

# GOES-R ORBIT AND INSTRUMENT ATTITUDE DETERMINATION

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## ABSTRACT

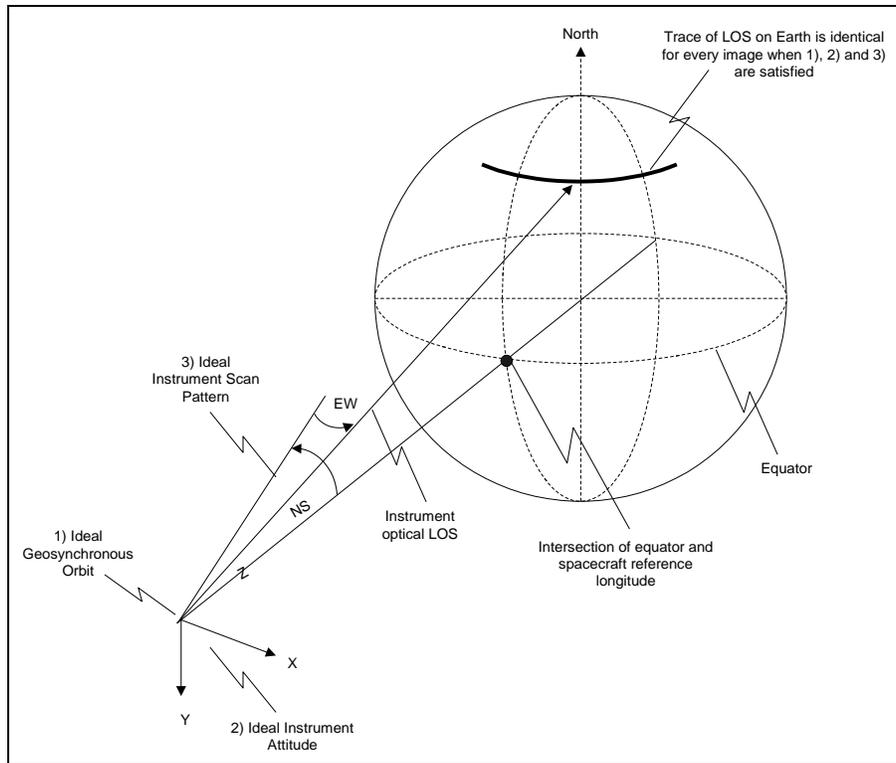
The Geosynchronous Operational Environmental Satellite (GOES) program provides visible and infrared imagery of the earth from geostationary orbit for weather forecasting, storm tracking and meteorological research. The current generation of satellites, designated the “NOP” series, will be replaced by the more capable “R” series starting in 2016. Imagery from GOES satellites is produced in the Fixed-Grid, meaning that pixels are associated with invariant earth locations as if the imagery were being acquired from an ideal geostationary orbit and an ideal attitude with respect to the earth. Image Navigation and Registration (INR) processing uses accurate knowledge of spacecraft orbit and instrument attitude to enable this objective. The INR implementation for GOES-R advances the state of the art over GOES-NOP in a number of important ways, including onboard orbit determination using GPS signals received from geostationary orbit. Kalman filtering is the estimation basis of the GOES-R INR capability. The ground system being developed by Harris Corporation, with Carr Astronautics as its INR subcontractor, implements an operational Kalman Filter executing in real time to drive image resampling. An INR Orbit and Attitude (O&A) determination capability -- also using a Kalman Filter -- is intended for validation and analysis of system performance. Both Kalman filters are discussed in this paper, with particular emphasis on the powerful analytical capabilities of the INR O&A Kalman filter.

## Introduction

The Geosynchronous Operational Environmental Satellite (GOES) system continuously provides visible and infrared images of the earth that have become essential for weather forecasting, storm tracking and meteorological research. The currently operational series of GOES satellites, designated GOES-NOP<sup>1,2</sup>, are Earth-pointing, three-axis stabilized spacecraft supporting two 2-axis scanning instruments: the 5-channel visible/infrared (IR) Imager and the 19-channel IR Sounder. The Imager and Sounder scan in an east-west (EW) direction with north-south (NS) steps at the end of east-west swaths. Other GOES-NOP instruments include a solar X-ray imager, space environment monitor and a magnetometer. Use of a three-axis stabilized spacecraft and 2-axis scanning instruments enable the sensors to continuously observe the Earth. The GOES Imager and Sounder image clouds, monitor Earth surface temperature and water vapor, and sound the atmosphere vertical thermal and vapor profiles. These capabilities allow tracking of dynamic atmospheric phenomena, particularly severe local storms and tropical cyclones.

The next generation of GOES satellites, designated GOES-R<sup>3,4,5,6,7,8</sup>, are expected to be launched in 2016. These are also three-axis stabilized spacecraft with a 2-axis scanning instrument: the Advanced Baseline Imager (ABI). The ABI senses 16 visible and infrared (IR) bands, and has resolutions of 0.5, 1.0 and 2.0 km (depending on the band), with a total image data rate approximately 20 times that of the GOES NOP instruments. Other instruments on GOES-R include the Extreme ultraviolet and X-ray Irradiance Sensor (EXIS), Solar Ultraviolet Imager (SUVI), Space Environment In-Situ Suite (SEISS), Magnetometer (MAG), and Geostationary Lightning Mapper (GLM).

It is important that weather phenomena be accurately located on the earth, and therefore the longitude/latitude coordinates of each image pixel must be known (*image navigation*). Weather phenomena are dynamic by nature and therefore pixels from consecutive images should view the same point on the earth (*image registration*), which facilitates accurate measurement of winds from cloud tracers. In accord with these objectives, GOES products are rendered in a system with fixed pixel locations<sup>9</sup> referred to as the “Fixed-Grid”. The Fixed Grid is referenced to an ideal location in space located in the earth’s equatorial plane at the radius from the earth center (42164.16 km) that allows a satellite to remain at a nearly fixed position with respect to the earth. The reference longitude is also fixed for each GOES satellite: GOES-E is nominally at 75 degree west longitude, and GOES-W is nominally at either 135 degree west (GOES-NOP) or 137 degree west (GOES-R). The imaging Fixed-Grid axes are defined with the x-axis parallel to the equator (+x = east), the y-axis parallel to the earth’s spin axis (+y = south), and the z-axis pointing to the earth center. These axes are shown in Figure 1, which also shows the ground trace of an east-west scan by an imaging instrument.

