COUPLED OPTIMIZATION OF LAUNCHER AND ALL-ELECTRIC SATELLITE TRAJECTORIES

M. Verlet[(1)], B. Slama[(1)], S. Reynaud[(1)], and M. Cerf[(1)]
[(1)] *Airbus Defence and Space, 66 Route de Verneuil, 78133 Les Mureaux, France*
Email: mathieu.verlet@astrium.eads.net
Email: brice.slama@astrium.eads.net
Email: stephane.reynaud@astrium.eads.net
Email: max.cerf@astrium.eads.net

Abstract: *In recent years, the maturity and performance of electric thruster have increased. In that context, as the main European launcher prime contractor, Airbus-DS is analyzing the impact of all-electric satellites on its launchers trajectories and optimal injection orbit. Airbus-DS has developed a fast, accurate and automatic trajectory optimization tool to minimize fuel consumption or transfer time duration for Electric Orbit Raising of an all-electric satellite using the Pontryagin Maximum Principle and an innovative way to prevent difficulty in initializing the problem. Airbus-DS also developed, starting with performance table generated by the previous tool, a fast and accurate optimization tool to solve the coupled problem of the launcher and all-electric satellite trajectories optimization minimizing the mass consumption for a given transfer duration or minimizing the transfer duration for a given satellite mass. Results of the tool for Ariane5-ES, the launcher currently untrusted in missions such as the Automated Transfer Vehicle and the Galileo Navigation system, will be presented along with the analysis of the impact on the launcher injection strategy.*

Keywords: *Low-thrust, Electric-Orbit-Raising, optimization, launcher, satellite.*

1. Introduction

The maturity and performance of electric thruster technology have recently increased. Moreover, electric thrusters have already been used for missions involving interplanetary transfer, as well as for orbit insertion manoeuvers of satellites in geocentric trajectories, proving their reliability and effectiveness for high total impulse missions. This technology, currently used for Station Keeping purposes, is already foreseen for raising the orbit of a few GEO satellites. Indeed, the advantage of this technology in terms of mass savings for orbit-raising with regards to chemical propulsion and then a more affordable launch is deemed to outweigh the loss due to a longer orbit-raising duration. As a result, satellite operators are now considering longer orbit-raising durations before operating their payload. The use of Electric Propulsion (EP) as the main satellite propulsion system will probably become the preferred technology for a larger number of satellites in the near future.

Electric propulsion could imply a launcher injection strategy for GEO satellites different than the classic GTO strategy used for chemical satellites. This new injection strategy may have an impact on the launcher design and qualification domain. In this context, as prime of the ARIANE launchers family, Airbus-DS has developed its own electric orbit-raising (EOR) optimizer and a global launcher and satellite trajectories optimization process in order to study the impact of all-electric satellites on its launcher family and to ensure its competitiveness for the electric satellites market.

The problem of finding optimal orbit transfers using low-thrust propulsion has been investigated in great detail by Airbus-DS. Two main strategies are considered: minimum-fuel or minimum-time EOR transfer. An indirect method using the Pontryaguin Maximum Principle (PMP), which results from the calculus of variation, was used to solve the problem and will be presented in this article. Even if an indirect method is harder to solve than a direct one, it has the great advantage of being faster and more accurate. This solver has been developed to solve the transfer between every foreseen injection orbit of an ARIANE launcher toward a GEO. EOR results will be presented for several thrust-to-weight ratios and transfer durations.

It has been chosen not to optimize globally the coupled launcher-satellite trajectory since the optimization duration will be too important. Therefore, very complete EOR tables were built with the optimal control optimizer either for minimum duration or for minimum consumption for several transfer durations and for several satellite configurations. These tables have been included in the launcher optimizer process. The process and the results of the optimization applied on the ARIANE5-ES launcher will be presented along with the main hypotheses, satellite assumptions and transfer duration constraints.

First, the paper focuses on the EOR optimizer, both theoretical principles and application cases. Then, the coupled optimizer is described and finally an application case on ARIANE5-ES is given.

2. EOR optimization using indirect method of optimal control command

First, we state the optimal control problem to solve, along with the necessary conditions given by the PMP. In a first stage impact, it has been decided in advanced project not to take into account slew rate limitation, third body perturbation or eclipses. In order to be more realistic, those ignored perturbations have to be taken into account later to build an operational trajectory.

