NUMERICAL AND ANALYTICAL SPACECRAFT ATTITUDE PREDICTION

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Abstract: *This paper presents the comparison between the numerical and analytical results of a spacecraft attitude prediction for a spin stabilized satellite. Some external torques are introduced in the equations of the motion and the comparisons are done considering that these torques are acting together, which are: gravity gradient, aerodynamic, solar radiation, magnetic residual and eddy current. In the numerical approach it is used the quaternion to represent the attitude. This numerical approach can be applied for any kind of satellite. The analytical approach is applied directly for a spin stabilized satellite and the equations of motion are described in terms of the spin velocity, spin axis right ascension and declination angles. An analytical solution of these equations is presented and valid for one orbit period. The solution for the spin velocity has exponential variation and the temporal variations on the right ascension and declination of the spin axis produce the precession and drift of the spin axis. Applications are developed considering the Brazilian spin stabilized satellites SCD1 and SCD2. The comparisons are important to validate some simplifications that are required in the analytical approach. The results show that the average components of the external torque are sufficient to observe the main influence of the considered torques.*

Keywords: *attitude propagation, quaternion, Euler angles, external torques, spin* axis.

1. Introduction

The goal of this paper is to compare the numerical and analytical results of a spacecraft attitude prediction. Some external torques are introduced in the equations of the motion and the comparisons are done considering all torques acting together. The considered torques are: gravity gradient, aerodynamic, solar radiation and magnetic torques.

In the numerical approach it is used the quaternion to represent the attitude and the 8th order Runge Kutta method to integrate the equation of motion. The dynamic equations of the satellite's rotational motion are described by the Euler equations and the four kinematic equations for the attitude quaternion. The Euler equations give the variation rate of the components of the satellite's spin velocity that depend on the components of the external torques in the satellite fixed system. The simulations are developed in FORTRAN language.

Applications are developed considering the Brazilian spin stabilized satellites SCD1 and SCD2, which are quite appropriated for verification and comparison of the theory with the real data acquired and processed by the Satellite Control Center of National Institute Space Research from Brazil (SCC/INPE). Spin stabilized satellites has the spin axis along the biggest principal moment of inertia's axis A spherical coordinate system fixed in the satellite is used to locate the satellite spin axis in relation to the terrestrial equatorial system. The spin axis direction is specified by its right ascension and the declination angles and is represented in Fig. 1. The time evolution of these angles is gotten from the numerical results of the quaternion attitude propagation.

The analytical approach is applied directly for a spin stabilized satellite and the equations of motion are described in terms of the spin velocity and the spin axis right ascension and declination angles. In this paper, the averages of the components of each torque over an orbital period are used in order to get an analytical solution of these equations. The solution for the spin velocity has exponential variation due to the eddy currents and gravity gradient torques. The temporal variations on the right ascension and declination of the spin axis produce the precession and drift of the spin axis. The numerical implementation in this case is done with software MATLAB.

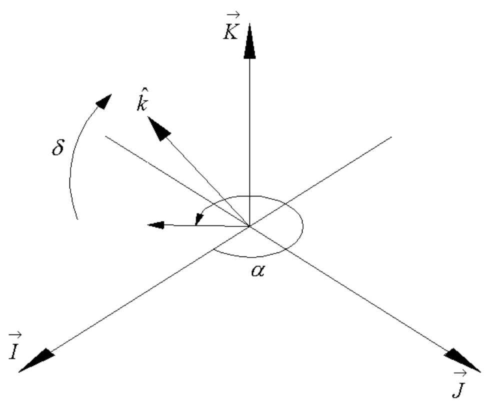

Figure 1 –Equatorial System ($\hat{I},\hat{J},\hat{K}$), spin axis orientation ($\hat{k}$), right ascension (α) and declination (δ) of the spin axis.

In the comparisons of the results it is important to observe the deviation between the actual spin axis data supplied by INPE and the computed spin axis for each satellite. Here this deviation is called pointing deviation and is given by the angle between the actual spin axis and the computed spin axis. The pointing deviation is calculated bydot product between the actual and computed spin axis.

The comparisons are important to validate some simplifications that were required in the analytical approach.

2. Considerations about External Torques

Classical models [7, 10] are assumed for each considered torque in terms of the quaternion parametrization for the numerical approach and in terms of the right ascension and declination of the spin axis for the analytical approach.

The gravity gradient torque is generated by the difference of the Earth gravity force direction and intensity actuating on each satellite mass element. This torque depends on the principal moments of inertia of the satellite and is inversely proportional to the cube of the satellite geocentric distance. The adopt mathematical model in terms of the quaternion attitude is given in [16] and for the analytical approach is given in [6,15].

The aerodynamic torque is created by the interactions of rarefied air particles with the satellite surface and it has the predominant orbit perturbation effect in LEO orbit satellites. This torque depends on the atmospheric density, the distance between the centre of pressure and the mass center of the satellite, the magnitude of the satellite's velocity relative to the atmosphere, reference section area of the satellite and the aerodynamic coefficient. In this paper only the influence of the Drag force is considered for both approach. In order to estimate the influence of the aerodynamic torque in the rotational motion, in this paper it is assumed that the velocity is equal to the orbit velocity, the aerodynamic coefficient is fixed, and the thermosphere model TD-88 is used for the atmospheric density [8]. The adopted mathematical model for the analytical approach is given in [6, 15] and for the numerical approach in terms of the quaternion is given in [16].

Magnetic disturbance torques result from the interaction between the spacecraft's residual magnetic field, the Earth´s magnetic field and the eddy current. In this paper, it is assumed that the spacecraft is manufactured from material such that the primary sources of magnetic torques are the spacecraft magnetic moments and eddy currents, with other sources neglected.

The residual magnetic torque results from the interaction between the spacecraft's residual magnetic moment and the Earth magnetic field, and its main effect is to produce a spin axis orientation drift. It depends of the residual magnetic moment of the spacecraft and the geocentric magnetic field. In this paper the dipole model is assumed for the Earth's magnetic field [10] and the satellite is supposed to be in an elliptical orbit. The adopt mathematical model for this torque in terms of the quaternion attitude is given in [16] and in terms of right ascension and declination is given in [2,15].