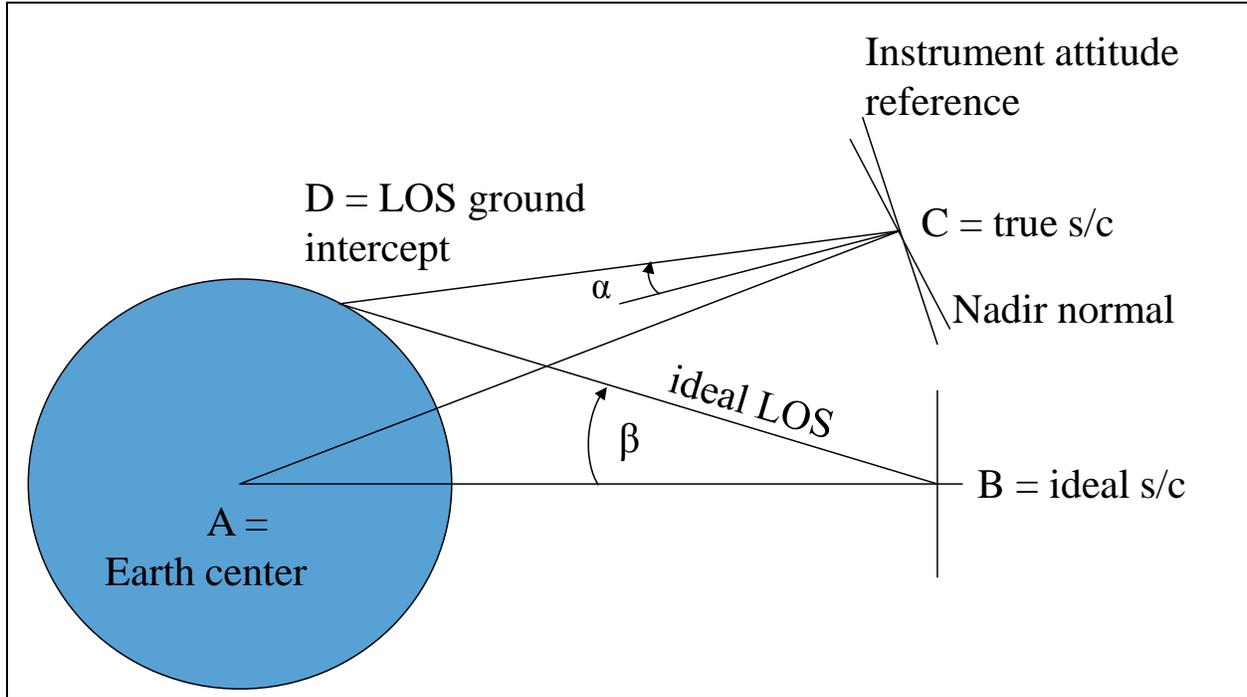


**Figure 1: GOES Fixed-Grid and EW Scan Line-of-Sight (LOS) Trace**

Unfortunately real satellites cannot maintain an ideal position and attitude alignment throughout the day: perturbing forces (solar and lunar gravitation, solar radiation pressure, and non-central earth gravity) cause the spacecraft to drift east or west, and orbit inclination to slowly grow with time. To compensate for this drift, the spacecraft must execute periodic E-W and N-S station-keeping maneuvers. Furthermore, the spacecraft cannot maintain perfect attitude alignment with the Fixed-Grid axes. Even if it could, the imaging instruments have external and internal misalignments that are affected by diurnal thermal deformation, which is driven by attitude with-respect-to the sun. Hence, the received images must be adjusted so that pixels are aligned on the Fixed-Grid. GOES Image Navigation and Registration (INR) is performed by determining actual pixel locations on the earth as a function of time (using best estimates of orbital position and image orientation), and then adjusting the received image pixels so that they appear as if obtained at the reference position; that is, registering the image pixel to the Fixed-Grid.

Figure 2 graphically shows how the image correction is computed for a simplified 2-dimensional case. Although the problem is inherently 3-D, the 2-D case is more easily understood. The “ideal” spacecraft position (at B) is exactly on the equator at the reference longitude, and the “ideal” attitude angles are defined relative to nadir. The real spacecraft is located at position C. The true nadir direction at point C is defined by line C-A, but because spacecraft and instrument attitude control is imperfect, instrument nadir deviates slightly from true nadir. Since instrument scan angles are defined with respect to the instrument attitude reference, the optical ray from C to ground location D has a measured scan angle of  $\alpha$ . Hence ground intercept point D can be computed given three quantities: angle  $\alpha$ , the angle by which the instrument reference deviates from the nadir normal, and the coordinates of point C. Notice that the “ideal” scan angle for

point D is  $\beta$ , which is not equal to  $\alpha$ . If the true spacecraft position and instrument attitude are known as a function of time, the navigation correction to measured scan angle  $\alpha$  is  $\beta - \alpha$ . Or conversely, given a desired pixel location represented by angle  $\beta$ , the instrument servo can be rotated to give scan angle  $\alpha$ . Since true orbit and instrument attitude are imperfectly known, they must be estimated from available measurements.



**Figure 2: Two-dimensional (EW only) True and Ideal Scan Angles**

The method used to perform INR for GOES-R is quite different from that used for GOES-NOP. The next section briefly discusses GOES-NOP INR because the ground INR software is the conceptual basis for the GOES-R INR O&A software. Then GOES-R INR is explained, and final sections describe INR O&A design and usage.

## GOES-NOP INR

In the legacy GOES-NOP system, image motion compensation (IMC) signals are applied by the spacecraft while the Imager and Sounder are performing east-west scans, i.e., the correction is an additive signal (analogous to  $\beta - \alpha$ ) to the instrument east and north servo commands. This correction is computed for the center of the north-south detector stack, and because the stack is compact (8 visible pixels) there is a small but negligible error for the northernmost and southernmost detectors. The IMC is computed using orbit and instrument attitude coefficients computed on the ground by the Orbit and Attitude Tracking System (OATS)<sup>10,11</sup>. The term “instrument attitude” refers to multiple parameters that include combined effects of satellite and instrument pointing errors. OATS uses the last day of observation data to compute the current satellite orbit and daily attitude profile. A set of coefficients defining the predicted orbit and daily attitude is calculated and uploaded to the satellite. Hence GOES-NOP INR implicitly assumes that the attitude profile for the next day is identical to that for the last day. Because

thermal deformation is a function of attitude relative to the sun, the error in the assumption is acceptably small, except near solar eclipses.

OATS uses three types of measurements to estimate the satellite orbit and instrument attitude: observed imaging angles to known stars, observed imaging angles to fixed earth landmarks, and 2-way ranges between the ground antenna and satellite. Because the orbital coordinate system (computed from the ground-uploaded predicted orbit) defines the NOP spacecraft reference attitude, star observations are only a function of instrument attitude relative to the orbit reference. The range measurements are only a function of satellite orbit, and the landmark measurements are a function of both orbit and attitude (O&A). Use of all three measurement types in a coupled O&A batch least squares solution provides observational redundancy, leading to accurate estimation of the O&A states. Carr<sup>12</sup> presents a brief history of GOES INR.

## GOES-R INR

The approach used in GOES-NOP of applying onboard IMC cannot be used for GOES-R because the ABI detector stack is extended (not compact) and spectral channels are separated in field. IMC computed at the center of the focal plane would be unacceptably inaccurate at its edges. Instead, the GOES-R system will perform all INR computations in ground software rather than onboard the spacecraft<sup>13,14,15</sup>. The ABI detector radiance samples are first “navigated” to determine their observed positions on the earth, and then a resampling kernel is applied to small blocks of pixels to interpolate the pixel radiance at intersections of the Fixed-Grid system. In this way, a Fixed-Grid product is created in the geometry of an ideal satellite, and pixels in the product ideally have invariant earth locations. The Fixed-Grid sites are oversampled by ABI, meaning that the sampling by the detectors is finer than the pixel sampling in the Fixed-Grid product, which improves the quality of the Fixed-Grid product.