2.1. Indirect method of optimal control command

Consider the problem of transferring a spacecraft from an initial low-Earth orbit (LEO), medium-Earth orbit (MEO) or geostationary transfer orbit (GTO) to final geostationary orbit (GEO) using low-thrust propulsion. The objective is to determine the minimum fuel or the minimum time trajectory that transfers a spacecraft from an initial orbit with an inclination to a final GEO with null eccentricity and inclination. In the remainder of this section, we describe the low-thrust optimal control problem for the aforementioned LEO to GEO transfer. First, the dynamics of the spacecraft, modelled as a point mass, are described using Cartesian coordinates together with a second-order oblate gravity model and a propulsion system that has a high specific impulse and a small thrust-to-mass ratio of O (10^{-4}). Second, we describe the boundary conditions for the orbit transfer in terms of both classical orbital elements and Cartesian elements. Finally, we describe the optimal control problem. It is assumed here that there is no chemical propulsion at all on the full electric satellite.

The state of the spacecraft is comprised of the Cartesian coordinates (x, y, z, v_x, v_y,v_z, m), where m is the mass of the spacecraft. The control is the thrust direction, u(t), where u is also expressed in Cartesian coordinate's u = (u_x, u_y, u_z). The differential equations of motion of the spacecraft are given as:

$$\dot{x} = f\left[x(t), u(t), t\right]; \quad x(t_0) \text{ given, } t_0 \leq t \leq t_f \tag{1}$$

Where x(t) is the state vector define above. f is defined in Cartesian coordinates as follow:

$$\vec{f} = \begin{pmatrix} \vec{v} \\ \frac{T}{m}\vec{u} + \vec{g} \\ -\frac{T}{Isv \times g_0} \end{pmatrix} \quad 0 \leq T \leq T_{max} \tag{2}$$

Where v is the speed of the spacecraft (v_x, v_y, v_z), T is the thrust (in N), g_0 is the Earth's gravitational acceleration at sea level (in m/s2), I_{sv} is the specific impulse of the thruster in vacuum (in s), and g represents the gravity vector. At that point the EOR tool takes into account perturbations due to Earth and also the second-order oblate gravity model

Consider then a performance index J.

The two problems of interest are, the minimum time transfer with:

$$J = \int_{t_0}^{t_f} \mathrm{dt} = \mathrm{t_f} - \mathrm{t_0}, \qquad \mathrm{L(x(t), u(t), t) = 1} \tag{3}$$

And the minimum consumption problem with:

$$J = \int_{t_0}^{t_f} \mathrm{q(t)dt} = \int_{t_0}^{t_f} \frac{\mathrm{T}}{\mathrm{Isv \times g0}} \mathrm{dt}, \qquad \mathrm{L(x(t), u(t), t)} = \frac{\mathrm{T}}{Isv \times g0} \tag{4}$$

The problem is to find the function u(t) that minimizes J.
Then, the multiplier function λ(t) is adjoined and defined by the following equation:

$$\dot{\lambda} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda \tag{5}$$

Starting from this point, only the minimum consumption problem will be considered. Equation 2 and Eq. 4 give the following Hamiltonian:

$$H = \frac{T}{\mathrm{Isv} \times \mathrm{g0}} + \vec{\lambda}_r^T \cdot \vec{v} + \vec{\lambda}_v \cdot \left(\frac{T}{m} \times \vec{u} + \vec{g} \right) + \vec{\lambda}_m \cdot \left(-\frac{T}{Isv \times g0} \right) \tag{6}$$

The initial conditions requested to bound the problem are given by the injection orbit and the final conditions are the following:

$$\begin{cases} perigee = fixed \\ apogee = fixed \\ inclination = fixed \end{cases} \qquad \begin{cases} \lambda_\omega = 0 \\ \lambda_\Omega = 0 \\ \lambda_\theta = 0 \\ \lambda_m = 0 \end{cases} \tag{7}$$

Where ω is the argument of periapsis, Ω is the longitude of the ascending node, m is the mass of the system and θ is the true anomaly. The final longitude stays free because it can be easily changed by tuning the launch date or slightly adjusting the transfer duration.

In order to solve the optimal control problem, the Hamiltonian has to be minimized at each point of the trajectory which means that u(t) is described by:

$$\vec{u} = -\frac{\vec{\lambda}_v}{\left\| \vec{\lambda}_v \right\|} \tag{8}$$

As the problem is stated, it gives the best solution but still remain the initialization issue of the multiplier functions λ. It is then not possible to directly address the problem.