The torque induced by eddy currents is caused by the spacecraft spin motion. It is known [2,13] that the eddy currents produce a torque which causes the precession in the spin axis and causes an exponential decay of the spin rate. It depends on the spacecraft's angular velocity vector, the geomagnetic field and of the constant coefficient (which depends on the spacecraft geometry and conductivity).The adopted mathematical model for the analytical approach is given in [13], and for the numerical approach, in terms of the quaternion, is given in [16].

The solar radiation pressure is created by the continuous photons collisions with the satellite surface, which can be able to absorb or reflect on this flow. The total change of the momentum of all the incident photons on the satellite surface originates from the solar radiation force and it can produce a torque. A Solar Radiation Torque model was developed in [9] for the case which the illuminated surfaces of the satellite are a circular flat surface and a portion of the cylindrical surface. It depends on the solar parameter, the Sun-Earth distance, the satellite geocentric distance, the Sun satellite distance, the specular and total reflection coefficients. For the analytical approach the model for this torque is presented in [5] and for the numerical approach it is presented in [1].

In the analytical approach only the mean components of each torque are considered over one orbit period. In order to obtain the mean torques, it is necessary to integrate the instantaneous torques over one orbital period. These procedure are discussed in [5,15], where it is also possible to get the mean components of each torque.

3. Numerical approach

For the numerical approach, the dynamic equations of the satellite's rotational motion are described by the Euler equations and the kinematic equations for the attitude quaternion. The Euler equations give the tax of variation of the components of the satellite's spin velocity and depend on the components of the external torques in the body system (satellite fixed system) [7,10]:

$$\dot{p} = q\,r\,(I_y - I_z)/I_x + N_x/I_x\,,$$
$$\dot{q} = p\,r\,(I_z - I_x)/I_y + N_y/I_y\,, \quad (1)$$
$$\dot{r} = p\,q\,(I_x - I_y)/I_z + N_z/I_z\,.$$

In these equations I_x, I_y, I_z are the Principal Moments of Inertia of the satellite, p,q,r and N_x, N_y, N_z are the components of the spin velocity and the external torques in the body system, respectively.
In this paper the kinematic equations are described in terms of the attitude quaternion q, which is a 4x1 vector given by [7]:

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4]^t = [\vec{q} \quad q_4]^t\,, \quad (2)$$

where t represents the transposed of the matrix and they can be expressed in function of the rotation angle (ϕ) and of the axis of rotation $\vec{\mathrm{n}}$:

$$\vec{q} = [q_1 \quad q_2 \quad q_3]^t = sin(\phi/2)\vec{n} \quad and \quad q_4 = cos(\phi/2). \quad (3)$$

The matrix of attitude in terms of the quaternion is presented in [7]. The kinematic equations that describe the tax of variation of the components of the quaternion of attitude, due to rotation of the satellite, are given by [7,10]:

$$\dot{q}_1 = \frac{1}{2}[p\,q_4 - q\,q_3 + r\,q_2], \qquad \dot{q}_2 = \frac{1}{2}[q\,q_4 - r\,q_1 + p\,q_3],$$
$$\dot{q}_3 = \frac{1}{2}[r\,q_4 - p\,q_2 + q\,q_1], \qquad \dot{q}_4 = -\frac{1}{2}[p\,q_1 + q\,q_2 + r\,q_3]. \quad (4)$$

The applications are developed for spin stabilized satellite which has the spin axis along the biggest principal moment of inertia's axis. The direction of the spin axis $\hat{k}$ is specified by its right ascension (α) and the declination (δ), which are represented in Fig. 1.This spherical coordinate can be obtained using the attitude quaternion q and the components of the spin velocity W. If the units vectors $(\hat{I}, \hat{J}, \hat{K})$ are associated with the equatorial system and the units vectors $(\hat{i}, \hat{j}, \hat{k})$ with the body system, then the spin velocity vector can be expressed by:

$$\overrightarrow{W_I} = P\hat{I} + Q\hat{J} + R\hat{K} \qquad \text{and} \qquad \vec{W} = p\hat{e_x} + q\hat{e_y} + r\hat{e_z}\,. \quad (5)$$

If the components of the attitude quaternion (q_1, q_2, q_3, q_4) and components of the satellite spin velocity (p, q, r) are known, the vectors $\vec{W}$ and $\overrightarrow{W_I}$ are related by [7]:

$$P = (q_1^2 - q_2^2 - q_3^2 + q_4^2)p + 2q(q_2q_1 - q_4q_3) + 2r(q_3q_1 + q_4q_2)$$
$$Q = 2p(q_1q_2 + q_4q_3) + (-q_1^2 + q_2^2 - q_3^2 + q_4^2)q + 2r(q_3q_2 - q_4q_1) \quad (6)$$
$$R = 2p(q_3q_1 - q_4q_2) + 2q(q_3q_2 + q_4q_1) + (-q_1^2 - q_2^2 + q_3^2 + q_4^2)r$$

and the magnitude of the spin velocity is given by:

$$W = (p^2 + q^2 + r^2)^{\frac{1}{2}}. \tag{7}$$

According to Fig. 1, the components of spin velocity P. Q, R can be obtained by:

$$P = W \cos \delta \cos \alpha, \quad Q = W \cos \delta \sin \alpha, \quad R = W \sin \delta. \tag{8}$$

By the Eq. (8), the right ascension (α) and declination (δ) of the spin velocity can be computed by:

$$\sin \delta = \frac{R}{W}, \quad \cos \alpha = \frac{P}{W \cos \delta}, \quad \sin \alpha = \frac{Q}{W \cos \delta} \tag{9}$$

with $0 \leq \delta \leq 90°$ and $0 \leq \alpha \leq 360°$.

Then in order to compare the numerical and analytical results, the right ascension and declination of the spin velocity will be computed using the numerical results obtained for components of the spin velocity and attitude quaternion by the numerical integration of the Eq. 1 and Eq. 4.

