A SYSTEMATIC STUDY ABOUT SUN PERTURBATIONS ON LOW ENERGY EARTH-TO-MOON TRANSFERS

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Abstract: *In this work, a systematic study about the perturbation of the Sun on the problem of transferring a space vehicle from a circular low Earth orbit (LEO) to a circular low Moon orbit (LMO) with minimum fuel consumption is presented. The optimization criterion is the total characteristic velocity. The optimization problem has been formulated using the classic planar circular restricted three-body problem (PCR3BP) and the planar bi-circular restricted four-body problem (PBR4BP), and, it has been solved using a gradient algorithm in conjunction with Newton-Raphson method. Numerical results are presented for several final altitudes of a clockwise or counterclockwise circular low Moon orbit and for a specified altitude of a counterclockwise circular low Earth orbit, taking the initial position of Sun as a parameter.*

Keywords: *Bi-circular problem, low energy transfers, optimal trajectories, three-body problem*

1. Introduction

In the last decades, new types of Earth-to-Moon transfers have been designed and used in lunar missions which could not be possible using traditional approaches [1, 2]. New methods make use of the nonlinear dynamics of the circular restricted three-body problem and of the bi-circular restricted four-body problem to design low energy transfers.

Low energy Earth-to-Moon transfers can be classified into exterior or interior, according to the geometry [3]. In the exterior transfers the spacecraft is injected into an orbit with large apogee which crosses the Moon orbit. The apogee distance is approximately four times the Earth-Moon distance. This kind of trajectories exploits the Sun’s gravitational attraction [4, 5]. In the interior transfers most part of the trajectory occurs within the Moon orbit.

In this work, a systematic study about the perturbation of the Sun on the problem of transferring a space vehicle from a circular low Earth orbit (LEO) to a circular low Moon orbit (LMO) with minimum fuel consumption is presented considering only interior transfers. It is assumed that the velocity changes are instantaneous, that is, the propulsion system is capable of delivering impulses. Two-impulse trajectories are considered in the analysis: a first accelerating velocity impulse (Δv_{LEO}) tangential to the space vehicle velocity relative to Earth is applied at a circular low Earth orbit, and, a second braking velocity impulse (Δv_{LMO}) tangential to the space vehicle velocity relative to Moon is applied at a circular low Moon orbit [6]. The minimization of fuel consumption is equivalent to the minimization of the total characteristic velocity which is defined by the arithmetic sum of velocity changes [7], that is, $\Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$.

In order to study the perturbation effects of Sun in such maneuvers, two formulations are used: the optimization problem has been formulated using the classic planar circular restricted three-body problem (PCR3BP) and the planar bi-circular restricted four-body problem (PBR4BP). Numerical results are obtained for several final altitudes of a clockwise or counterclockwise circular low Moon orbit and for a specified altitude of a counterclockwise circular low Earth orbit. Direct ascent trajectories, with time of flight of approximately 4.5 days, and multiple revolution trajectories with moderate time of flight (lesser than 60 days), are considered in this study. The initial position of the Sun θ_{S0} is taken as a parameter and the major parameters of the optimal trajectories – first delta-v, second delta-v, time of flight and initial position of the spacecraft – are calculated as function of θ_{S0}.

The main results show that Sun perturbation effects are significant for trajectories with multiple revolutions: the fuel consumption and the second delta-v can vary significantly according the initial position of the Sun. Swing-by maneuvers with the Moon are made for the trajectories with multiple revolutions and fuel can be saved, for the both dynamical models.

2. Formulation of the optimization problem

The eccentricity of the Earth's orbit is about 0.0167 and the mean value of eccentricity of the Moon's orbit is about 0.0549 (small periodic changes in eccentricity occur at intervals of 31.8 days). The inclination of the Moon's orbit to ecliptic varies between $4°59'$ and $5°18'$, with mean value of $5°8'$. In view of these small values for eccentricities and inclination, the planar circular restricted three-body problem and the planar bi-circular restricted four-body problem dynamics are used in the analysis.

2.1 Bi-circular restricted four-body problem model

The optimization problem based on the planar bi-circular restricted four-body problem – PBR4BP – is formulated as described below. The following assumptions are employed:

1. Earth and Moon move in circular orbits around the center of mass of the Earth-Moon system;

2. The center of mass of Earth-Moon system moves in circular orbit around the center of mass of the Sun-Earth-Moon system;

3. The flight of the space vehicle takes place in the Moon orbital plane;

4. The space vehicle is subject only to the gravitational fields of Earth, Moon and Sun;

5. The gravitational fields of Earth, Moon and Sun are central and obey the inverse square law;

6. The class of two-impulse trajectories is considered. The impulses are applied tangentially to the space vehicle velocity relative to Earth (first impulse) and Moon (second impulse).

Consider a moving reference frame *Bxy* contained in the Moon orbital plane: its origin is the center of mass of Earth-Moon system; the *x*-axis points towards the Moon position at the initial time $t_0 = 0$ and the *y*-axis is perpendicular to the *x*-axis. See Fig.1.

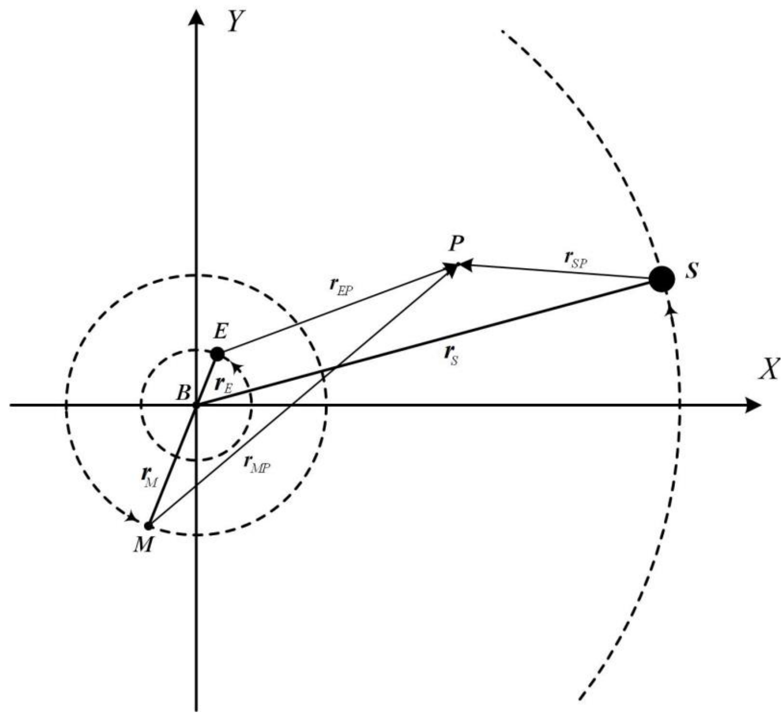


Figure 1. The reference frame *Bxy*

In *Bxy* reference frame, the motion of the space vehicle is described by the following set of differential equations:

$$\frac{dx_P}{dt} = u_P$$

$$\frac{dy_P}{dt} = v_P$$

$$\frac{du_P}{dt} = -\frac{\mu_S}{r_{SP}^3}(x_P - x_S) - \frac{\mu_E}{r_{EP}^3}(x_P - x_E) - \frac{\mu_M}{r_{MP}^3}(x_P - x_M) - \frac{\mu_S}{r_S^2}\cos(\omega_S t + \theta_{S0})$$

$$\frac{dv_P}{dt} = -\frac{\mu_S}{r_{SP}^3}(y_P - y_S) - \frac{\mu_E}{r_{EP}^3}(y_P - y_E) - \frac{\mu_M}{r_{MP}^3}(y_P - y_M) - \frac{\mu_S}{r_S^2}\sin(\omega_S t + \theta_{S0}), \qquad (1)$$