2.2. Resolution process to overcome initialization issue

The main issue using indirect optimal control law is to correctly initialize the multiplier functions λ. In order to overcome this issue and automatically solve all type of initial and final orbit, a specific process was implemented. The idea is to start from an easy problem and to go through intermediate step to reach the targeted problem. The main issue about minimum consumption is the detection of on/off commutation of thrusters. To solve this issue, the problem is first solved in minimum energy and then converted to a minimum consumption problem. It allows smoothening the control law and easing the numerical solution. Switching to Minimum Energy also changes a bit the problem formulation that becomes:

$$J=\int_{t_0}^{t_f} \mathrm{q(t)dt} = \int_{t_0}^{t_f} \left(\frac{\mathrm{T}}{\mathrm{Isv}\times \mathrm{g0}}\right)^2 \mathrm{dt}\ , \qquad \mathrm{L(x(t),u(t),t)} = \left(\frac{\mathrm{T}}{Isv\times g0}\right)^2 \tag{9}$$

The process is segmented in three steps. At each step of the process, the optimal control problem is slightly modified in order to correspond to the current step. Each step has its own Hamiltonian formulation and its own commutation function formulation resulting from the modification of the problem. The first step is to solve the problem in minimum energy with high trust. The second step initialized by the end of the first is to solve the problem in minimum energy with low thrust and the last step initialized by the end of the second is to solve the problem in minimum consumption with low thrust.

The great advantage of being in a minimum energy problem is to have a continuous thrust law, allowing adjusting it very easily when getting from one problem to another whether the minimum consumption problem is discontinuous and require to be correctly initialized. The hard point of the process is the switch between minimum energy and minimum consumption problems.

The objective of having a tool usable by a non-expert on the subject was achieved with acceptable computation time (typically less than 1 hour).

3. EOR tool application

In this part, it will be presented application of the EOR tool on the 3 following test cases:

- A transfer from a classic GTO for a launch from Kourou to GEO.
- A transfer from a medium orbit with medium inclination to GEO.
- A transfer from a super synchronous transfer orbit with medium inclination to GEO.

For these cases, assumptions are Hall-Effect Thrusters (HET) with a thrust of 0.58 N and a specific impulse of 1800s. For the two first cases, satellite has an initial mass of 2000 kg and for the third case, it has an initial mass of 4600 kg.

3.1. Description of the application cases

Today, European launchers use GTO orbit to inject commercial chemical satellites toward GEO. The interest of this case is to have a direct comparison of performance between an all-electric satellite and a chemical one. The transfer duration is fixed to 180 days in order to get a great difference between minimum duration and the fixed duration transfers and then have the most impact on the trajectory characteristics..

The second case shows a transfer from a lower initial eccentricity orbit to study the impact of extra time on the couple low eccentricity / medium inclination. The transfer duration is fixed to 180 days in order to get a great difference between minimum duration and the fixed duration as in the first case.

The third case shows the result of the transfer from a super-synchronous orbit toward GEO. Indeed, in order to correct a medium or high inclination, it could be more interesting to have a higher apogee at injection. The transfer duration is here fixed to 160 days in order to get this time a very small difference between minimum duration and the fixed duration.

3.2. Performance obtained on described cases

The following Tab. 1 resumes performance for all three cases described in §3.1.

Table 1: Performance obtained with EOR tool

	Case 1	Case 2	Case 3
Injection apogee (km)	35786	10000	65000
Injection perigee (km)	250	5000	11497
Injection inclination (degree)	5	13	13.8
Initial mass (kg)	2000	2000	4600
Thrust (N)	0.58	0.58	0.58
I_{SV} (s)	1800	1800	1800
Minimum duration (days)	82	99	132
Consumption (kg)	234	282	376
Fixed duration (days)	180	180	160
Minimum Consumption (kg)	171	263	329
Consumption gain (%)	27.0	7.0	12.5

Consumption gain is less important on case 2 because the initial eccentricity is low. In fact, with no initial eccentricity and a low inclination orbit, there is no difference between the problem that minimizes duration and the one that minimizes consumption because the optimum for consumption is to activate thrusters during all the transfer.