Navigation of detector samples requires knowledge of orbital position, spacecraft attitude, and a few other parameters internal to the ABI. The GOES-R satellite orbital position is computed onboard the satellite (from GPS signals), and then sent to the ground as telemetry; it does not depend on ground computation using range and landmark data. The time-varying ABI attitude states are computed continuously on the ground in an operational Kalman filter that processes ABI star measurements. High-rate spacecraft angular rate samples are also telemetered to the ground to allow the operational Kalman filter (and image resampling) to follow spacecraft attitude changes between star observations. Landmark measurements are not used operationally for INR purposes in the baseline system.

While the GOES-R INR approach involves much greater ground computation than the NOP system, it provides better pointing accuracy and stability, greater scan flexibility, operational simplicity, and greatly reduced operational burden. For example, the GOES-NOP sample 3-sigma visible navigation requirement was 55 microradian<sup>16,17</sup>, while the equivalent GOES-R requirement is 28 microradian<sup>18</sup>. Note that 14 microradian in optical angle (the ABI visible detector size) is about 0.5 km on the Earth surface at nadir. GOES-R INR computations are nearly real-time in that they do not depend on a 24 hour prediction of orbit or attitude (which complicates GOES NOP operations and limits flexibility). The ABI can operate autonomously for 7 days because the spacecraft supplies real-time orbit and spacecraft attitude information, and the ground uploads 7-day star selection and Line-of-sight Motion Compensation (LMC) tables.

The ABI uses onboard orbit and spacecraft attitude information to compute Orbit Motion Compensation (OMC) and Spacecraft Motion Compensation (SMC) corrections to the ABI scan angles. The LMC adjustment corrects for optical misalignment with respect to an ideal alignment defined relative to the spacecraft. The sum of the OMC, SMC and LMC is equivalent to the GOES-NOP IMC correction, but unlike GOES-NOP, the ABI cannot and does not depend on the onboard correction to meet INR requirements. Rather, onboard OMC, SMC and LMC are used to minimize over-scanning of the earth and star scenes, resulting in efficient scan patterns.

To appreciate the importance of the INR correction, consider that the maximum correction for orbit errors is about 290  $\mu\text{rad}$  when the spacecraft is located at the edge of the allocated  $\pm 0.1^\circ$  GOES-R latitude or longitude station-keeping box. The maximum combined correction for GOES-R spacecraft plus ABI attitude error is probably about 120  $\mu\text{rad}$ , which is much smaller than that of the GOES-NOP satellites.

## **GOES-R INR O&A and LMC Support Software**

The complexity and novelty of the GOES-R INR design elevates the importance of its validation in orbit. Furthermore, system performance characteristics that affect INR may not be completely understood before launch. To mitigate this risk and to provide a post-launch analysis capability for anomalous behavior, INR O&A and LMC Support software is included in the GOES-R ground system. Other INR O&A functions include ephemeris generation for ABI Star Selection computations, and generation of Line-of-sight Motion Compensation (LMC) coefficient tables for ABI attitude correction. The INR O&A estimation process is similar to that of GOES-NOP OATS in that star, landmark and range measurements are used to estimate O&A states, but GOES-R continuously (for life of mission) solves for those states using a Kalman filter, rather than using batch least squares. This is practical because the spacecraft provides high-rate angular rate measurements to the ground, landmark measurements are computed using image data obtained before resampling, and a stochastic model of ABI misalignments handles both non-eclipse and eclipse cases. The following sections describe the models and algorithms used in the INR O&A software.

### ***Orbit Model***

The geosynchronous spacecraft orbit model used in the INR O&A software is similar to models used in many orbit determination packages<sup>19</sup>. The 10 orbit states include six J2000 cartesian epoch orbit elements (x, y, z, xdot, ydot, zdot), a solar radiation pressure coefficient, and three states modeling maneuver delta-v in an orbit-referenced coordinate system. Cartesian orbit element states are used rather than equinoctial elements because perturbing effects, state transition matrices and random process noise are more easily modeled in a Cartesian system. The convergence advantages of equinoctial elements for fixed epochs are unimportant in this application because the Kalman filter uses a moving epoch and initial orbit estimates are quite accurate; extensive simulated and actual GOES NOP orbit determination has confirmed this. All orbit states can be designated as “adjusted” or not, although the 6 orbit elements must be treated as a group.

Modeled perturbing effects include 8<sup>th</sup> degree-and-order geopotential, solar and lunar gravitation, “cannonball” solar pressure, and either maneuver delta-v for short-duration impulsive

maneuvers, or orbit-referenced maneuver acceleration for longer maneuvers. More complicated panel-based solar pressure models were tested for GOES-NOP spacecraft, and not found to provide significant accuracy improvement compared to cannonball models. All GOES-R maneuvers are necessarily low thrust. Maneuvers of less than 5-10 minutes are treated as impulsive, where the orbit is integrated up to the maneuver, the delta-v (transformed from orbit to J2000 coordinates) is applied, and integration proceeds. Longer maneuvers are handled by converting the orbital-referenced delta-v to acceleration, and then numerically integrating the acceleration (also transformed from orbit to J2000 coordinates) in the orbit integration.

Numerical orbit integration is performed using the adaptive-stepsize Burlirsch-Stoer extrapolation method<sup>20</sup>. The orbit partition of the state transition matrix is computed using central-difference perturbation of the epoch states followed by integration, rather than numerical integration of the variational equations. The perturbational approach has the advantages of greater flexibility, simpler implementation, faster computation, and possibly greater accuracy. The state process noise matrix is computed using a linear approximation, followed by iterative interval doubling<sup>21</sup>.

### ***Attitude Model***

The ABI uses fixed detector arrays and separate east-west and north-south scan mirrors, where the north-south scan mirror is the one closer to space. For this generic design type, misalignments that affect the line-of-sight viewed by an individual detector include:

- rotations of the ABI mounting with respect to the satellite,
- misalignments of the mirror rotational axes with-respect-to the ABI mounting frame,
- mirror rotational sensing error,
- misalignments of the mirrors with-respect-to the rotational axes,
- rotational and translational misalignments of the detector arrays (including telescope distortion),
- detector plate scale error,
- misalignments of the individual detectors on the focal plane arrays, and
- misalignments between the three focal plane modules.

Detector mounting errors can be accurately measured on the ground and used when processing the image data. Other misalignments can also be measured, but in orbit, they are affected by thermal deformation that can change during the day as internal temperatures vary with the sun angle. Of these effects, a total of 17 fundamental misalignments are of concern for ABI modeling.

To model these errors in the INR O&A software, it is necessary to compute the sensitivity of the line-of-sight to each of the misalignment types. The sensitivity is determined by following the ray from the detector origin to space, accounting for mirror reflections, and linearizing the equations for perturbations of the misalignments. When defining attitude states to be used in the INR O&A Kalman filter, it is undesirable to include misalignments or linear combinations of misalignments that have nearly the same effect on the line-of-sight. Since these redundant states will not be independently observable, inclusion of redundant states will lead to nearly singular covariance matrices in the Kalman filter. Thus INR O&A uses a reduced-order attitude model.

