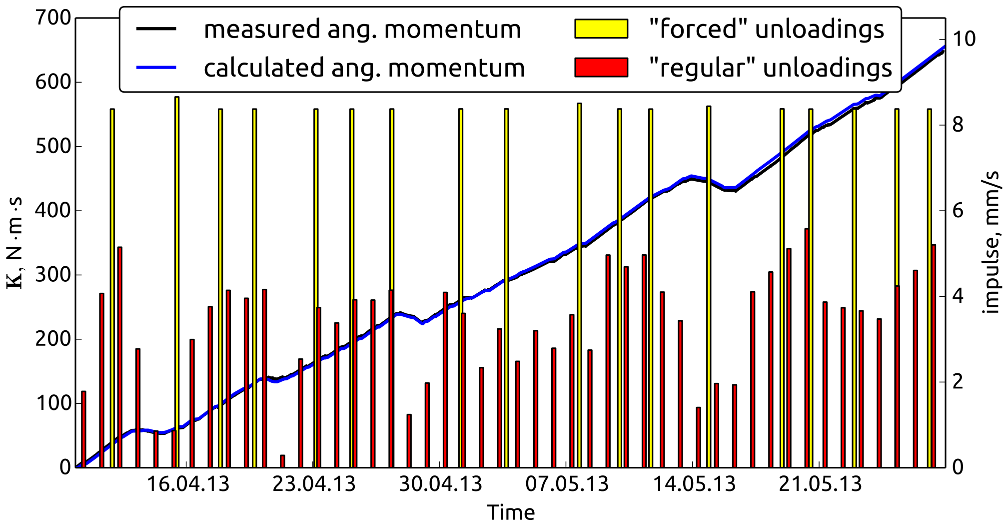

Figure 4. Angular momentum of reaction wheels and predicted unloadings

where q — is the number of thrusters. In case of the Radioastron Eq. (6) cannot be resolved with respect to p_j. But with additional expression $\sum_{i=1}^{q} p_j \to \min$ and obvious conditions $p_j \geq 0,\ i = 1, \ldots, q$ the solution $\{p_j^*\}_{i=1}^{q}$ can be found. Such solution will correspond to the unloading performed with minimum fuel cost. The optimal in that manner impulse of unloading is described as follows

$$\Delta \mathbf{v}_{opt}(\mathbf{K}) = -\frac{\sum_{j=1}^{q} p_j^* \mathbf{e}_j}{M} \tag{7}$$

where M — is current mass of the spacecraft. In case of the Radioastron a direction of an impulse of unloadings is always known since an unloading is performed with co-alligned thrusters. We will look for actual value of the unloading impulse as function of Δv_{opt}. The analysis of almost 700 unloadings of the Radioastron showed that the relation is well described by a linear function (see Fig. 3).

Described method was tested using the results of the previous section. Orbital parameters, determined on the first interval (from 20-Feb-2013 to 10-Apr-2013) where used to predict spacecraft's motion on the second interval (from 10-Apr-2013 to 30-May-2013). Accumulation of angular momentum on the second interval was calculated using known attitude of the spacecraft and parameters α_1, μ_1 and α_2 estimated on the first interval. Predicted and actual angular momentum of reaction wheels are shown on Fig. 4.

Direction of each predicted impulse of unloading is defined by current spacecraft's attitude. Predicted impulses of unloadings were used in turn to predict spacecraft's motion. There were calculated four predictions:

- Simple prediction, which neither takes account for unloadings or complex solar radiation pressure.
- Prediction with "forced" unloadings.
- Prediction with "regular" unloadings.
- Prediction with actual unloadings.

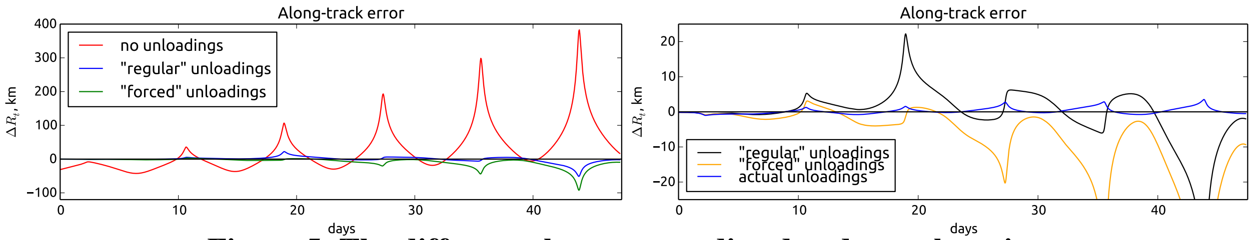

Figure 5. The difference between predicted and actual motion

Last three predictions uses complex SRP model and appropriate parameters of solar radiation pressure.