4. Analytical approach

For the analytical approach, the motion for spin stabilized artificial satellites are described by the variations of the spin velocity, the declination and the right ascension of the spin axis and are given by the Euler equations in spherical coordinates [13]:

$$\frac{dW}{dt} = \frac{1}{I_z} N_z, \qquad \frac{d\delta}{dt} = \frac{1}{I_z W} N_y, \quad \frac{d\alpha}{dt} = \frac{1}{I_z W \cos \delta} N_x, \tag{10}$$

where I_z is the moment of inertia along the spin axis, N_x, N_y, N_z are the components of the external torques in the body system and here are given by the sum of the mean components of gravity gradient, aerodynamic, solar radiation, residual magnetic and eddy current torques.

The solution of these equations were gotten in [4,6], and for one orbital period they are given by:

$$W = \left(W_0 + \frac{N_{gzm}}{N_{izm}}\right) e^{\frac{N_{izm}}{I_z} t} - \frac{N_{gzm}}{N_{izm}}. \tag{11}$$

$$\delta = \frac{t}{I_z}\left(N_{iym} - \frac{N_{tym} N_{izm}}{N_{gzm}}\right) + \frac{N_{tym}}{N_{gzm}} \ln\left(\frac{W}{W_0}\right) + \delta_0, \tag{12}$$

$$\alpha = \frac{t}{I_z \cos \bar{\delta}}\left(N_{ixm} - \frac{N_{txm} N_{izm}}{N_{gzm}}\right) + \frac{N_{txm}}{N_{gzm} \cos \bar{\delta}} \ln\left(\frac{W}{W_0}\right) + \alpha_0, \tag{13}$$

where N_{ixm}, N_{iym}, N_{izm} are the mean components of eddy currents torques and $N_{tym} = N_{Ay} + N_{sy} + N_{gym} + N_{rym}$ and $N_{txm} = N_{Ax} + N_{sxm} + N_{gxm} + N_{rxm}$, being N_{Axm}, N_{Aym}, N_{Azm}, N_{sxm}, N_{sym}, N_{szm}, N_{gxm}, N_{gym}, N_{gzm}, N_{rxm}, N_{rym}, N_{rzm} are the mean components of the aerodynamic, solar radiation, gravity gradient and residual magnetic torques, respectively, and $\bar{\delta} = \frac{\delta - \delta_0}{2}$, δ is the computed declination, W_0, δ_0 and α_0 are the initial values for spin velocity, declination and right ascension of the spin axis.

The numerical implementation of these analytical solution is developed for the real data of Brazilian satellite SCD1 and SCD2 in order to compare with the numerical results of Eqs. (1),(4), (7) and (9).

5. Pointing Deviation

For the tests, it is important to observe the deviation between the actual attitude data supplied by CCS/INPE and the computed attitude for each satellite. Here this deviation is called pointing deviation and given by the angle θ between the actual spin axis $\hat{k}$ and the computed spin axis $\overline{k_c}$. It can be computed by [14,16]:

$$\cos\theta = \hat{k} \cdot \overline{k_c}\,, \tag{14}$$

where ($\cdot$) indicates the dot product between actual spin axis $\hat{k}$ and computed spin axis $\overline{k_c}$.
The unit vectors $\hat{k}$ and $\overline{k_c}$ can be obtained using the right ascension and declination of the spin axis as:

$$\hat{k} = \cos\alpha\cos\delta\,\hat{I} + \sin\alpha\cos\,\delta\,\hat{J} + \sin\delta\,\hat{K}\,, \tag{15}$$

$$\overline{k_c} = cos\ \alpha_c\ cos\ \delta_c\hat{I} + sin\ \alpha_c\ cos\ \delta_c\,\hat{J} + sin\ \delta_c\,\hat{K}, \tag{16}$$

with α and δ supplied by SCC/INPE and α_c and δ_c computed by the analytical or numerical approach.

6. Numerical simulation and results comparison

The numerical and analytical approaches have been applied to the spin stabilized Brazilian Satellite (SCD1 and SCD2) for verification and comparison of the approaches against data generated by the SCC/INPE.

The 8th Runge Kutta method is used to determine the numerical solution for Eq. 1 and Eq. 4. The numerical solutions give the components of the attitude quaternion and the spin velocity, which are used to compute the spin velocity, right ascension and declination of the spin axis by using Eq. 7 and Eq. 9. Then these numerical values are compared with the numerical values gotten by the analytical solution, given by Eqs. 11 – 13, and with real data supplied by SCC/INPE in order to check the precision of the presented approaches. That is also important to observe the deviation between the actual spin axis and the computed spin axis, that is, the pointing deviation computed by Eq. 14.

Two simulations are presented. In the first one, the propagated (analytical and numerical) attitude is daily updated with the actual satellite data. In the second simulation, the daily updates of the attitude data has not been performed in the propagation process. In both simulations, the orbital elements are updated, taking into account the main influences of the Earth oblateness.

Initial conditions for the attitude were taken from CSS/INPE supplied data [5,14]. The simulations were made for 16 days for each satellite.

6.1 First simulation: daily updated data

For SCD1, the actual values, numerical and analytical results for the temporal variations of the right ascension, declination, spin velocity and the pointing deviation are shown in Fig. 2 to Fig 5. In the Fig. 6 to Fig. 8 are shown the results for the deviation error between the computed values and actual values of the right ascension, declination, spin velocity and for the pointing deviation. In Tab. 1 are shown the mean deviation errors of each parameter for this simulation.