The reduced-order states were selected by the following procedure:

1. The equations for EW and NS view angle sensitivity to misalignments were examined to identify groups with identical sensitivities. Several misalignments were immediately identified as redundant and tentatively combined.
2. This computation was repeated using numerically-computed partial derivatives for a range of detector locations on the focal plane. Because the detector arrays are large, this revealed a nonlinear effect that could not be ignored.
3. Several different linear combinations of misalignments with nearly the same effect on line-of-sight were examined for possible state reduction. Again numerical evaluation revealed that one combination produced slightly smaller pointing errors than others.
4. Another factor affecting state selection was a need to include the same primary states (loosely defined as roll, pitch, yaw and primary orthogonality) used in the operational star-based ABI Kalman filter that supports image product generation. This is desirable so that attitude estimates for the two Kalman filters could be compared.

Details of the attitude state selection process are beyond the scope of this paper. The final ABI attitude model used in INR O&A consists of 22 states: 11 misalignment angles and 11 time derivatives of those angles. The 11 angle states include the four primary variables defined above, and seven variables that model linear combinations of secondary misalignments.

### ***Measurement Data***

ABI star measurements are obtained by commanding the EW and NS mirror angles so that a given target star will cross a designated subset of the detector array as the spacecraft rotates to match earth rotation rate, with added EW mirror rotation to reduce the required duration of the star scenes<sup>22,23</sup>. Ground software then processes the time sequence of detector samples to determine the centroid time of the sensed star, and to determine the fractional detector NS position at which the star energy is maximum. The mirror positions and detector coordinates are then converted to EW and NS scan angles at the calculated star sense time.

The Kalman filter then differences these measured scan angles with predicted scan angles ( $e, n$ ) calculated from the Kalman filter state vector, i.e.,

$$\begin{bmatrix} \Delta e \\ \Delta n \end{bmatrix} = \begin{bmatrix} e \\ n \end{bmatrix}_{measured} - \begin{bmatrix} e \\ n \end{bmatrix}_{predicted}$$

The predicted scan angles are computed as

$$\begin{bmatrix} e \\ n \end{bmatrix}_{predicted} = \begin{bmatrix} \sin^{-1}(\hat{s}_{imf-x}) \\ \tan^{-1}(-\hat{s}_{imf-y} / \hat{s}_{imf-z}) \end{bmatrix}$$

where  $\hat{\mathbf{s}}_{imf} = \begin{bmatrix} \hat{s}_{imf-x} \\ \hat{s}_{imf-y} \\ \hat{s}_{imf-z} \end{bmatrix}$  is the scan unit vector in the instrument mounting frame (IMF). The

$\hat{\mathbf{s}}_{imf}$  vector is computed as follows:

1. The right ascension and declination from the star catalog is corrected for proper motion, parallax and annual aberration to compute the star unit vector in J2000 coordinates.
2. That J2000 star vector is rotated to the spacecraft orbit reference frame (ORF), which is computed using the onboard orbit position and velocity. The ORF is the steering reference used by the spacecraft attitude control system.
3. The *a priori* Kalman filter state vector estimate ( $\hat{\mathbf{x}}_{i/i-1}$ ) at time  $t_i$  based on measurements up to and including time  $t_{i-1}$  is used to rotate the ORF star vector to the ABI virtual mounting frame (VMF). That is, the first three filter states -- denoted as roll, pitch and yaw -- define a rotation from the ORF to the VMF. The VMF is described as “virtual” because it is not physical: the roll, pitch and yaw states include spacecraft attitude error, error in the ABI mounting with respect to the spacecraft, and linear combination of various internal ABI misalignments that behave as 3-axis rotations.
4. The star vector in the VMF is corrected for the remaining ABI misalignment states in  $\hat{\mathbf{x}}_{i/i-1}$  to obtain  $\hat{\mathbf{s}}_{imf}$ . (For numerical reasons, these misalignments are actually used to compute corrections to  $[e \ n]_{predicted}^T$ , rather than directly to  $\hat{\mathbf{s}}_{imf}$ ).

The Extended Kalman filter also requires the partial derivative matrix  $\mathbf{H}_{i/i-1} = \frac{\partial [e \ n]_{predicted}^T}{\partial \hat{\mathbf{x}}_{i/i-1}}$ .

These partials are obtained by a combination of analytic and numerical methods.

Landmark measurements are obtained from the un-resampled ABI image data by correlating transitions in image sample radiances with expected land-water boundaries. The chosen landmarks are typically islands, peninsulas or bays. ABI ground processing will then determine the ABI mirror angles and fractional detector positions corresponding to a reference position for each landmark. Then INR O&A processing is similar to that for star measurements. That is, the mirror and detector angles are converted to scan angles, which are compared with predicted scan angles. The primary difference for landmarks is that the known landmark position is defined in *earth-centered-fixed* (ECF) coordinates, and rotated to the J2000 frame. The scan unit vector is computed from the difference between the landmark position and the filter-estimated spacecraft position (states in  $\hat{\mathbf{x}}_{i/i-1}$ ). Thus the landmark measurement model uses both the onboard spacecraft position and the filter-estimated position. The landmark position is computed from shoreline databases with corrections for planetary aberration.

The range measurements used in INR O&A are obtained from the round-trip transit time of the GOES Re-Broadcast (GRB) signals uploaded to GOES-R and transmitted to users. The measured time shift between transmitted and received Pseudo-Random Noise (PRN) signals is converted to a 2-way range. The range bias corrections for different antennas are included as

Kalman filter states. The filter-computed range is computed as the difference in ground antenna position (rotated from earth-centered-fixed coordinated to J2000) and the filter-estimated spacecraft position, with corrections for ionospheric and tropospheric group delay, transponder delay, and known range bias.

It should be noted that the measurement model equations can be written in several equivalent forms. The above descriptions were chosen to facilitate explanation. Equations actually coded are slightly different.

Two other aspects of the INR O&A measurement processing are noteworthy. Early in the GOES-R conceptual design, it was recognized that very accurate attitude control and small attitude angular rates are difficult to simultaneously maintain in 3-axis stabilized spacecraft. Thus control requirements on the spacecraft were relaxed, and instead, the spacecraft must provide measured 3-axis angular rates as telemetry to ground processing. The filter-estimated ABI roll, pitch and yaw states (updated as each measurement is processed) must be propagated to the time of the next measurement using not only the estimated ABI angle rates, but also using the change in spacecraft orientation during the time interval. That "known" correction is computed by integrating the measured angular rates, and is added to the angular change computed from the estimated ABI misalignment rate states. A small additive process noise term is included in the state covariance propagation to account for noise in the measured angular rates.

The other unusual aspect of the filter implementation involves delayed measurement processing. Unlike the GOES-NOP system, GOES-R spacecraft will continue imaging operations during station-keeping maneuvers, which are intentionally frequent and small. There is some uncertainty in the orbit change caused by a planned orbit maneuver, so ground pre-maneuver estimates of the orbit change cannot be completely trusted as accurate. Post-maneuver estimates of the orbit change, calculated from maneuver jet firing telemetry, should be significantly more accurate. Thus INR O&A stores all incoming measurements (including angular rate telemetry) in a buffer, and delays processing for about 15 minutes so that post-maneuver calculations of the delta-v may be available for E-W and momentum-dumping maneuvers. N-W station-keeping will generally be much longer than acceptable processing delays, so pre-maneuver estimates must be used for these maneuvers, and the filter state noise model is increased accordingly.