3.5. Trajectory display on GTO toward GEO case

For the first case, display of evolution of apogee, perigee, inclination, mass consumption and thrust versus time are given. Figure 2 shows the evolution of perigee and apogee (in km), propellant mass consumption (kg), inclination (degree) and Thrust value (N) function of time (days)

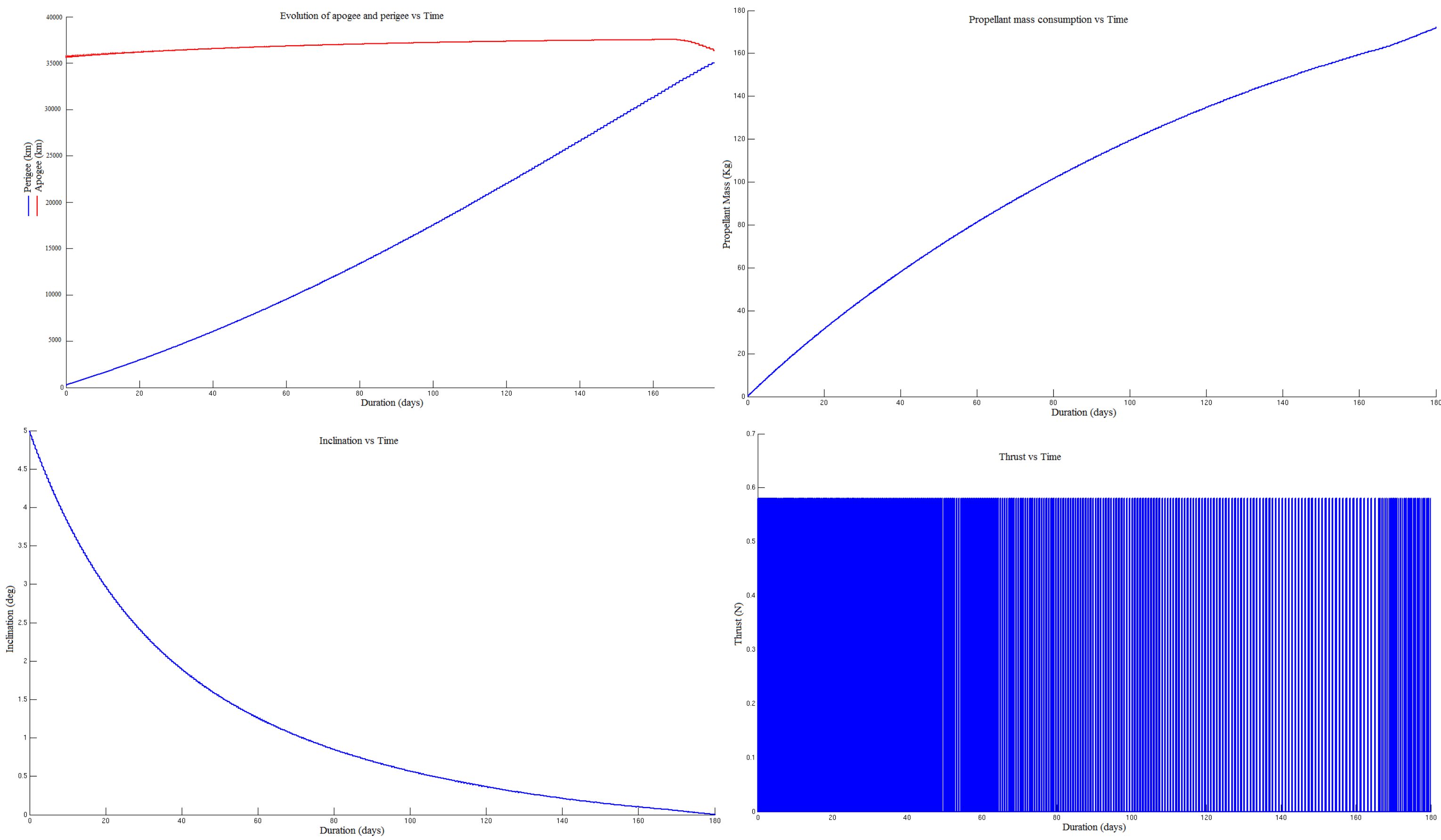

Figure 2: evolution of apogee/perigee (top left), inclination (bottom left), propellant mass consumption (top right) and thrust (bottom right) function of time (days)

At some time of the transfer, there is one arc of thrust at perigee and one at apogee per orbit (mainly at the end) and at other times, there is only one arc of thrust per orbit at apogee to simultaneously correct perigee and inclination.