where μ_E, μ_M and μ_S are the gravitational parameters of Earth, Moon and Sun, respectively; r_{EP}, r_{MP} and r_{SP} are the distances of the space vehicle from Earth (*E*), Moon (*M*) and Sun, respectively; that is,

$$r_{EP}^2 = (x_P - x_E)^2 + (y_P - y_E)^2$$

$$r_{MP}^2 = (x_P - x_M)^2 + (y_P - y_M)^2$$

$$r_{SP}^2 = (x_P - x_S)^2 + (y_P - y_S)^2 . \tag{2}$$

The distance from *B* to Earth, Moon and Sun are denoted by r_E, r_M and r_S, respectively. So, the position vectors of Earth, Moon and Sun are defined in the reference frame B*xy* by the equations

$$x_E(t) = -r_E \cos\theta_M(t) \quad y_E(t) = -r_E \sin\theta_M(t), \tag{3}$$

$$x_M(t) = r_M \cos\theta_M(t) \quad y_M(t) = r_M \sin\theta_M(t), \tag{4}$$

$$x_S(t) = r_S \cos\theta_S(t) \qquad y_S(t) = r_S \sin\theta_M(t), \tag{5}$$

where:

$$\theta_M(t) = \theta_0 + \sqrt{(\mu_E + \mu_M)/D^3}\, t , \tag{6}$$

$$\theta_S(t) = \theta_{S0} + \omega_S\, t , \tag{7}$$

$\omega_S = \sqrt{(\mu_S + \mu_E + \mu_M)/r_S^3}$, $\theta_0 = 0$ is the initial phase of the Moon, θ_{S0} is the initial phase of the Sun, $r_E = \mu D/(1+\mu)$ and $r_M = D/(1+\mu)$, $\mu = \mu_M / \mu_E$ and *D* is the distance from the Earth to the Moon.

The initial conditions of the system of differential equations (1) correspond to the position and velocity vectors of the space vehicle after the application of the first impulse. The initial conditions $(t_0 = 0)$ can be written as follows

$$x_P(0) = x_E(0) + r_{EP}(0)\cos\theta_{EP}(0)$$

$$y_P(0) = y_E(0) + r_{EP}(0)\sin\theta_{EP}(0)$$

$$u_P(0) = -\left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_{LEO}\right]\sin\theta_{EP}(0) + \dot{x}_E(0)$$

$$v_P(0) = \left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_{LEO}\right]\cos\theta_{EP}(0) + \dot{y}_E(0), \tag{8}$$

where Δv_{LEO} is the velocity change at the first impulse, $r_{EP}(0) = h_0 + a_E$ and $\theta_{EP}(t)$ is the angle which the position vector $\boldsymbol{r}_{EP}$ forms with x-axis. h_0 is the altitude of LEO and a_E is the Earth radius. It should be noted that $\boldsymbol{r}_{EP}(0)$ and $\boldsymbol{v}_{EP}(0)$ are orthogonal (impulse is applied tangentially to LEO). From Eqns. (3), (4) and (6), one finds

$$x_E(0) = -\mu \frac{D}{1+\mu} \qquad y_E(0) = 0$$

$$\dot{x}_E(0) = 0 \qquad \dot{y}_E(0) = -\mu \frac{D}{1+\mu} \dot{\theta}_M \,. \tag{9}$$

The final conditions of the system of differential equations (1) correspond to the position and velocity vectors of the space vehicle before the application of the second impulse. The final conditions $(t_f = T)$ can be put in the following form:

$$(x_P(T) - x_M(T))^2 + (y_P(T) - y_M(T))^2 = (r_{MP}(T))^2 \,, \tag{10}$$

$$(u_P(T) - \dot{x}_M(T))^2 + (v_P(T) - \dot{y}_M(T))^2 = \left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_{LMO} \right]^2 , \tag{11}$$

$$\begin{aligned} &(x_P(T) - x_M(T))(v_P(T) - \dot{y}_M(T)) - (y_P(T) - y_M(T))(u_P(T) - \dot{x}_M(T)) \\ &= \mp r_{MP}(T) \left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_{LMO} \right]. \end{aligned} \tag{12}$$

where Δv_{LMO} is the velocity change at the second impulse, $r_{MP}(T) = a_M + h_f$, h_f is the altitude of LMO and a_M is the Moon radius. The upper sign refers to clockwise arrival to LMO and the lower sign refers to counterclockwise arrival to LMO. From Eqns. (4) and (6), one finds

$$x_M(T) = \frac{D}{1+\mu} \cos\theta_M(T) \qquad y_M(T) = \frac{D}{1+\mu} \sin\theta_M(T)$$

$$\dot{x}_M(T) = -\frac{D\dot{\theta}_M}{1+\mu} \sin\theta_M(T) \qquad \dot{y}_M(T) = \frac{D\dot{\theta}_M}{1+\mu} \cos\theta_M(T) . \tag{13}$$

The problem defined by Eqns. (1) – (13) involves five unknowns Δv_{LEO} , Δv_{LMO}, T , $\theta_{EP}(0)$ and θ_{S0} that must be determined in order to satisfy the three final conditions – Eqns. (10), (11) and (12). This problem has two degrees of freedom, so an optimization problem can be formulated as follows: Determine Δv_{LEO} , Δv_{LMO} , T , $\theta_{EP}(0)$ and θ_{S0} which satisfy the final constraints (10) – (12), and, minimize the total characteristic velocity $\Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$.

This problem has been solved in [8] by using an algorithm based on gradient method [9] in conjunction with Newton-Raphson method [10].

2.1 Circular restricted three-body problem model

In this section, the optimization problem based on the planar circular restricted three-body problem – PCR3BP – is formulated as described below. The following assumptions are employed:

1. Earth and Moon move around the center of mass of the Earth-Moon system;
2. The eccentricity of the Moon orbit around Earth is neglected;
3. The flight of the space vehicle takes place in the Moon orbital plane;
4. The space vehicle is subject only to the gravitational fields of Earth and Moon;
5. The gravitational fields of Earth and Moon are central and obey the inverse square law;
6. The class of two impulse trajectories is considered. The impulses are applied tangentially to the space vehicle velocity relative to Earth (first impulse) and Moon (second impulse).

These assumptions are very similar to those stated in the preceding section. Now, the reference frame *Bxy* is inertial. By taking out the terms related do Sun in Eq. (1), one finds that the motion of the space vehicle is described by the following set of differential equations:

$$\frac{dx_P}{dt} = u_P$$

$$\frac{dy_P}{dt} = v_P$$

$$\frac{du_P}{dt} = -\frac{\mu_E}{r_{EP}^3}\left(x_P - x_E\right) - \frac{\mu_M}{r_{MP}^3}\left(x_P - x_M\right)$$

$$\frac{dv_P}{dt} = -\frac{\mu_E}{r_{EP}^3}\left(y_P - y_E\right) - \frac{\mu_M}{r_{MP}^3}\left(y_P - y_M\right). \tag{14}$$

The initial conditions of the system of differential equations (14) are given by Eqns. (8) and (9), and, the final conditions are given by Eqns. (10) – (13).

The boundary value problem is now defined by Eq. (14) in the place of Eq. (1) and it involves four unknowns Δv_{LEO} , Δv_{LMO} , T and $\theta_{EP}(0)$ that must be determined in order to satisfy the three final conditions – Eqns. (10), (11) and (12). Since this problem has one degree of freedom, an optimization problem can be formulated as follows: Determine Δv_{LEO} , Δv_{LMO} , T

and $\theta_{EP}(0)$ which satisfy the final constraints (6) – (8) and minimize the total characteristic velocity $\Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$. This optimization problem has been solved using the same algorithm described in the preceding section.