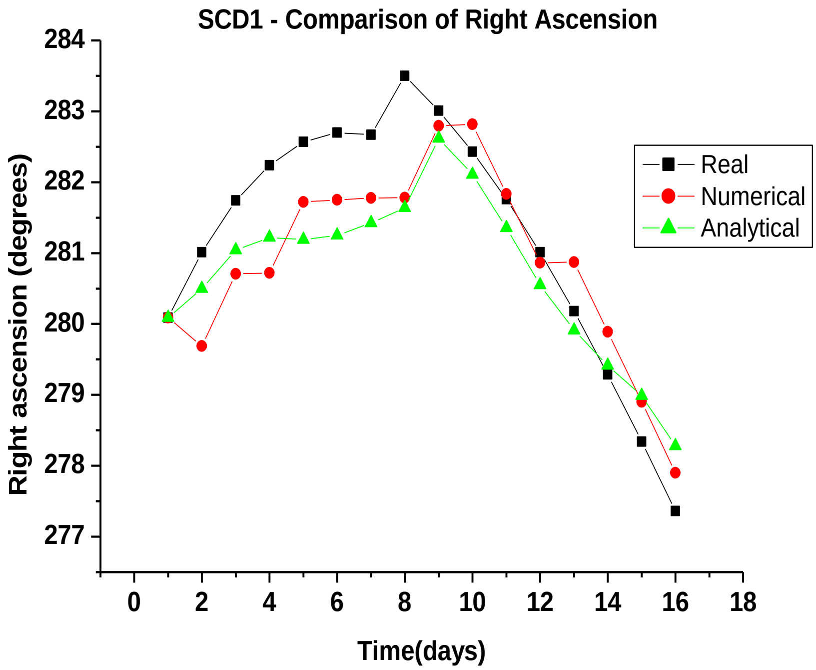

Figure 2 - Temporal variation for the actual, analytical and numerical right ascension of the spin axis for SCD1, with the daily updated data.

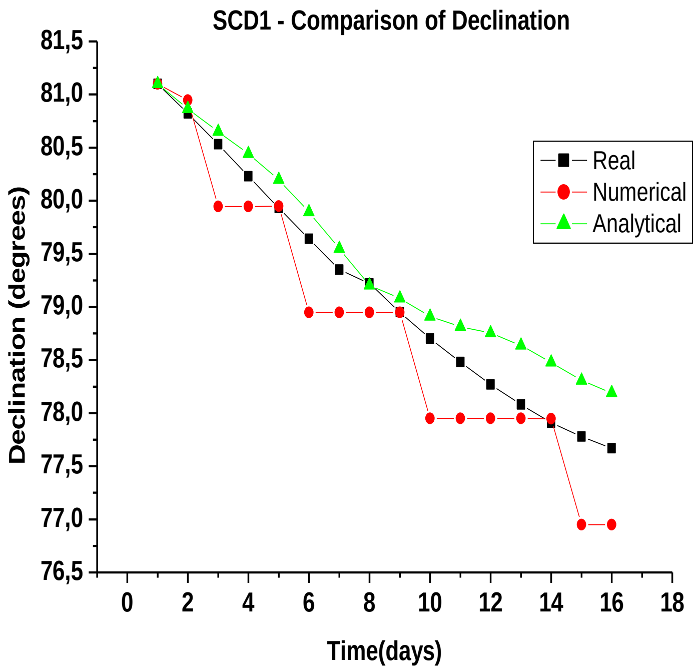

Figure 3 - Temporal variation for the actual, analytical and numerical declination of the spin axis for SCD1, with the daily updated data.

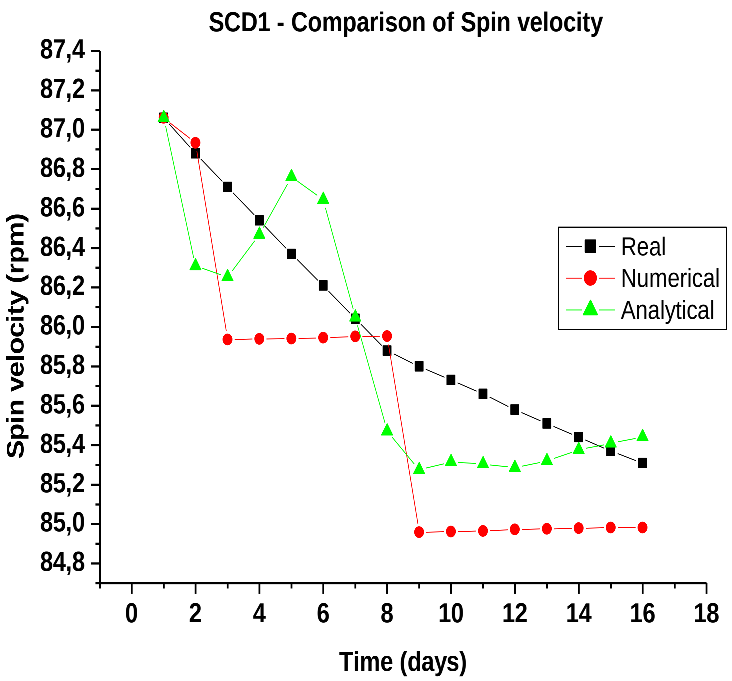

Figure 4 - Temporal variation for the actual, analytical and numerical spin velocity for SCD1 and with the daily updated data.

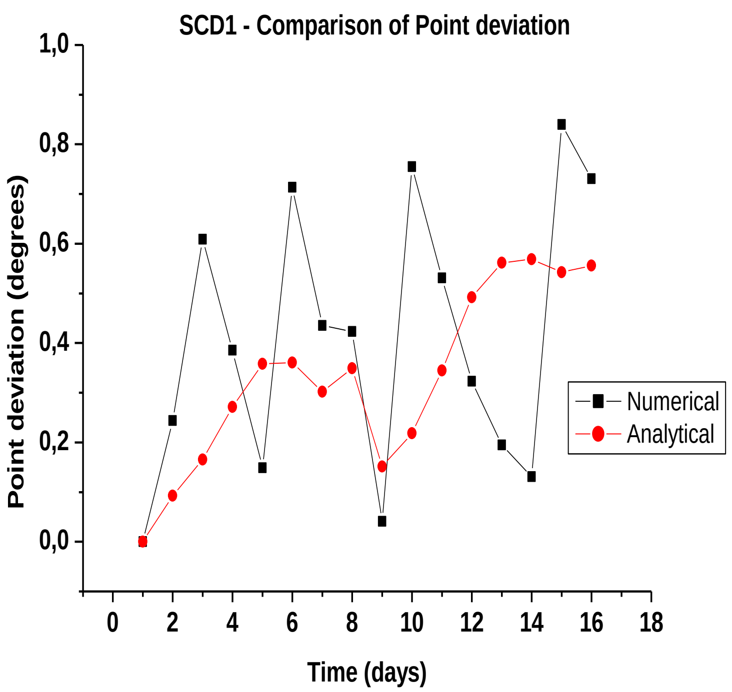

Figure 5 - Temporal variation for the pointing deviation for SCD1, with the daily updated data.