In order to better understand what happens along the transfer trajectory, Fig. 3 is provided representing the thrust power all along transfer. Thrusting periods are represented by red parts of the trajectory while ballistic parts are represented in blue.

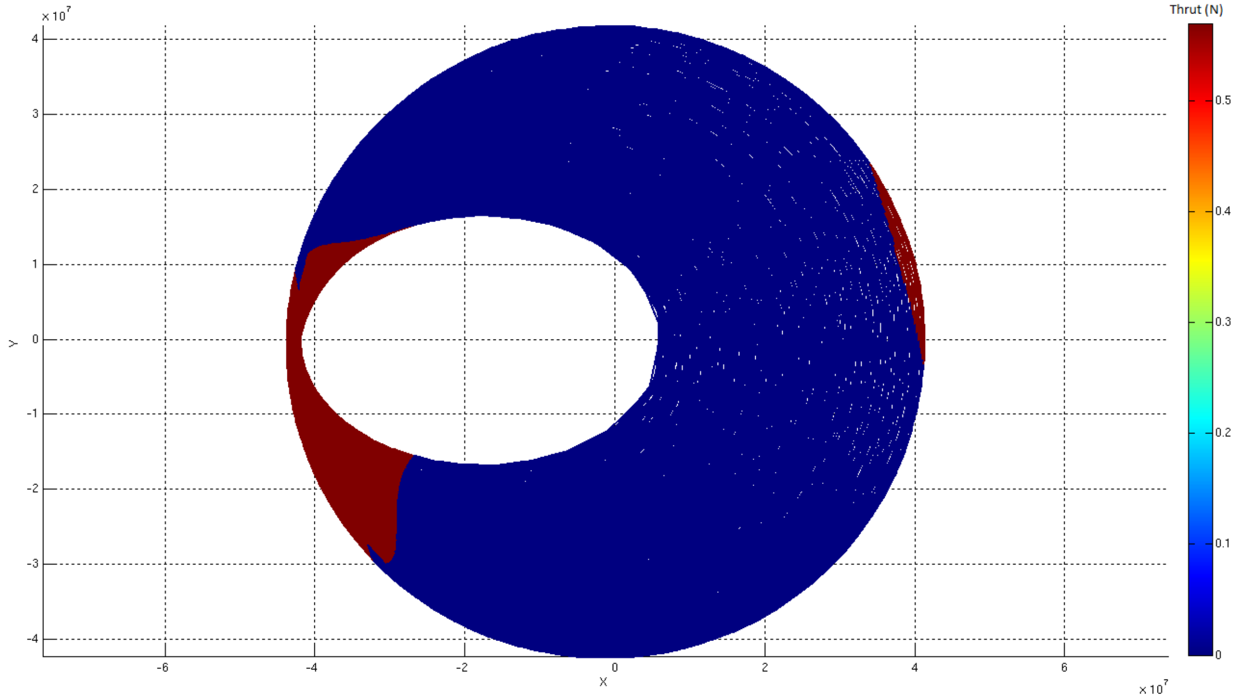

Figure 3: Thrust activation (red) and ballistic phase (blue) along the transfer trajectory between GTO toward GEO represented as a projection on the equatorial frame.

As a matter of fact, very long durations of ballistic arcs are present on each period due to the great difference between the minimal time require for this transfer and the available transfer duration.

4. Coupled electric satellite and launcher trajectory optimization using EOR tables

This part describes how EOR performances are used to optimize the coupled transfer trajectory of one launcher and one or several electric satellites. The objective of the global optimization tool is to define the optimal injection orbit of a given launcher for a given electric satellite. Two different type of mission are studied: first, to maximize the satellite mass at Beginning of Life (BOL) in GEO with transfer duration fixed by the satellite operator (this case is equivalent to minimize the propellant consumption during the transfer) or second, to minimize the transfer duration for a given satellites.