3. Results

The analysis of the perturbations of the Sun on lunar missions is split into three parts: in the first one, the initial position of the Sun is taken as a parameter and the optimal two-impulse trajectories are computed by solving the optimization problem with one degree of freedom defined by the initial position of the spacecraft, as described in the preceding sections. In the second part, the optimization problem with two degrees of freedom – initial position of the spacecraft and initial position of the Sun – is solved and the best optimal two-impulse trajectory is determined. In the third part, a briefly discussion about the use of a sub-optimal trajectory is presented. The "sub-optimal trajectory" term is used for optimal two-impulse trajectory calculated to an initial position of the Sun different from the position corresponding to the best optimal trajectory.

The following data are used in the analysis:

$\mu_S = 1.327\times10^{11}\ \text{km}^3/\text{s}^2$

$\mu_E = 3.986\times10^{5}\ \text{km}^3/\text{s}^2$

$\mu_M = 4.903\times10^{3}\ \text{km}^3/\text{s}^2$

$r_S = 1.496\times10^{8}$ km

$r_E = 4.678\times10^{3}$ km

$r_M = 3.803\times10^{5}$ km

$D = 384400$ km (distance from the Earth to the Moon),

$a_E = 6378$ km (Earth radius)

$a_M = 1738$ km (Moon radius)

$h_0 = 167$ km

$h_f = 100, 200, 300$ km.

3.1 Influence of the initial position of the Sun on local optimal trajectories

Figures 2 – 5 show the behavior of major parameters – $\Delta v_{Total}, \Delta v_{LEO}$, Δv_{LMO}, T and $\theta_{EP}(0)$ – as function of the initial position of the Sun θ_{S0} for optimal two-impulse trajectories. Lunar missions with direct ascent trajectories and time of flight of approximately 4.5 days, and, lunar missions with multiple revolutions maneuver and time of flight of approximately 58.5 days are considered for clockwise arrival and counterclockwise arrival, and, $h_f = 100$ km.

From the results presented in Figs 2 - 5, the major comments are:

1. For direct ascent trajectories and maneuvers with multiple revolutions, with clockwise or counterclockwise arrivals, there exist two initial positions of the Sun, separated by approximately $180°$, which give the best optimal two-impulse trajectories, taking into account the pre-specified accuracy used in the calculation of the local optimal

solutions. On the other hand, also there exist two initial positions of the Sun, separated by approximately $180°$, which give the worst local optimal two-impulse trajectories.

2. Perturbation effects of the Sun are too small for direct ascent trajectories. The difference in Δv_{Total} between the best two-impulse optimal trajectories and the worst two-impulse optimal trajectories is of order of 4 m/s.

3. Perturbation effects of the Sun are significant for maneuvers with multiple revolutions. The difference in Δv_{Total} between the best two-impulse optimal trajectories and the worst two-impulse optimal trajectories is of order of 50 m/s.

4. The first impulse of magnitude Δv_{LEO} is almost the same for all local optimal direct ascent trajectories, with clockwise or counterclockwise arrivals. Similar results are valid for maneuvers with multiple revolutions.

5. The second impulse of magnitude Δv_{LMO} varies significantly with the initial position of the Sun for all local optimal trajectories concerning to maneuvers with multiple revolutions, with clockwise or counterclockwise arrivals.