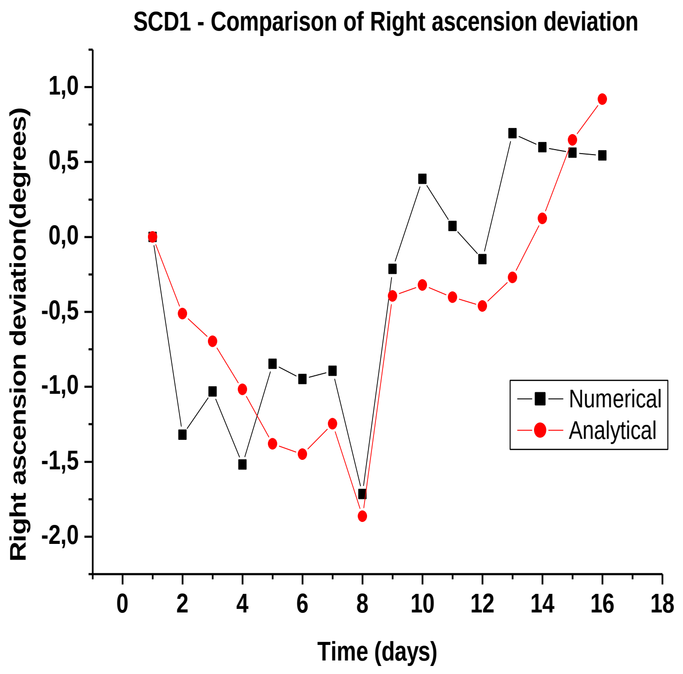

Figure 6 - Temporal variation for the deviation between the analytical or numerical and actual right ascension of the spin axis for SCD1, with the daily updated data.

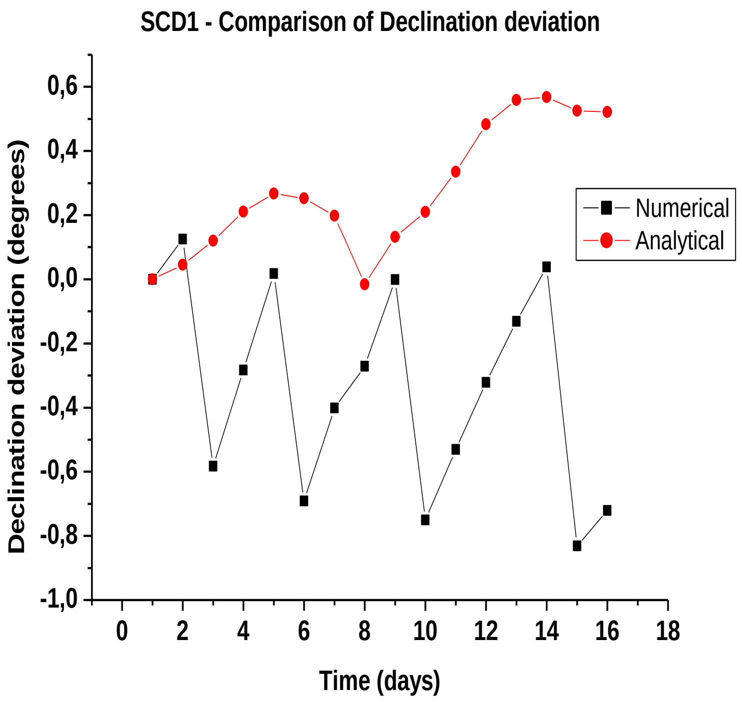

Figure 7 - Temporal variation for the deviation between the analytical or numerical and actual declination of the spin axis for SCD1, with the daily updated data.

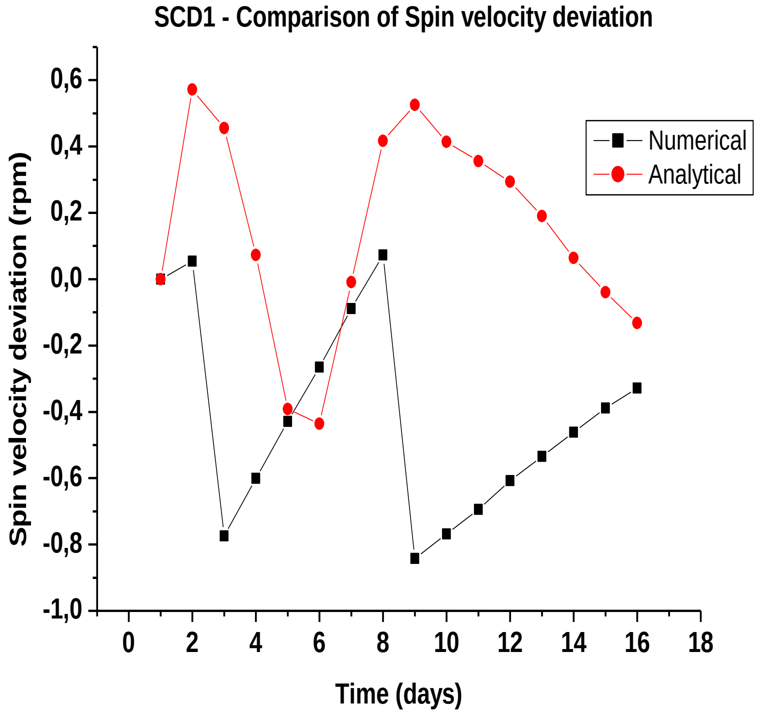

Figure 8 - Temporal variation for the deviation between the analytical or numerical and actual spin velocity for SCD1, with the daily updated data.