### ***Filter Data Editing and Diagnostics***

The INR O&A filter is a standard Extended Kalman Filter (EKF) implemented in the U-D factored form for greater numerical accuracy<sup>24,25</sup>. The EKF is well-suited to mildly nonlinear systems driven by random inputs. The EKF has another feature that is very useful when processing data from physical systems: the capability to detect outlying measurements. These outliers may be caused by occasional erroneous measurements not consistent with given noise models, or the underlying measurement or dynamic models may not be consistent with reality. Because the Kalman filter theoretically has "infinite memory", it is very important that erroneous measurements not be processed in the filter. This automated editing logic is easily included in a Kalman filter, and has proven to be very effective when processing real data for a number of different applications.

The Kalman filter measurement update equations naturally provide the basis for outlier detection. The EKF state measurement update may be written as

$$\hat{\mathbf{x}}_{i/i} = \hat{\mathbf{x}}_{i/i-1} + \mathbf{K}_i \tilde{\mathbf{z}}$$

where

$\tilde{\mathbf{z}} = \mathbf{z}_{meas} - \mathbf{h}_i(\hat{\mathbf{x}}_{i/i-1})$  is the measurement residual

$\mathbf{z}_{meas}$  is the measurement vector

$\mathbf{h}_i(\hat{\mathbf{x}}_{i/i-1})$  is the nonlinear measurement model equation: a function of the *a priori* filter state

$\mathbf{K}_i = \mathbf{P}_{i/i-1} \mathbf{H}_i^T (\mathbf{H}_i \mathbf{P}_{i/i-1} \mathbf{H}_i^T + \mathbf{R}_i)^{-1}$  is the Kalman gain matrix

$\mathbf{P}_{i/i-1}$  is the *a priori* state error covariance matrix

$\mathbf{R}_i$  is the measurement noise covariance matrix.

$\mathbf{H}_i = \left. \frac{\partial \mathbf{h}_i(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_{i/i-1}}$  is the measurement partial derivative matrix

The term  $\mathbf{H}_i \mathbf{P}_{i/i-1} \mathbf{H}_i^T + \mathbf{R}_i$  is easily shown to be the error covariance matrix of the measurement residual  $\tilde{\mathbf{z}}$ . If the filter models are accurate, the metric  $\tilde{\mathbf{z}}_i^T (\mathbf{H}_i \mathbf{P}_{i/i-1} \mathbf{H}_i^T + \mathbf{R}_i)^{-1} \tilde{\mathbf{z}}_i$  is a scalar Chi-squared distributed variable with degrees-of-freedom equal to the dimension of  $\mathbf{z}_{meas}$ . When the U-D filter is used, the measurements are sequentially processed as scalars, so  $\mathbf{z}_{meas}$  and  $\mathbf{H}_i \mathbf{P}_{i/i-1} \mathbf{H}_i^T + \mathbf{R}_i$  are also scalars. Hence, if  $|\tilde{z}_i| / \sqrt{\mathbf{H}_i \mathbf{P}_{i/i-1} \mathbf{H}_i^T + \mathbf{R}_i}$  is greater than perhaps 4 or 5, one may suspect that the measurement is an outlier and should be ignored. However if a consecutive sequence of measurements is rejected as outliers, one may suspect that the filter models have diverged from reality, and the filter state and covariance should be reset. (In this case, an analyst should examine the data more carefully to determine why divergence occurred.) This logic is implemented in the INR O&A software, and an example is presented later in this paper.

Another useful property of Kalman Filter residuals is “whiteness”. That is, the residuals will be uncorrelated in time if the filter model is correct. Patterns in residuals are a cause for concern. Usefulness of this property will be discussed later in the paper.

Other diagnostic information is also available in the INR O&A software. The difference between the onboard-computed and INR O&A-computed orbit can also be used to test statistical significance. Because the two estimates are independent, the metric

$$(\hat{\mathbf{o}}_{onboard\_i} - \hat{\mathbf{o}}_{filter\_i})^T (\mathbf{P}_{orbit\_onboard\_i} + \mathbf{P}_{orbit\_filter\_i})^{-1} (\hat{\mathbf{o}}_{onboard\_i} - \hat{\mathbf{o}}_{filter\_i})$$

should also be a Chi-squared distributed variable with 6 degrees-of-freedom if models are correct. Here

$\hat{\mathbf{o}}_{orbit\_onboard\_i}$  is the onboard-computed J2000 orbit position and velocity

$\hat{\mathbf{o}}_{orbit\_filter\_i}$  is the filter-computed J2000 orbit position and velocity

$\mathbf{P}_{orbit\_onboard\_i}$  is the error covariance of the onboard-computed orbit position and velocity

$\mathbf{P}_{orbit\_filter\_i}$  is the error covariance of the filter orbit position and velocity (partition of the full filter covariance).

Unfortunately information on  $\mathbf{P}_{orbit\_onboard\_i}$  will not be available, but GOES-R spacecraft requirements, converted to a variance, provide an upper bound on the diagonal elements of the covariance. When testing statistical significance, note that the mean of a Chi-squared distribution is equal to the degrees-of-freedom, with variance equal to 2 times the degrees-of-freedom.

Finally, we can difference the operational ABI INR filter estimate of the primary attitude states with the INR O&A attitude estimates. **If the state estimates are independent**, the metric

$$(\hat{\mathbf{a}}_{operational\_i} - \hat{\mathbf{a}}_{filter\_i})^T (\mathbf{P}_{a\_operational\_i} + \mathbf{P}_{a\_filter\_i})^{-1} (\hat{\mathbf{a}}_{operational\_i} - \hat{\mathbf{a}}_{filter\_i})$$

should be a Chi-squared distributed variable with 4 degrees-of-freedom if the models are correct and attitude is limited to the 4 primary variables. Here

$\hat{\mathbf{a}}_{operational\_i}$  is the ABI attitude as computed by the operational filter

$\hat{\mathbf{a}}_{filter\_i}$  is the INR O&A filter-computed ABI attitude

$\mathbf{P}_{a\_operational\_i}$  is the error covariance of the operationally-computed ABI attitude

$\mathbf{P}_{a\_filter\_i}$  is the error covariance of the INR O&A filter-computed ABI attitude.

Unfortunately the above equation is not exactly correct because both estimates include the same star measurements in the respective filters. The INR O&A estimate should be the more accurate because it also uses landmark measurements, but the error correlation between estimates cannot be easily quantified. The INR O&A solution will use many more landmark measurements than star measurements, but most will be infrared (IR) having a noise sigma about 16 times larger than that of the visible star measurements. Even the visible landmarks will have a noise sigma about 4 times larger than that of the visible stars, although the number of landmarks will be several times greater than the number of stars. For lack of better information, we simply use the operational Kalman error covariance to define the expected error of the difference, although this is probably larger than actual differences. Thus INR O&A uses the following approximation:

$$(\hat{\mathbf{a}}_{operational\_i} - \hat{\mathbf{a}}_{filter\_i})^T (\mathbf{P}_{a\_operational\_i})^{-1} (\hat{\mathbf{a}}_{operational\_i} - \hat{\mathbf{a}}_{filter\_i})$$

While these Chi-squared metrics are useful for testing statistical significance, the INR O&A workstation displays plots of measurement residuals and O&A differences as either un-normalized or normalized by the expected 3-sigma. That is, the normalized measurement residuals are computed as  $|\tilde{z}_i| / (3\sqrt{H_i P_{i/i-1} H_i^T + R_i})$ , the normalized orbit differences are computed as  $(\hat{\mathbf{o}}_{\text{onboard}_i} - \hat{\mathbf{o}}_{\text{filter}_i}) / (3\sqrt{\text{diag}(\mathbf{P}_{\text{orbit\_onboard}_i} + \mathbf{P}_{\text{orbit\_filter}_i})})$ , and the normalized attitude differences are computed as  $(\hat{\mathbf{a}}_{\text{PG}_i} - \hat{\mathbf{a}}_{\text{filter}_i}) / (3\sqrt{\text{diag}(\mathbf{P}_{\text{a\_PG}_i})})$ . Normalized residuals or differences should almost always be less than 1.0, so outliers are easily identified on the plots.