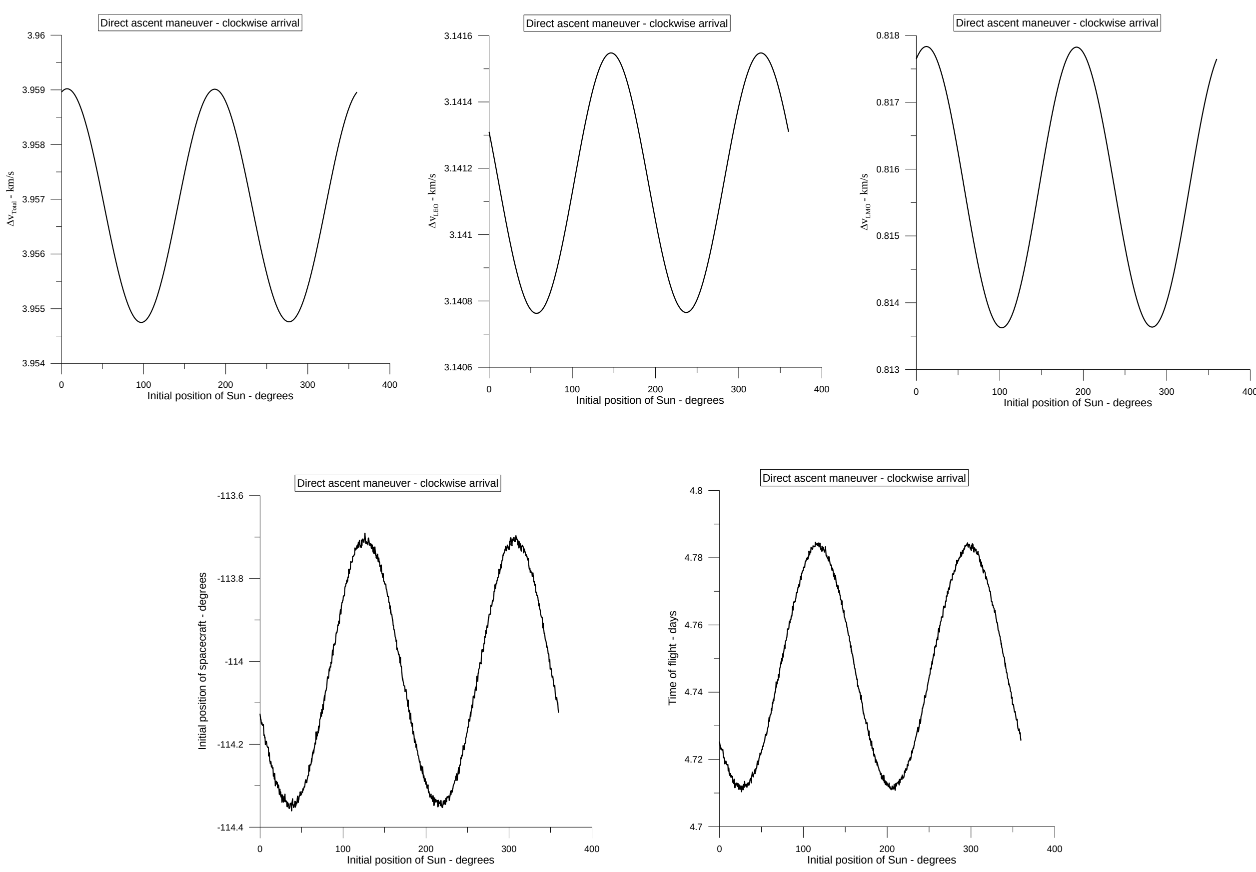


Figure 2 – Major parameter for direct ascent trajectory with clockwise arrival

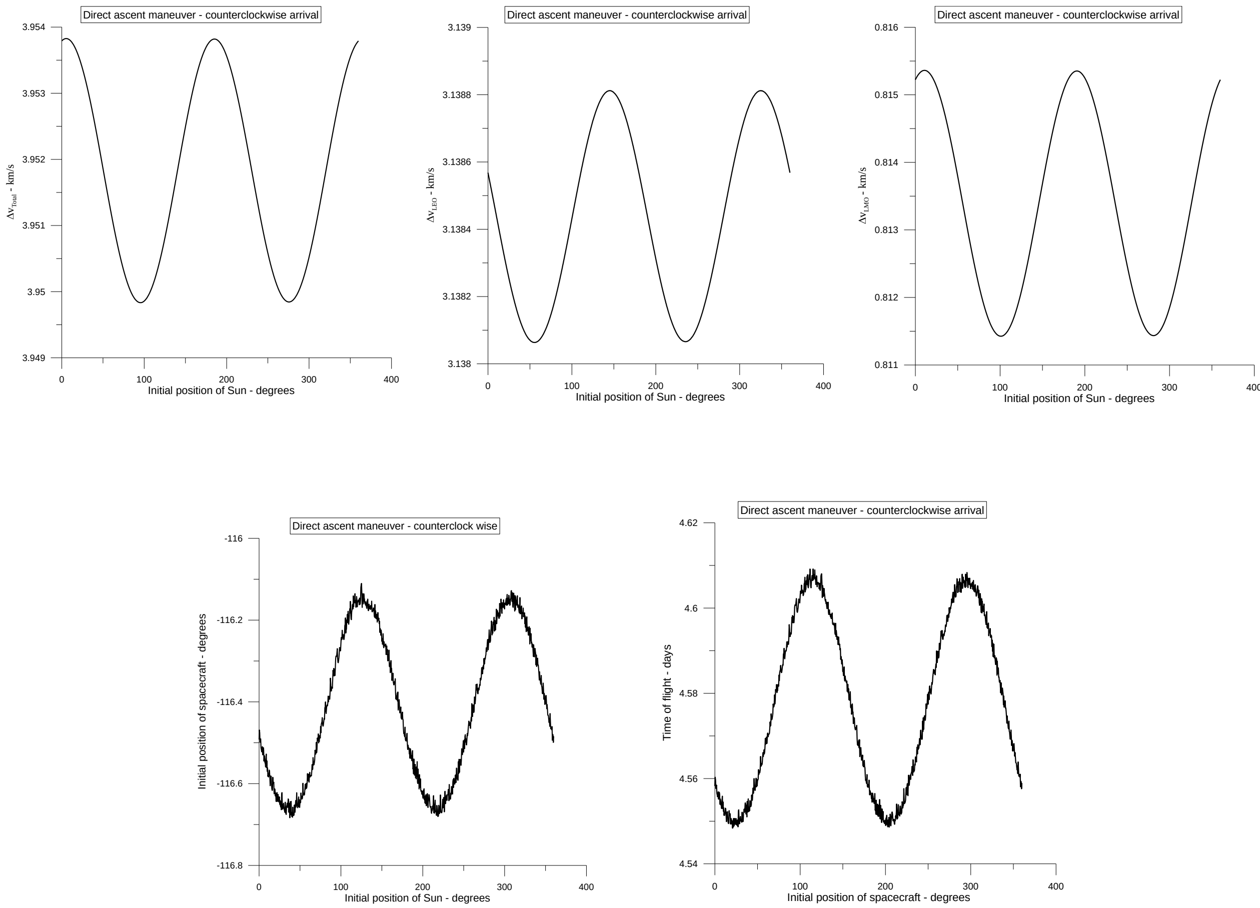


Figure 3 – Major parameter for direct ascent trajectory with counterclockwise arrival

6. For direct ascent trajectories, the initial position of the spacecraft varies around –114° for maneuvers with clockwise arrival, and, –116.4° for maneuvers with counterclockwise arrival.

7. For trajectories with multiple revolutions, the initial position of the spacecraft varies around 224° for maneuvers with clockwise arrival, and, 223° for maneuvers with counterclockwise arrival.

8. For direct ascent trajectories, the time of flight varies around 4.75 days for maneuvers with clockwise arrival, and, 4.58 days for maneuvers with counterclockwise arrival.

9. For trajectories with multiple revolutions, the time of flight varies around 58.4 days for maneuvers with clockwise arrival, and, 58.5 days for maneuvers with counterclockwise arrival.

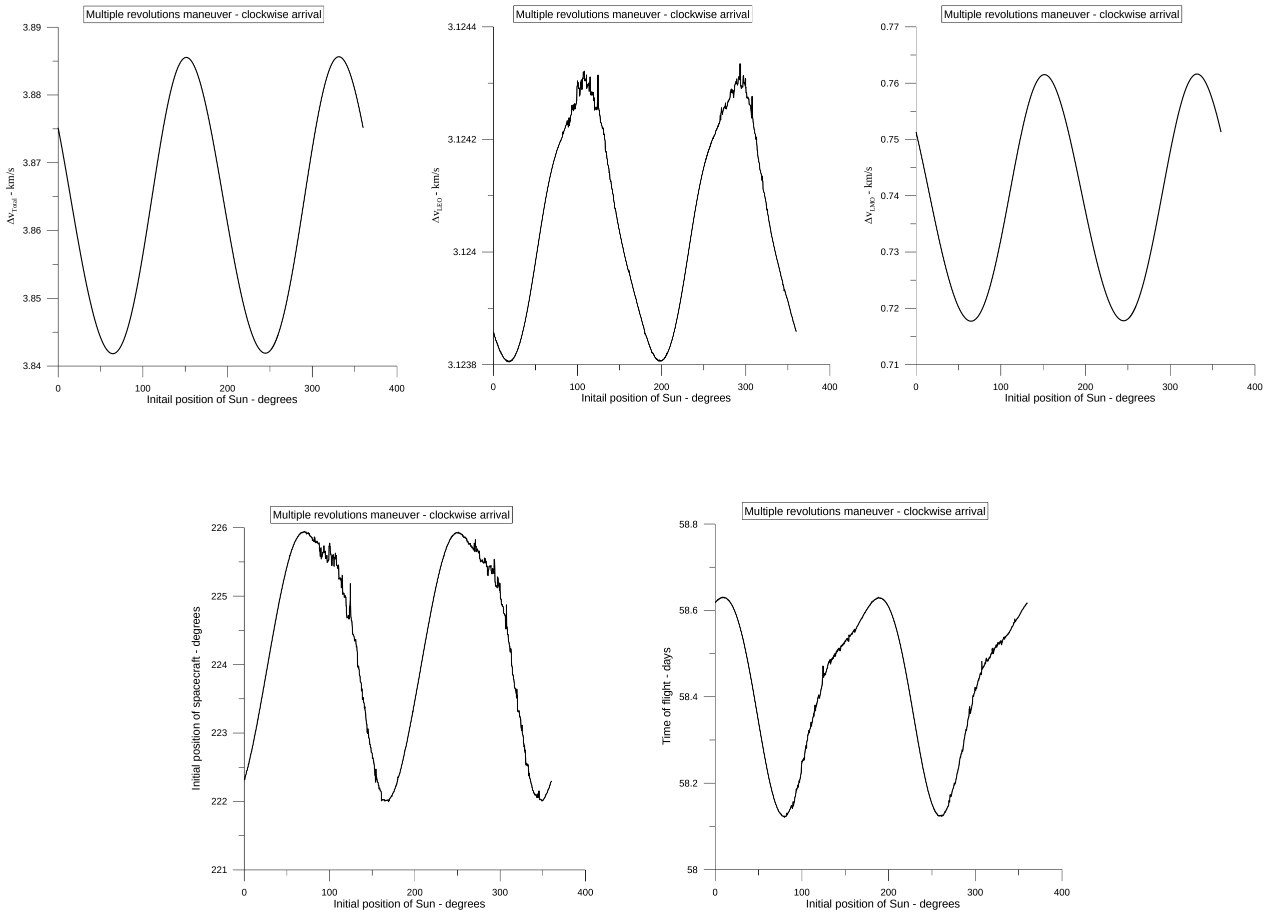


Figure 4 – Major parameter for multiple revolutions maneuver with clockwise arrival

10. For direct ascent trajectories, lunar missions with clockwise LMO arrival spend more fuel than lunar missions with counterclockwise LMO arrival for a same initial position of the Sun.