Table 1 – Mean deviation errors for SCD1 simulations with daily updated data.

	Numerical	**Analytical**
Right Ascension Deviation (degrees)	-0.3616	-0.5206
Declination Deviation (degrees)	-0.3336	0.2757
Spin Velocity Deviation (rpm)	-0.4161	0.1471
Pointing Deviation (degrees)	0.4066	0.3335

For this test period, the results showed that mean deviation error in right ascension, declination, spin velocity and pointing deviation are within the dispersion range of the attitude determination system performance of CSS/INPE, which is 0.5^{o} for the angles and 0.5rpm for the spin velocity. In Tab. 1 it is possible to observe that only for the right ascension the mean deviation errors for the numerical approach are better than the analytical approach.

For the SCD2, the actual values, numerical and analytical results of the right ascension, declination and spin velocity and the pointing deviation are shown in Fig. 9 to Fig.12. The deviation between the computed values and actual values of the right ascension, declination and spin velocity are shown in Fig 13 to Fig. 15. The discontinuities in these figures occur due to the attitude control corrections applied by SCC/INPE. In Tab. 2 are shown the mean deviation errors for this simulation. It is important to note that when the attitude control actuates, the computed values are assumed to be equal to the real data because the control system is not included in the proposed theory.

For the test period of 16 days, the mean deviation errors are also within the dispersion range of the attitude determination system of SCC/INPE. In Tab.2 it is possible to observe that all mean deviation errors in the analytical approach are smaller than these errors for numerical approach.

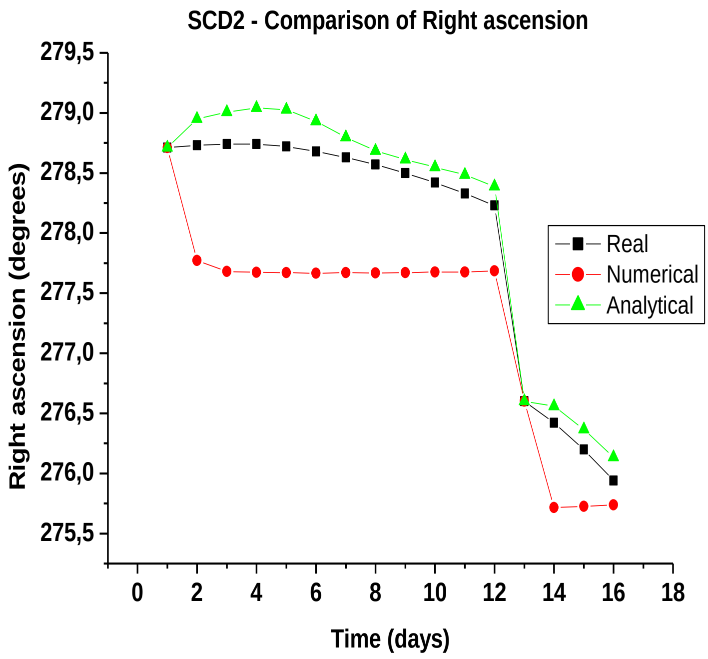

Figure 9 - Temporal variation for the actual, analytical and numerical right ascension of the spin axis for SCD2, with daily updated data.

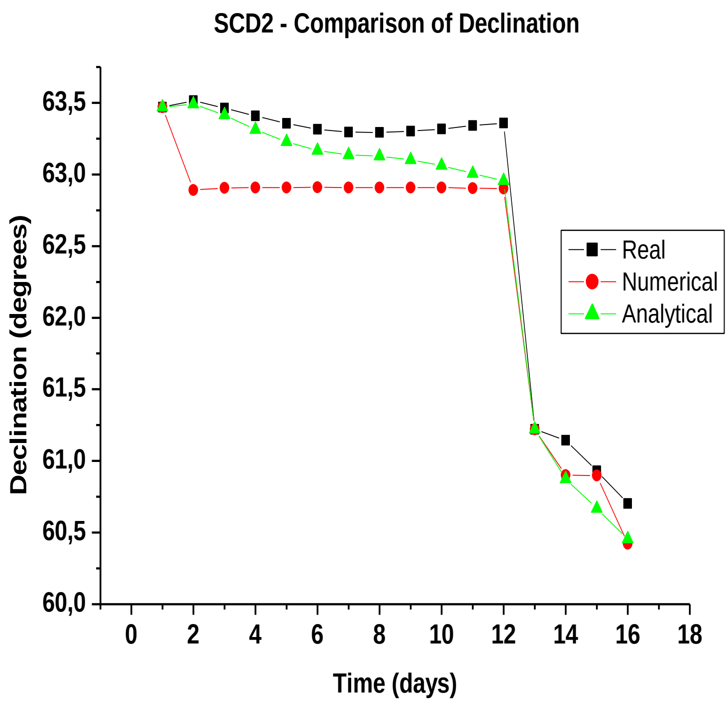

Figure 10 - Temporal variation for the actual, analytical and numerical declination of the spin axis for SCD2, with daily updated data.

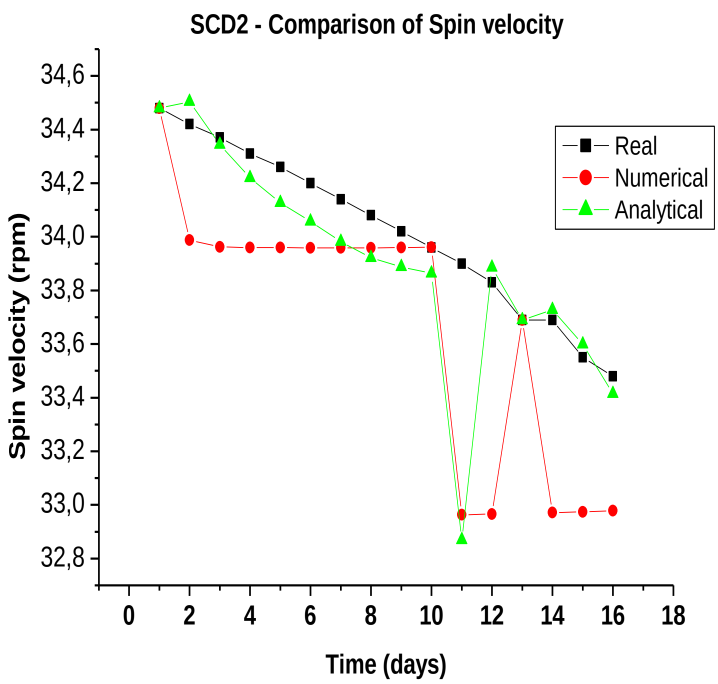

Figure 11 - Temporal variation for the actual, analytical and numerical spin velocity for SCD2, with daily updated data.