The orbit determined on the second interval by using model with adjustable unloadings and complex SRP was considered as actual motion of the spacecraft. This actual motion was compared with calculated predictions. The result of comparison in along-track direction, which contains the major part of the total positional error, is shown in Fig. 5. The simplest model showed deviation from the actual orbit more than 380 km on 50 days interval. Prediction with "forced" unloadings showed maximum error about 90 km. Prediction with "regular" unloadings turned to be most accurate, its maximum deviation from actual motion is 51 km. The most likely the reason why prediction with "regular" unloadings showed better result is that directions of predicted impulses more often were similar to directions of actual unloadings occurred on the interval. Generally during a morning the Radioastron does not perform any observations, instead it conducts service tasks. During the service time the spacecraft is always in specific orientation, which among other depends on position of the Earth, the Moon and the Sun; it changes relatively slow. Unloadings are likely conducted in the same orientation. Prediction with actual unloadings does not go further than 3.5 km from the actual motion of the spacecraft. From one side it demonstrates the potential of such prediction model, which can be achieved with more complex models of prediction of unloading times or through activities coordinated with mission control center. From another side it shows the quality of obtained orbits.

5. Conclusion

A motion model for the Radioastron spacecraft has been developed. The model takes account for unloadings of the spacecraft's reaction wheels and complex solar radiation pressure, which depends on spacecraft's attitude and optical properties of its surface. Developed solar radiation pressure model uses simplified form of the spacecraft consisting of a space radio telescope, a spacecraft bus ans solar panels. The model was tested during the Radioastron orbit determination. Overall accuracy of the obtained orbits is satisfactory. Orbits are enough accurate to perform successful correlation of interferometric data. Despite the fact that adjustable impulses of unloadings partially hide unmodeled accelerations their variation remained on relatively low level. Estimated parameters of solar radiation pressure were used for prediction of accumulated angular momentum of reaction wheels and parameters of resulting unloadings. This data was utilized for the Radioastron orbit prediction. Although prediction of unloadings improves accuracy of the orbit, the improvement could be much better, if predicted times of unloadings were closer to actual ones.

6. References

[1] Kardashev, N. S., Kovalev, Y. Y., and Kellermann, K. I. “RadioAstron: An Earth-Space Radio Interferometer with a 350,000 km Baseline.” The Radio Science Bulletin No 343, p. 22-29, Vol. 343, pp. 22–29, Dec. 2012.

[2] Rodriguez-Solano, C. J., Hugentobler, U., and Steigenberger, P. “Adjustable box-wing model for solar radiation pressure impacting GPS satellites.” Advances in Space Research, Vol. 49, No. 7, pp. 1113–1128, Apr. 2012. ISSN 0273-1177. doi:10.1016/j.asr.2012.01.016.

[3] Marshall, J. and Luthcke, S. “Modeling radiation forces acting on Topex/Poseidon for precision orbit determination.” Journal of Spacecraft and Rockets, Vol. 31, pp. 99–105, 1994. ISSN 0022-4650. doi:10.2514/3.26408.

[4] Berthias, J., Broca, P., Ferrier, C., and Gratton, S. “JASON-1: a New Reference for Precise Orbit Determination.” “IAF abstracts, 34th COSPAR Scientific Assembly,” 2002.

[5] Fliegel, H. F., Gallini, T. E., and Swift, E. R. “Global Positioning System Radiation Force Model for geodetic applications.” Vol. 97, pp. 559–568, Jan. 1992. doi:10.1029/91JB02564.

[6] Zeibart, M. “Generalized Analytical Solar Radiation Pressure Modeling Algorithm for Spacecraft of Complex Shape.” Vol. 41, No. 5, pp. 840–848, 2004.