11. For trajectories with multiple revolutions, lunar missions with counterclockwise LMO arrival spend more fuel than lunar missions with clockwise LMO arrival for a same initial position of the Sun.

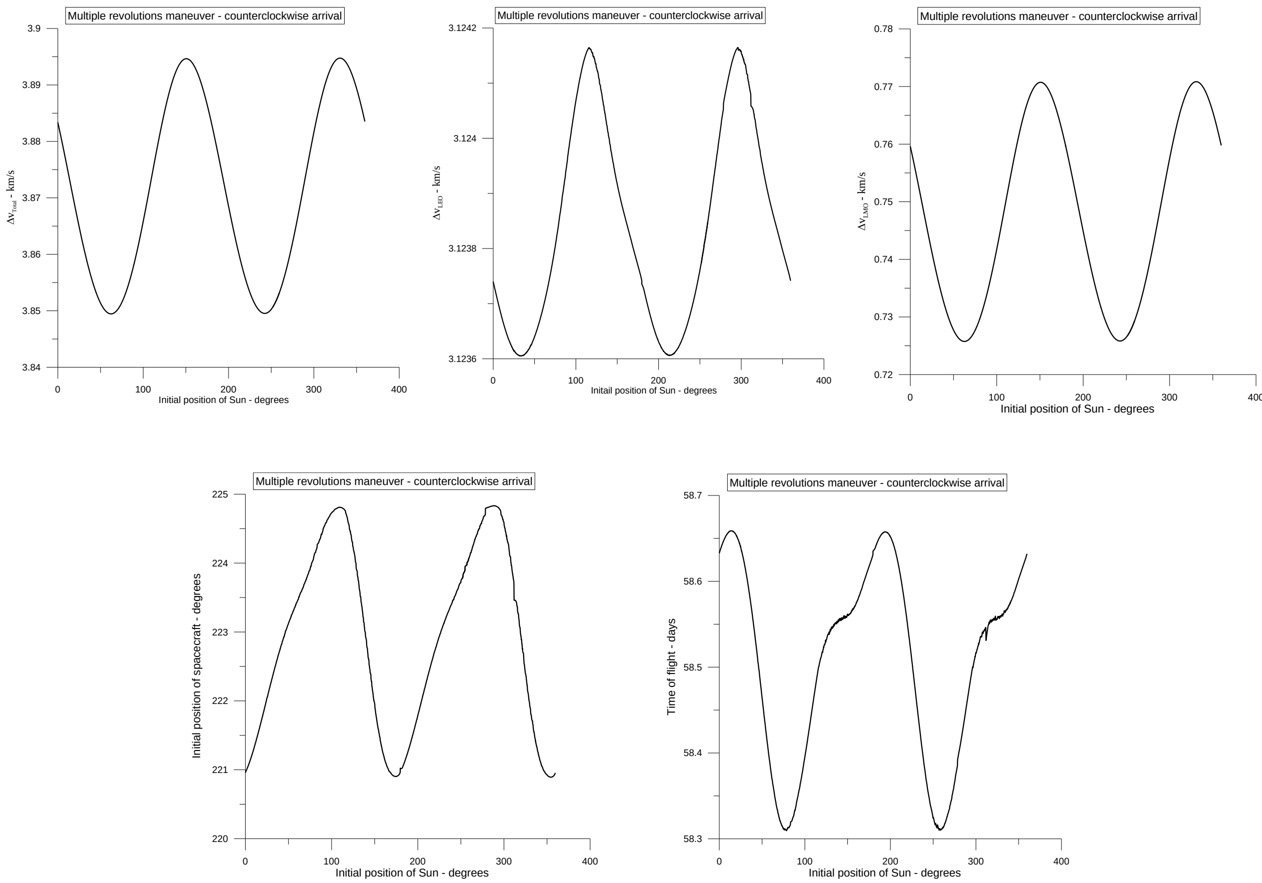


Figure 5 – Major parameter for multiple revolutions maneuver with counterclockwise arrival

3.2 Comparison of the two dynamical models

In this section is presented a comparison between the optimal trajectories which are computed by using the two dynamical models - PCR3BP and PBR4BP. For the PBR4BP is only presented one of the best solutions (see discussion in the preceding section).

Tables 1 and 3 show the major parameters for the lunar missions involving a direct ascent trajectories with time of flight of approximately 4.5 days, and, a maneuver with multiple revolutions with time of flight of approximately 58.5 days, respectively, considering clockwise or counterclockwise arrival at the Moon, for the PCR3BP model. In Tables 2 and 4 similar results are presented for the PBR4BP model. The best the initial position of Sun is included.

Figures 6 - 9 depict two maneuvers with five revolutions for the PCR3BP model, viewed from the Earth-centered reference frame and from the rotating reference frame, respectively. Figures 10 and 11 depict two maneuvers with five revolutions for the PBR4BP model, viewed only from the Earth-centered reference frame. In both cases, clockwise and counterclockwise arrivals are considered.

Table 1 – Lunar missions with direct ascent, major parameters - $h_{LEO} = 167\ km$ – PCR3BP

Maneuver	***LMO altitude*** *km*	Δv_{Total} *km/s*	Δv_{LEO} *km/s*	Δv_{LMO} *km/s*	T *days*	$\theta_{EP}(0)$ *degrees*
Clockwise	100	3.9570	3.1413	0.8157	4.762	–113.84
	200	3.9429	3.1413	0.8016	4.766	–113.81
	300	3.9300	3.1413	0.7887	4.771	–113.77
Counterclockwise	100	3.9519	3.1386	0.8133	4.571	–116.47
	200	3.9378	3.1385	0.7991	4.569	–116.51
	300	3.9245	3.1385	0.7860	4.567	–116.55

Table 2 – Lunar missions with direct ascent, major parameters - $h_{LEO} = 167$ *km* – PBR4BP