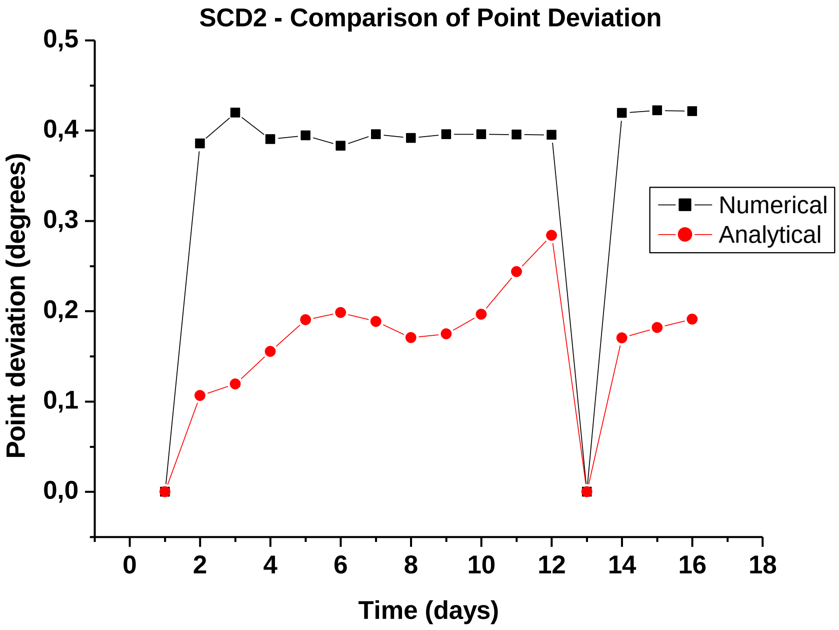

Figure 12 - Temporal variation for the pointing deviation for SCD2, with daily updated data.

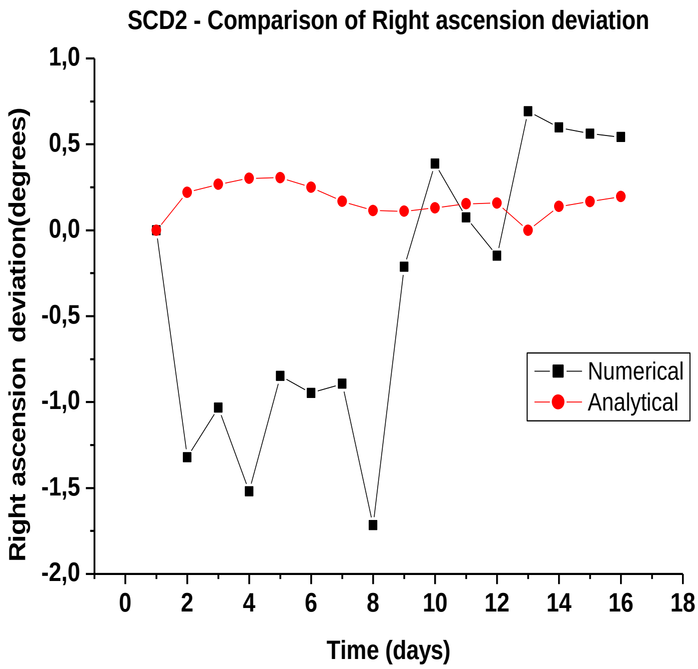

Figure 13 - Temporal variation for the deviation between the analytical or numerical and actual right ascension of the spin axis for SCD2, with daily updated data

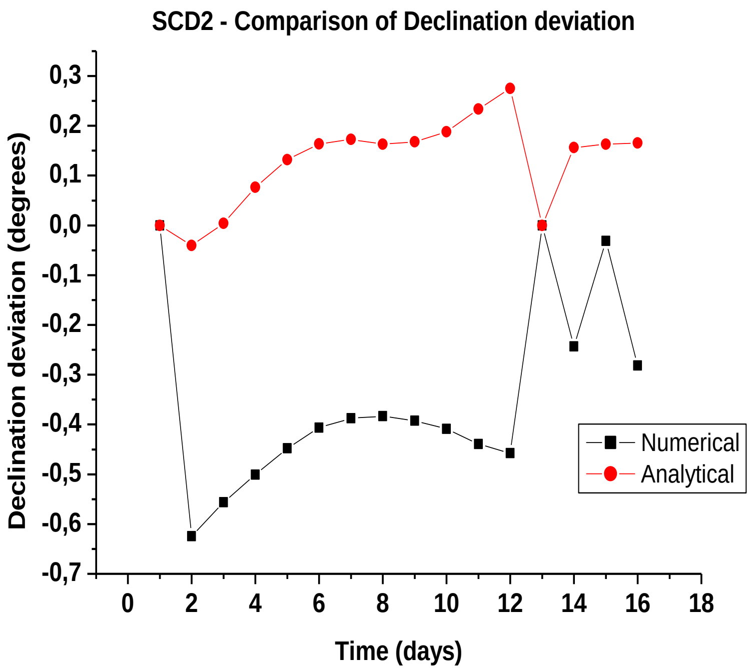

Figure 14 - Temporal variation for the deviation between the analytical or numerical and actual declination of the spin axis for SCD2, with daily updated data.