The launcher phase is optimized "classically" using control laws as described in [3] and [4]. In order to process end-to-end simulations and global trajectory and vehicle optimizations, the flight is divided into legs and sequences. Specific models are selected for each flight sequence and leg. The electric transfer phase uses EOR performance tables generated by above EOR tool. The launcher injects the satellite on a transfer orbit and the consumption for the EOR transfer is then linearly interpolated in a performance table. The possibility to extrapolate has not been given to the tool because the risk of obtaining wrong performance is too high. Two types of performance tables were created:

- Minimum consumption tables: those tables take as input the orbital parameters, the initial satellite mass and the transfer duration. The BOL mass in GEO is then the only output of this type of table.
- Minimum transfer time tables: those tables take only as input the orbital parameters and the initial mass. Two outputs are given by those tables: the transfer time to reach GEO orbit and the corresponding BOL mass in GEO.

With those tables, the coupled optimization has about the same complexity level as a usual launcher optimization problem. It enables to optimize very accurately and very fast the launcher trajectory and then to analyze the impact on the launcher's market of the coming of all-electric satellites.

An example is presented in next part, the application of that optimization process of one of Airbus-DS in house launcher: Ariane5-ES.

5. Application on ARIANE5-ES

For minimum transfer duration, the goal is to minimize the transfer duration when launching in dual launch two satellites, one of 3.5 tons and one of 2 tons. For minimum consumption, the idea is to maximize the Beginning of Life (BOL) satellite mass while fixing the transfer duration (90 days and 180 days).

5.1. Hypothesis concerning electric thrusters

For this study, two different thrusters were considered:

- Arcjet: 900 s of I_{sv} and a Thrust of 1.5 N at 10 kW.
- HET: Hall Effect Thruster: 1800 s of I_{sv} and a Thrust of 0.58 N at 10 kW.

5.2. Hypothesis regarding the launcher

Ariane5-ES has the possibility to bring two satellites in orbit at the same time with a structure separating the two satellites. This structure called "Sylda" weights around 700 kg and is subtracted from the total injected mass. This configuration of the launcher will be the one considered for this study.

5.3. Performance obtained

As explained above, three different performances are assessed for each thruster. The following Tab. 2 presents those performances.

Table 2: Performance obtained with Global optimization tool

Mission	Thruster I_{sv} (s)	Launcher injection orbit (perigee, Apogee, Inclination)	Performance GEO BOL
90 days transfer	900	1700 km * 15000 km*5°	2* 3.4 t
	1800	11000 km * 15000 km, 5°	2* 2.4 t
180 days transfer	900	300 km * 8500 km, 4.5 °	2* 4.5 t
	1800	7300 km * 14050 km* 5°	2*2.7 t
Minimum transfer duration (3.5 tons and 2 tons in GEO)	900	2800km * 25000 km * 4.9°	64 days (3.5 t) and 38 days (2t)
	1800	6600 km – 20000 km * 5°	139 days (3.5 t) and 80 days (2 t)

Those orbits are not the classical one used for Ariane5-ES. Therefore, the launcher will have to be qualified on those new orbits if they are retained for future electric satellite launch. In order to reduce new development costs, sensitivity has been done to an injection orbit near to GTO. The result is presented in the following Tab. 3.

Table 3: Performance obtained with apogee fixed at 35786 km

Mission	Thruster I_{sv} (s)	Launcher injection orbit (perigee, Apogee, Inclination)	Performance GEO BOL
Minimum transfer duration (3.5 tons and 2 tons in GEO)	900	590 km * 35786 km, 5°	66 days (3.5 t) and 39 days (2t)

The loss between this injection strategy and the one without fixing apogee at 35786 km is really low (3 %). Keeping a strategy with a near classic GTO injection could be probably preferred but it will have to be confirmed taking into account operational constraints.

6. Conclusion

The problem of minimum fuel consumption during a transfer between a launcher injection orbit and an operational orbit has been considered for all electric satellite. The problem of minimum duration has also been considered for comparison purpose. The tool developed by Airbus-DS, based on indirect optimal control problem, can be used by a non-expert on the subject with acceptable computation time (typically less than 1 hour).

The problem of finding the optimal injection orbit of Ariane5-ES to launch an all-electric satellite has also been explained. The results of this paper regarding Ariane5-ES may lead to consider new injection orbits for that launcher. The optimal injection orbit is in most of the cases an intermediate nearly circular one.

As a perspective of evolution, similar studies shall have to be performed with all operational constraints not taken into account in this paper such as eclipses, solar panel illumination, communication constraints or slew rates limitations.

7. References

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