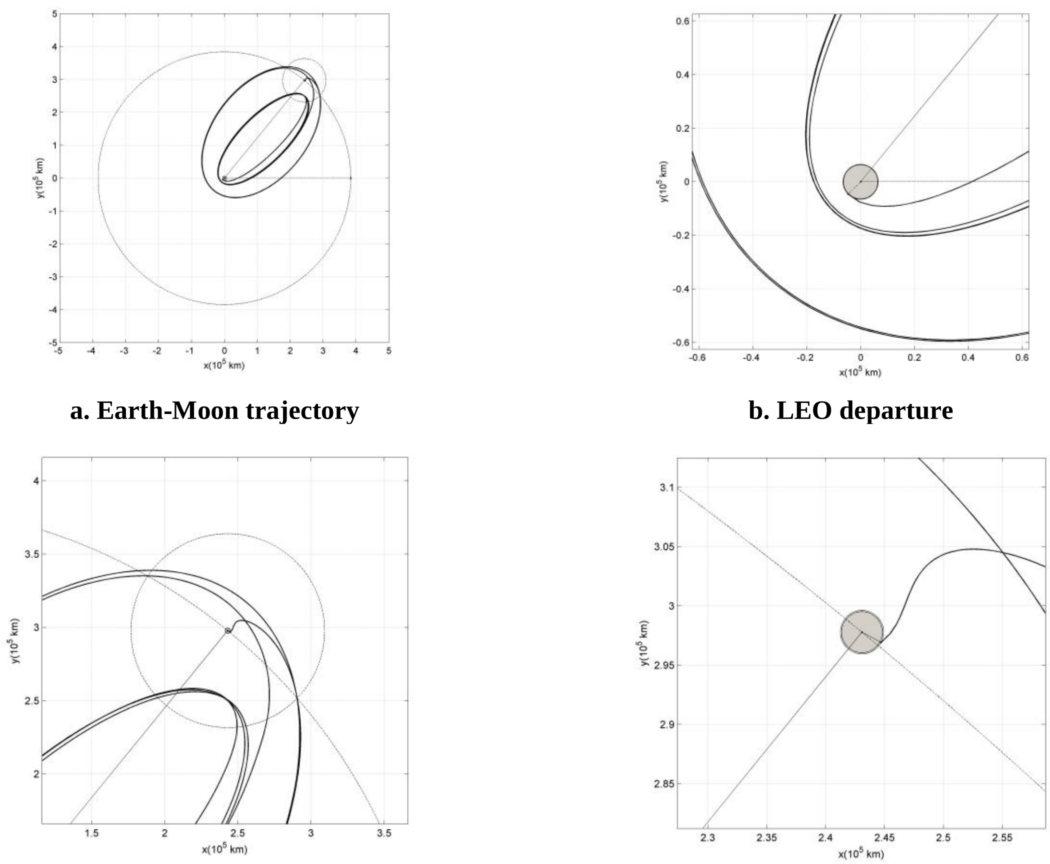
Maneuver	***LMO altitude*** *km*	Δv_{Total} *km/s*	Δv_{LEO} *km/s*	Δv_{LMO} *km/s*	T *days*	$\theta_{EP}(0)$ *degrees*	$\theta_{S}(0)$ *degrees*
Clockwise	100	3.9547	3.1410	0.8137	4.795	–113.65	96.84
	200	3.9406	3.1411	0.7995	4.799	–113.62	96.98
	300	3.9277	3.1412	0.7865	4.804	–113.57	97.13
Counterclockwise	100	3.9498	3.1384	0.8114	4.597	–116.33	95.28
	200	3.9354	3.1383	0.7971	4.595	–116.37	95.39
	300	3.9223	3.1383	0.7840	4.594	–116.41	95.60

Table 3 – Lunar missions with multiple revolutions, major parameters - $h_{LEO} = 167\ km$ – PCR3BP

Maneuver	***LMO altitude*** *km*	Δv_{Total} *km/s*	Δv_{LEO} *km/s*	Δv_{LMO} *km/s*	T *days*	$\theta_{EP}(0)$ *degrees*
Clockwise	100	3.8641	3.1244	0.7397	58.42	224.23
	200	3.8460	3.1241	0.7219	58.38	223.89
	300	3.8310	3.1241	0.7069	58.37	223.86
Counterclockwise	100	3.8750	3.1246	0.7504	58.60	224.41
	200	3.8563	3.1241	0.7322	58.39	221.49
	300	3.8407	3.1239	0.7168	58.46	222.44

Table 4 – Lunar missions with multiple revolutions, major parameters - $h_{LEO} = 167$ *km* – PBR4BP

Maneuver	*LMO altitude km*	Δv_{Total} *km/s*	Δv_{LEO} *km/s*	Δv_{LMO} *km/s*	*T days*	$\theta_{EP}(0)$ *degrees*	$\theta_S(0)$ *degrees*
Clockwise	100	3.8418	3.1241	0.7177	58.19	225.90	64.585
	200	3.8251	3.1241	0.7010	58.19	225.95	64.585
	300	3.8097	3.1241	0.6856	58.19	226.00	64.606
Counterclockwise	100	3.8494	3.1237	0.7257	58.37	223.54	62.397
	200	3.8332	3.1237	0.7095	58.37	223.52	62.394
	300	3.8182	3.1237	0.6945	58.37	223.50	62.389



a. Earth-Moon trajectory

b. LEO departure

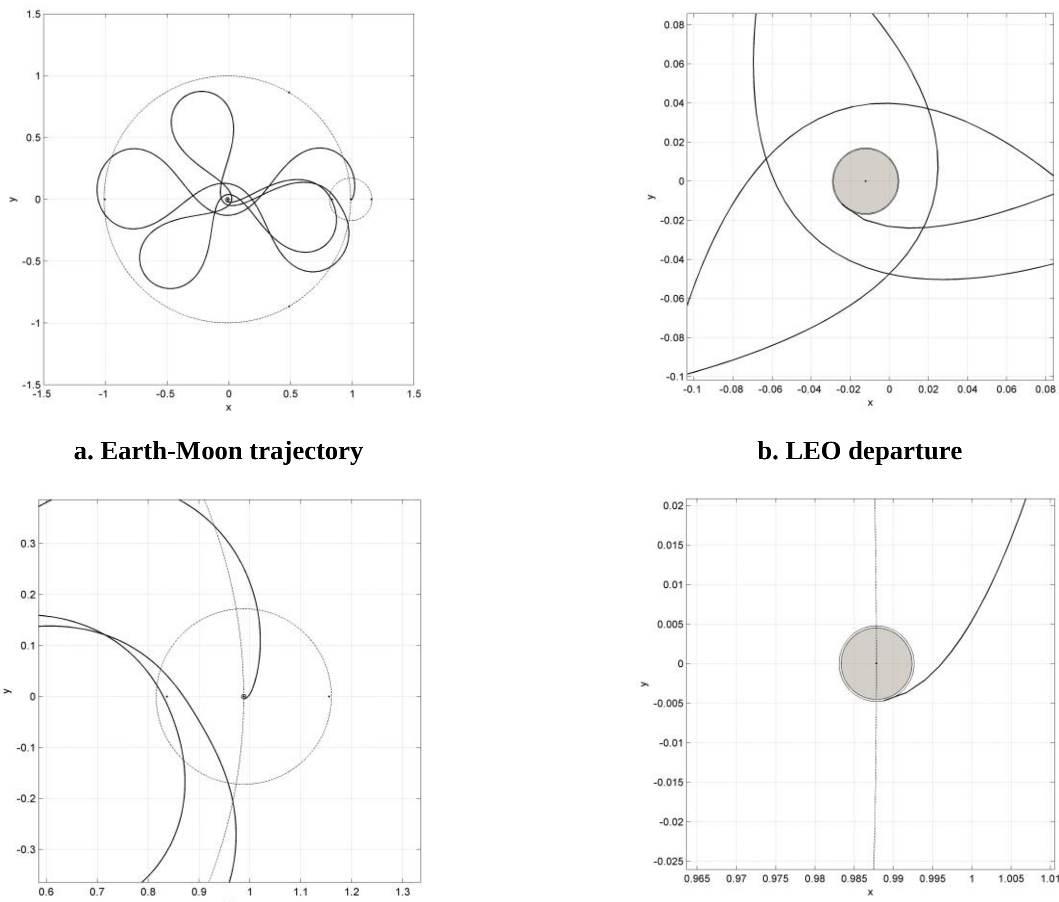
c. Swing-by

d. LMO arrival

Figure 6 – Trajectory with five revolutions – PCR3BP – clockwise arrival, Earth-centered reference frame

From the results presented in Tables 1 – 4, and, Figs 6 - 11, the major comments are:

1. The perturbation effects of the Sun are too small for direct ascent trajectories.
2. The differences in Δv_{LMO} between the two models are of order of 2 m/s for direct ascent trajectories.



a. Earth-Moon trajectory **b. LEO departure**

c. Swing-by **d. LMO arrival**

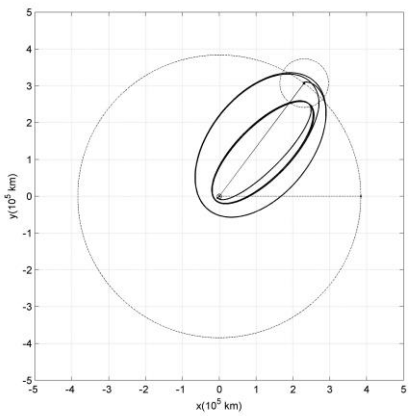
Figure 7 - Trajectory with five revolutions – PCR3BP – clockwise arrival, Rotating reference frame

3. The optimum initial position of the Sun is almost the same for all direct ascent trajectories, regardless the altitude of LMO. For the trajectories with clockwise arrival at the Moon, θ_{s0} is approximately $97°$. For the trajectories with counterclockwise arrival at the Moon, θ_{s0} is approximately $95.4°$.