## LMC Computation

The INR O&A software also has requirements to compute Line-of-sight Motion Compensation (LMC) coefficient tables. The ABI is designed to automatically point the mirrors so that the detectors either observe target stars or specified swath regions on the earth. The detector arrays include sufficient NS overlap regions to compensate for normal ABI and spacecraft pointing error. However, to minimize the risk that the overlap regions may not be sufficient, the ABI uses a 7-day LMC table consisting of four LMC variables, labeled for convenience as roll, pitch, yaw and primary orthogonality. This table may be recomputed and uploaded each day, or the table may be reused if LMC values are constant.

INR O&A internally saves the filter-computed LMC angles as each star observation is processed. When commanded to compute LMC, the LMC history for the last several days is retrieved, the component of the Fixed-Grid LMC angles due to orbit deviation from the reference position is removed, LMC samples for the past days are interpolated to the desired LMC sample times, and averaged. The 1-day prediction is then repeated for another several days to fill the table, and the LMC file is made available for upload to the ABI.

## Simulation Results

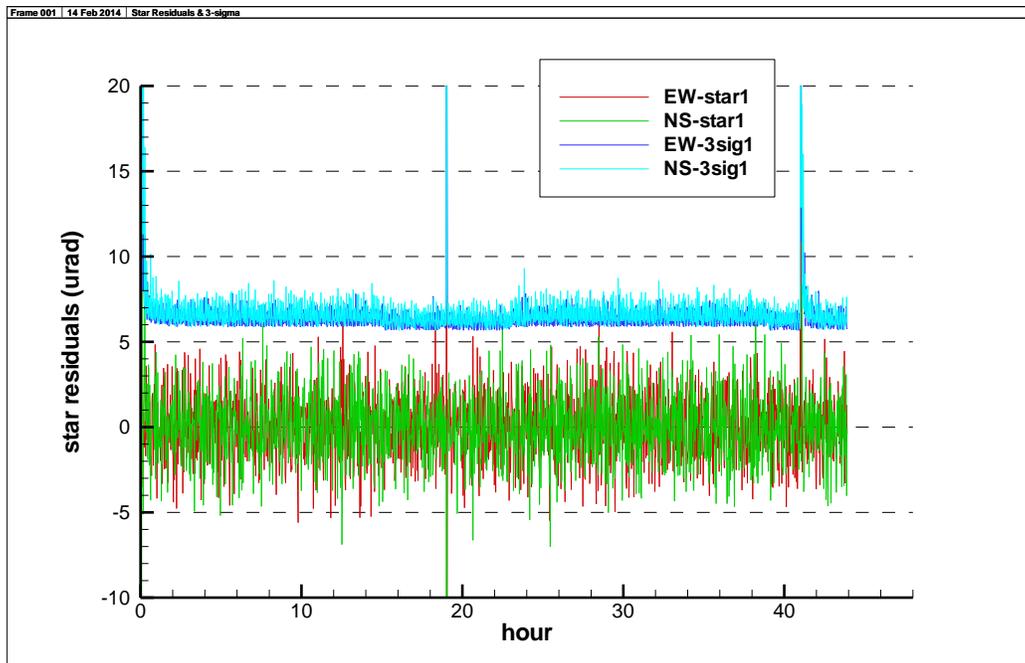
For INR O&A software development and testing purposes, a high-fidelity simulation was created. The simulation uses high-fidelity first-principles models of equal or higher accuracy than those used in INR O&A to produce ABI star and landmark measurements, GRB range measurements, 100 Hz spacecraft angular rate telemetry data, and 1 Hz spacecraft orbit and attitude telemetry data. Because no code is common to both the simulation and INR O&A, the simulated data can be treated as a nearly independent check on the INR O&A implementation.

The following plots show INR O&A performance for a simulated case centered near a winter solstice, where the simulated measurements contain Gaussian random measurement noise of magnitude matching that of the noise model in INR O&A filter. The simulated scenario includes

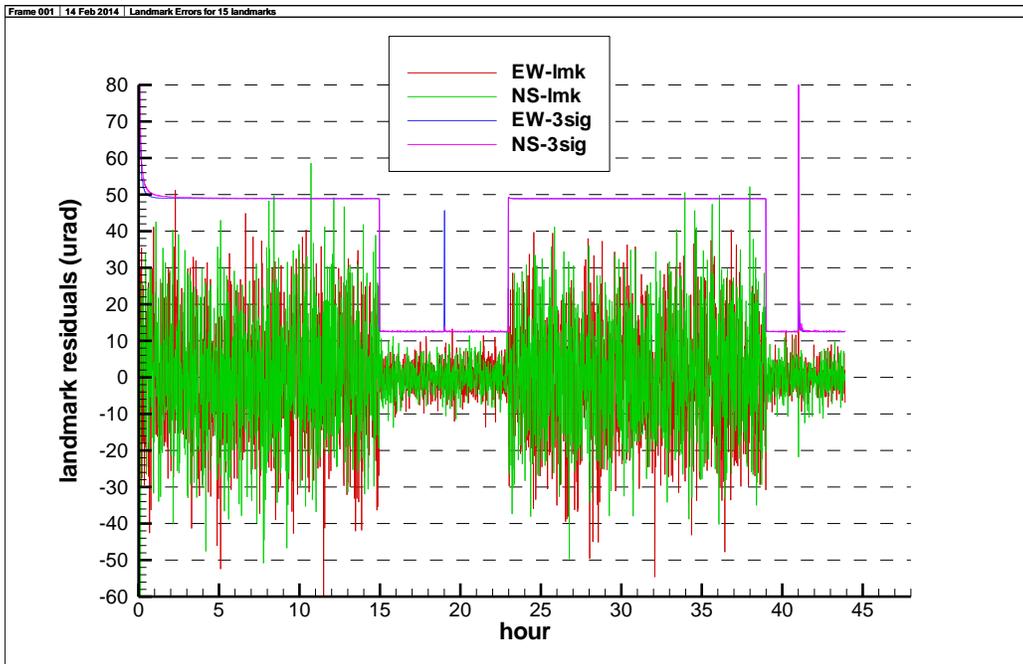
- a north-south station-keeping maneuver 24 hours after the beginning of the simulation,
- a yaw-flip maneuver 41 hours after simulation start,
- a 30 second gap in 100 Hz angular rate data introduced 19 hours after simulation start,
- a 10 minute step jump of +30 meters added to the range measurements starting 26 hours after simulation start.