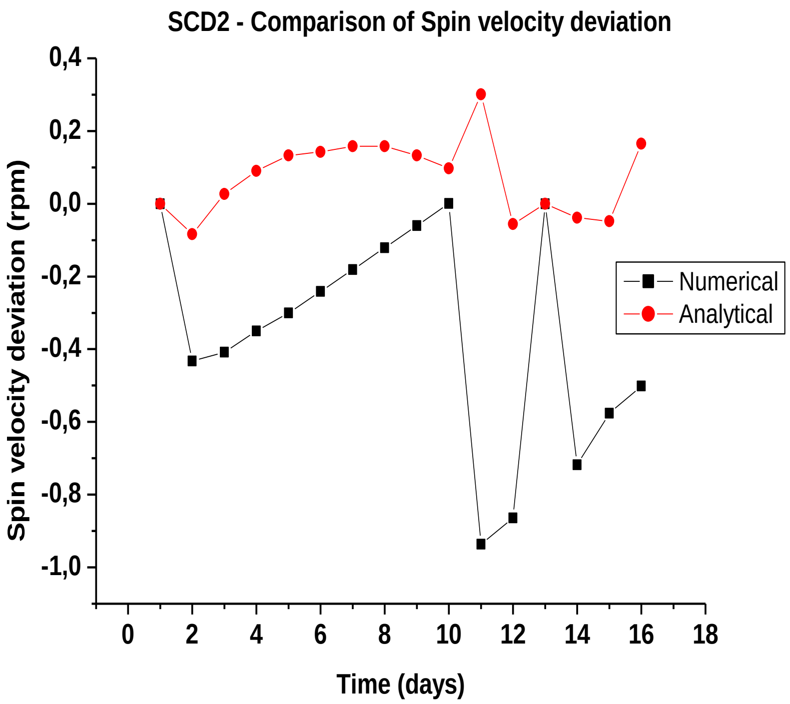

Figure 15 - Temporal variation for the deviation between the analytical or numerical and actual spin velocity for SCD2, with daily updated data.

Table 2 – Mean deviation errors for SCD2 simulations with daily updated data.

	Numerical	**Analytical**
Right Ascension Deviation (degrees)	-0.3616	0.1675
Declination Deviation (degrees)	-0.3475	0.1263
Spin Velocity Deviation (rpm)	-0.3556	0.0736
Pointing Deviation (degrees)	0.3506	0.1608

6.2. Second simulation: without daily updated data

Many tests were developed for SCD1 and SCD2 satellites when data are not daily update, using different intervals of time. The results show a good agreement between the computed values and the actual satellite behavior only for 1 day simulation. For more than 1 day, the mean deviation error for one or more parameters is higher than the precision required for SCC/ INPE. Just for example, in Tab 3 and Tab. 4 are shown the results in term of the difference between computed and actual right ascension, declination, spin velocity and pointing deviation in each approach for the SCD1 and SCD2, respectively. It is important to observe that in general the analytical solution is also better than the numerical solution.

Table 3 - Deviation between analytical or numerical values and actual values for SCD1, without daily updated data.
(subscript *na* – numerical approach and *aa* – analytical approach)

DATA	**08/25/1993**	**08/26/1993**	**08/27/1993**
$\alpha_{na} - \alpha_{inpe}$ $(^\circ)$	0	0.3873	0. 0768
$\alpha_{aa} - \alpha_{inpe}$ $(^\circ)$	0	0.1047	0.4949
$\delta_{na} - \delta_{inpe}$ $(^\circ)$	0	-0.7506	-1.5307
$\delta_{aa} - \delta_{inpe}$ $(^\circ)$	0	-0.2762	-0.6858
$W_{na} - W_{inpe}$ (rpm)	0	-0.7683	-1.6943
$W_{aa} - W_{inpe}$ (rpm)	0	0.5739	1.0735
θ_{na} $(^\circ)$	0	0,7547	1.5308
θ_{aa} $(^\circ)$	0	0.2769	0.6924

Table 4 - Deviation between analytical or numerical values and actual values for SCD2, without daily updated data.
(subscript *na* – numerical approach and *aa* – analytical approach)

DATA	02/24/2002	02/25/2002	02/26/2002
$\alpha_{na} - \alpha_{inpe}$ $(°)$	0	-0.7049	-1.4763
$\alpha_{aa} - \alpha_{inpe}$ $(°)$	0	-0.2096	-0.4257
$\delta_{na} - \delta_{inpe}$ $(°)$	0	-0.2431	-1.0333
$\delta_{aa} - \delta_{inpe}$ $(°)$	0	0.4712	1.0212
$W_{na} - W_{inpe}$ (rpm)	0	-0.7118	-1.5743
$W_{aa} - W_{inpe}$ (rpm)	0	0.1478	-0.3231
θ_{na} $(°)$	0	0.4191	1.2645
θ_{aa} $(°)$	0	0.4822	1.0427

7. Conclusions

In this paper numerical and analytical approaches were presented to the spin-stabilized satellite attitude propagation taking into account the residual, eddy current, aerodynamic, solar radiation and gravity gradient torques. The numerical approach is developed in terms of the quaternion parametrization and the analytical approach in terms of the spin velocity, right ascension and declination of the spin axis.

Two numerical simulations were presented to the spin stabilized Brazilian's satellites SCD1 and SCD2. In the first one, where the attitude and orbital data are daily updated with real attitude data supplied by INPE, the results show a good agreement between the computed and actual data during the simulated time interval. For both satellite, all the mean deviation are within the dispersion range of the attitude determination system used for these satellites.

In the second numerical simulation the attitude and orbital data are not daily updated. For SCD1 and SCD2, the obtained results showed a good agreement between the analytical or numerical solution and the actual satellite behavior only for one day simulation. For more than 1 day the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required for SCC/ INPE.

In general, the mean deviation errors of the analytical approach are smaller than the numerical approach. This can be explained by the fact that in the numerical approach there are many transformations of variables, which can cause small numerical errors.

The results show that the average components of the external torque are sufficient to propagate the spacecraft attitude. It is also important to observe that the time simulation for the analytical is faster than numerical approach.

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