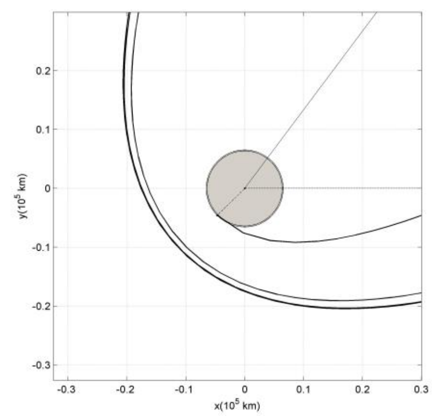
4. The perturbation effects of the Sun are significant for trajectories with five revolutions. Fuel consumption can vary significantly according the initial position of the Sun (see comment 3 in the preceding section).

5. Swing-by maneuvers with the Moon are made in the trajectories with five revolutions, for both dynamical models.

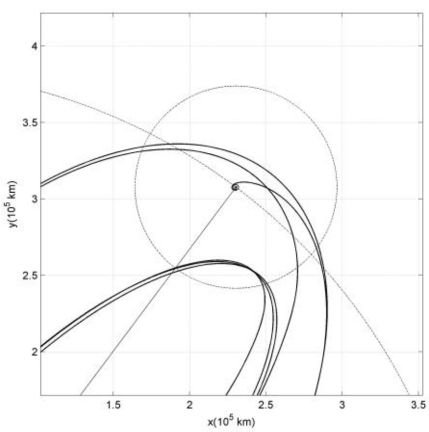
6. The velocity increment at the second impulse - Δv_{LMO} - is significantly affected by the presence of the Sun for trajectories with five revolutions (see comment 5 in the preceding section).



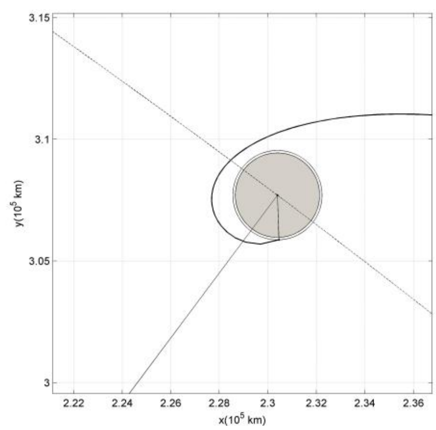
a. Earth-Moon trajectory



b. LEO departure

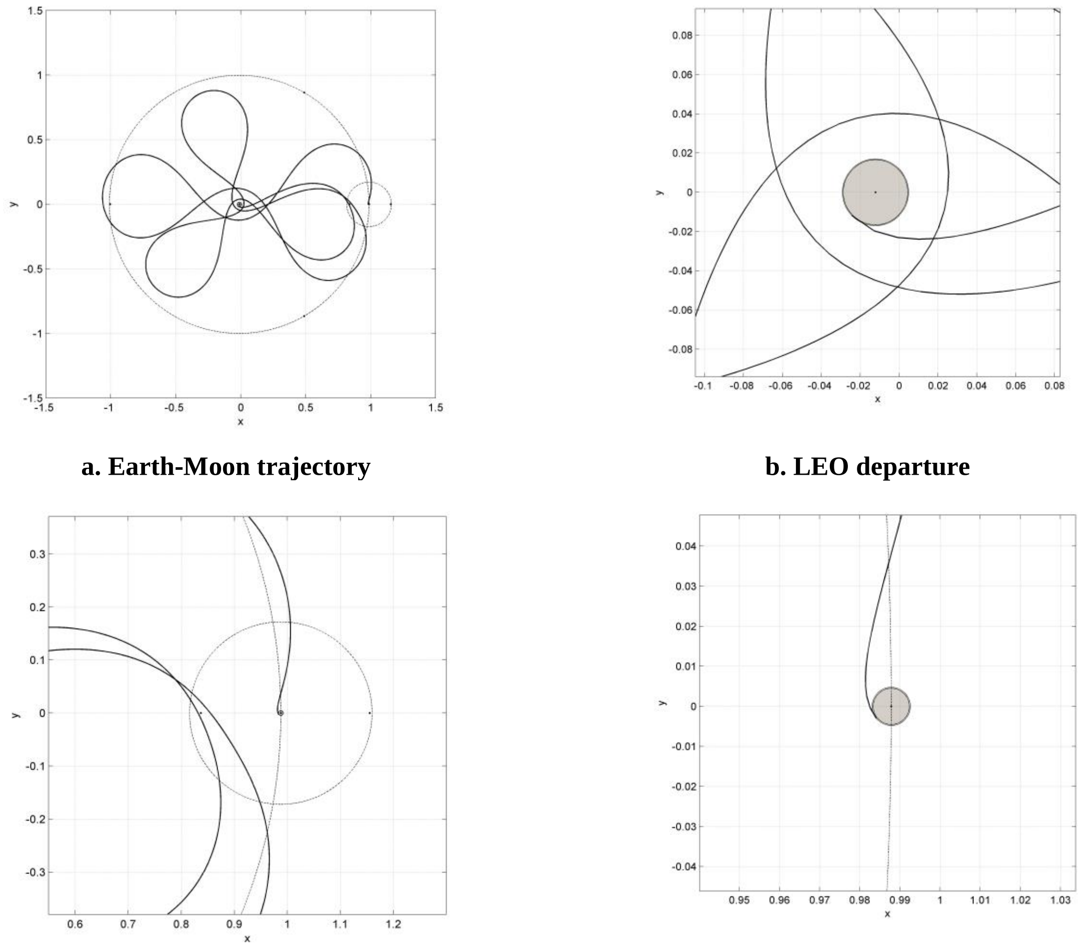


c. Swing-by



d. LMO arrival

Figure 8 – Trajectory with five revolutions – PCR3BP – counterclockwise arrival, Earth-centered reference frame



a. Earth-Moon trajectory

b. LEO departure

c. Swing-by

d. LMO arrival

Figure 9 - Trajectory with five revolutions – PCR3BP – clockwise arrival, Rotating reference frame

3.2 Sub-optimal trajectories

In this section, a brief discussion about the use of a sub-optimal trajectory is presented. In Table 5 the increments – Δv_{Total}, Δv_{LEO}, Δv_{LMO} – and the differences $\Delta_1 = \Delta v_{LEO} - \Delta v^*_{LEO}$ and $\Delta_2 = \Delta v_{LMO} - \Delta v^*_{LMO}$, where Δv^*_{LEO} and Δv^*_{LMO} are the velocity increments related to the best optimal solution given in Table 4, are presented for lunar missions considering some values of the initial position of the Sun and altitude of 100 km of the LMO (these values are extracted from the data sheet of Figs 4 and 5).