The data gap is used to validate processing logic that resets the first three ABI attitude states when a gap in angular rate data appears. The jump in range measurements is used to validate logic that detects and edits outlying measurements.

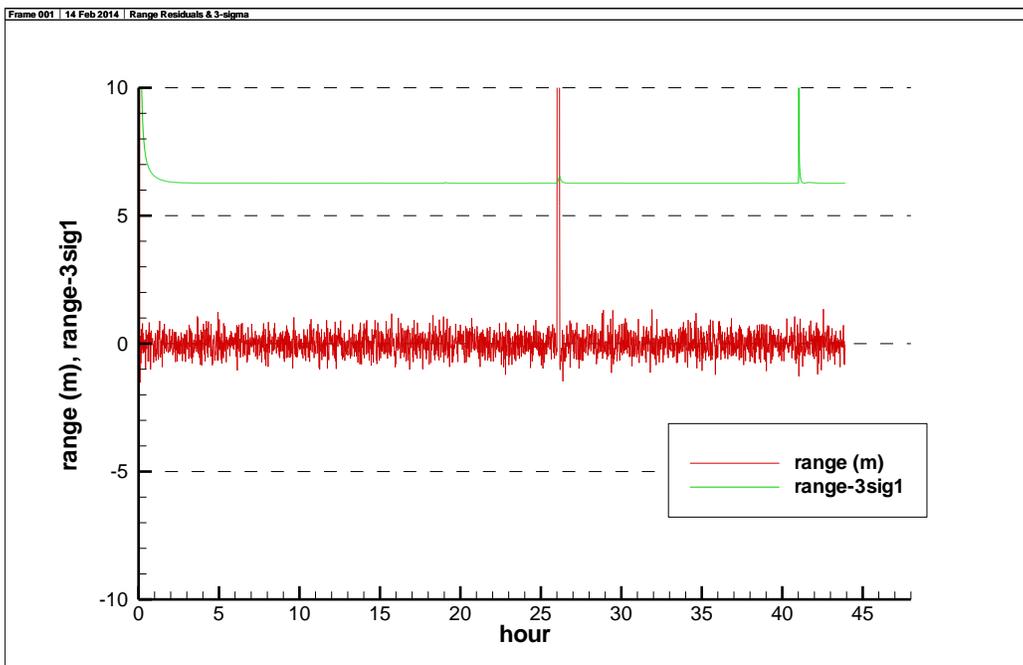
Figures 3 to 5 show the INR O&A filter star, landmark and range residuals and 3-sigma residual uncertainty. As expected, nearly all measurement residuals are smaller than the 3-sigma uncertainty; this includes the NSSK maneuver at 24 hours and the yaw-flip maneuver at 41 hours. The smaller landmark residuals from 15-23 hours and after 39 hours correspond to local daylight, when visible landmarks are available. Other night-time intervals use only IR landmarks. The reset of roll-pitch-yaw states (due to the gap in angular rate data) at 19 hours is visible in the star and landmark residuals, and the 10 minute jump in ranges at 26 hours is apparent in the range residuals.



**Figure 3: Visible Star Residuals and 3-sigma Uncertainty**

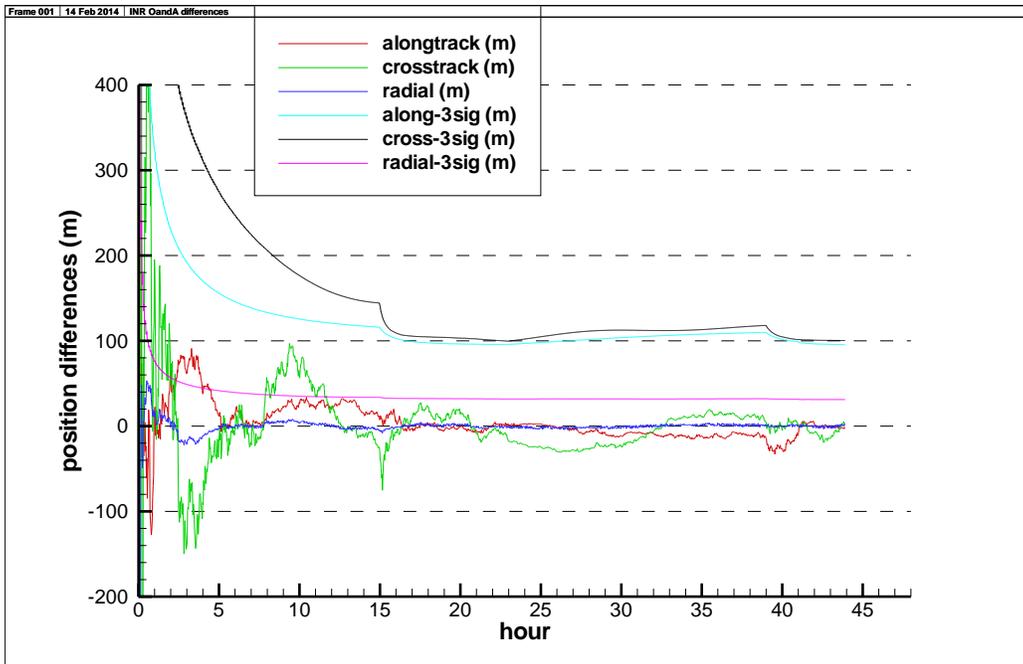


**Figure 4: Landmark Residuals and 3-sigma Uncertainty**

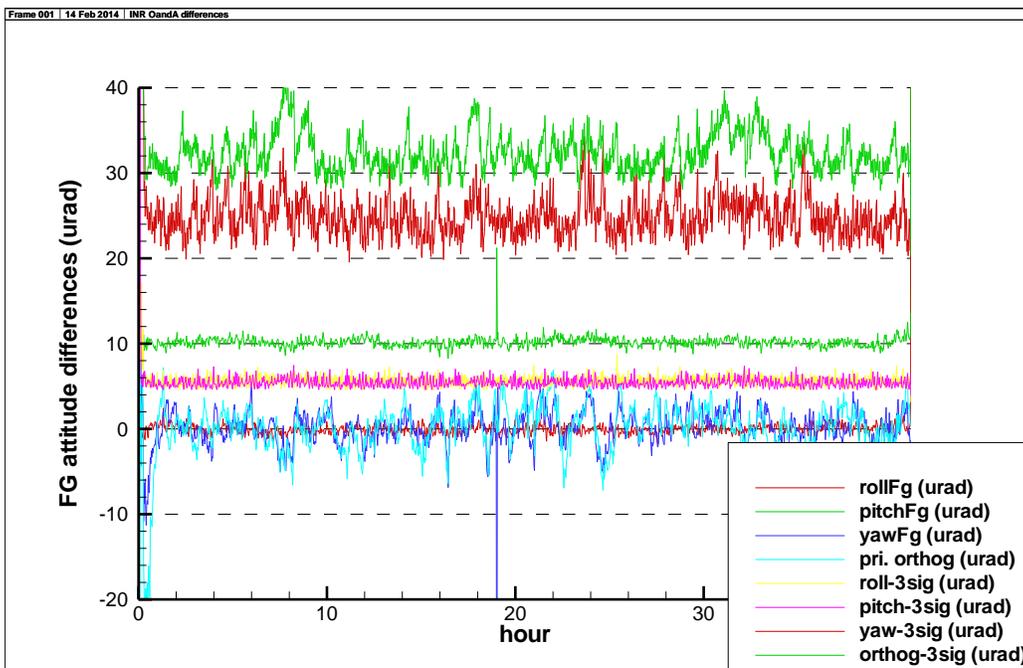


**Figure 5: Range Residuals and 3-sigma Uncertainty**

Figures 6 and 7 respectively show the orbit and Fixed-Grid attitude differences and expected 3-sigma uncertainty. Again the orbit and attitude differences are always much smaller than the 3-sigma uncertainty. The +10 microradian bias in the pitch difference (Figure 7) is caused by the difference in handling of the planetary aberration effect between the two filters.