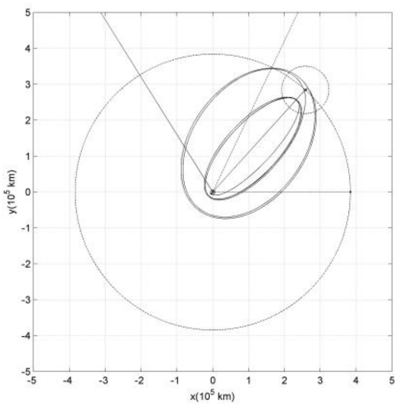
Table 5 – Lunar missions, major parameters - $h_{LEO} = 167\ km$ and $h_{LMO} = 100$

Maneuver	Δv_{Total} km/s	Δv_{LEO} km/s	Δv_{LMO} km/s	Δ_1 km/s	Δ_2 km/s	$\theta_S(0)$ $degree$
Clockwise	3.8752	3.1239	0.7513	–0.0002	0.0336	0.0
	3.8461	3.1239	0.7222	–0.0002	0.0045	45.0
	3.8496	3.1242	0.7254	0.0001	0.0077	90.0
	3.8816	3.1242	0.7574	0.0001	0.0397	135.0
	3.8751	3.1239	0.7512	0.0002	0.0335	180.0
Counterclockwise	3.8834	3.1237	0.7569	0.0	0.0312	0.0
	3.8532	3.1236	0.7296	–0.0001	0.0039	45.0
	3.8587	3.1239	0.7347	0.0002	0.0090	90.0
	3.8909	3.1240	0.7669	0.0003	0.0412	135.0
	3.8833	3.1237	0.7596	0.0	0.0339	180.0

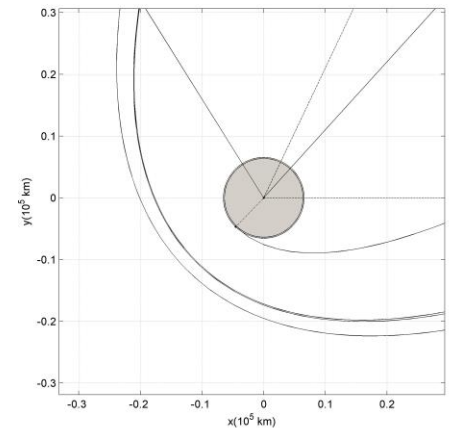
According to the results in Table 5, the main penalty in the use of a sub-optimal solution is an additional fuel consumption in the second impulse. The first impulse is not affected; the very small differences may be considered associated to the accuracy used in the calculations. The additional increment in the second impulse can be of order of 50 m/s (see comment 3 in the Section 3.1). For instance, for the initial position $\theta_S(0) = 45°$, near the best initial position of the Sun, which is given by $\theta_S(0) = 64.585°$ for clockwise arrival and $\theta_S(0) = 62.397°$ for counterclockwise arrival, the additional increment is of order of 5 m/s. On the other hand, for $\theta_S(0) = 135°$, the additional fuel consumption is of order of 40 m/s.

4. Conclusion

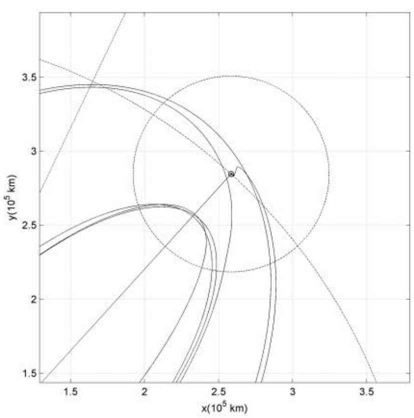
In this work, a systematic study about the perturbation of the Sun on optimal trajectories for Earth-Moon flight of a space vehicle is presented. The optimization problem has been formulated using the classic planar circular restricted three-body problem (PCR3BP) and the planar bi-circular restricted four-body problem (PBR4BP). Results presented for some lunar missions with time of flight of approximately 4.5 days show that the presence of the Sun causes small perturbations in the main parameters, which define the optimal solutions; but, some fuel can be saved if the duration of the transfer becomes larger (approximately 58.5 days), because swing-by maneuvers occur with the Moon. The presence of the Sun affects mainly the magnitude of the second impulse for maneuvers with multiple revolutions.



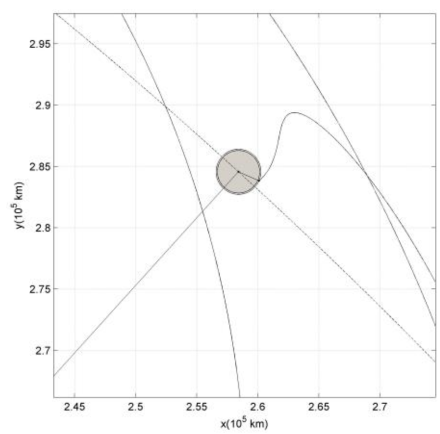
a. Earth-Moon trajectory



b. LEO departure



c. Swing-by



d. LMO arrival

Figure 10 – Trajectory with five revolutions – PBR4BP – clockwise arrival

5. Acknowledgements

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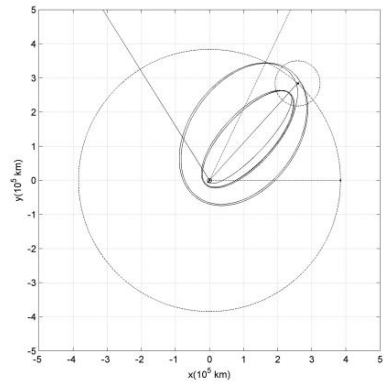
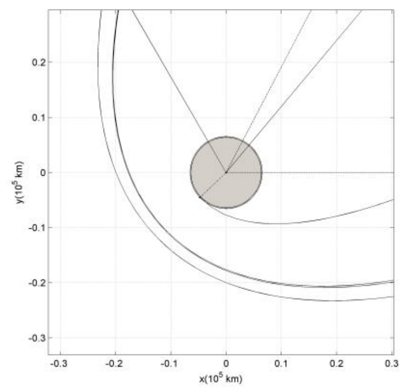
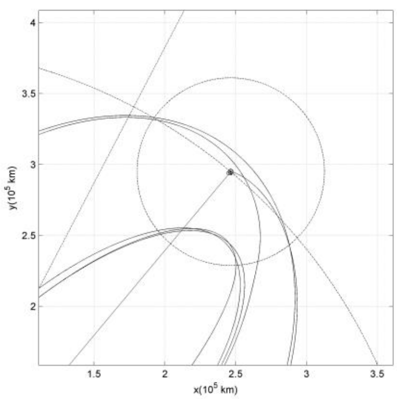


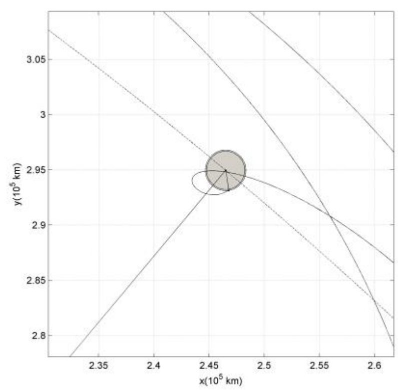
a. Earth-Moon trajectory



b. LEO departure



c. Swing-by



d. LMO arrival

Figure 11 – Trajectory with five revolutions – PBR4BP – counterclockwise arrival

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