**Figure 6: Orbit Differences and 3-sigma Uncertainty**



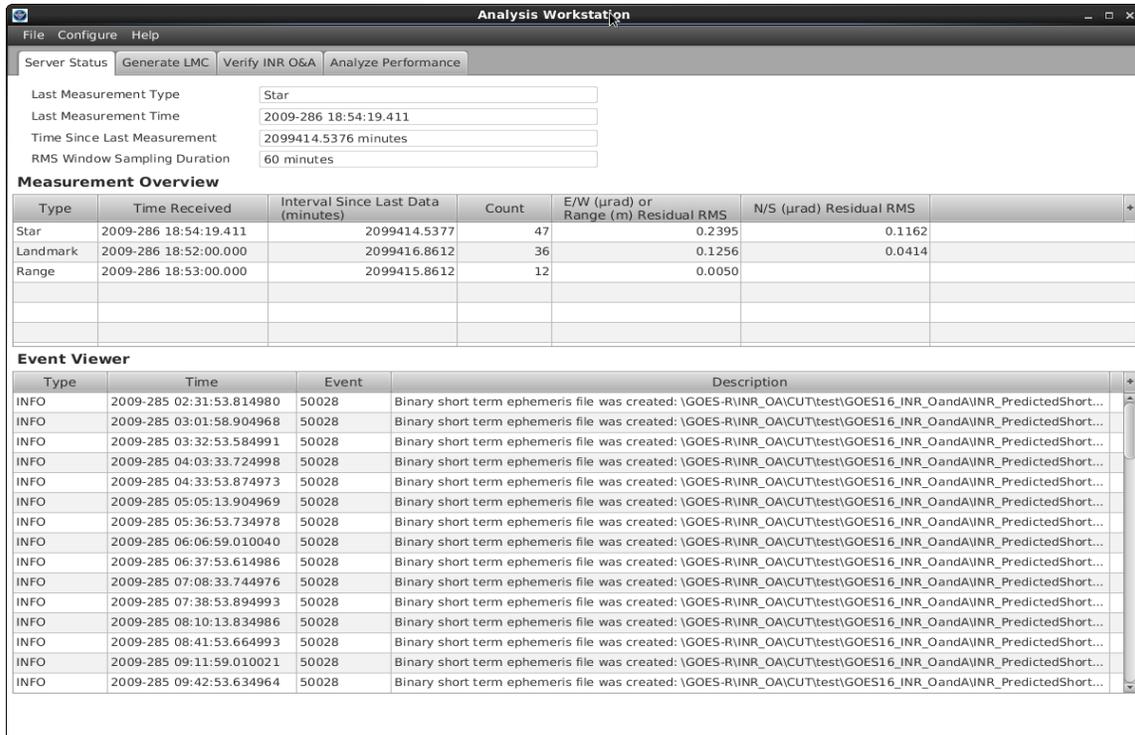
**Figure 7: Fixed-Grid ABI Attitude Differences and 3-sigma Uncertainty**

### INR O&A Workstation

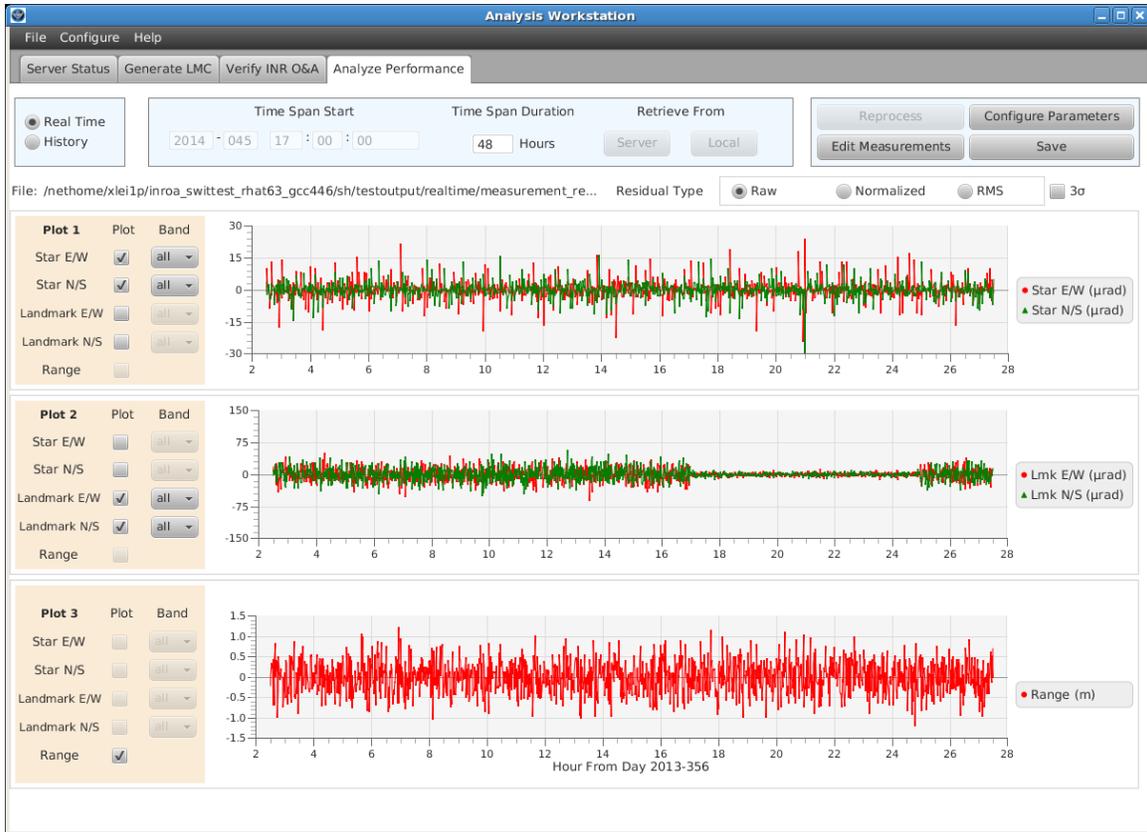
The INR O&A Kalman filter application is intended to continuously operate (while the ABI is functioning) without interruption for the spacecraft life-of-mission. A Backup INR O&A processes the same data as the Primary, and can be made Primary if the original Primary fails. The Primary writes files to mission life storage and sends messages to other processes, but does

not have direct interactive capability. Rather, a separate INR O&A Workstation application reads the filter output disk files, and provides analyst capability to display measurement residuals, display O&A differences, interactively edit outlying measurements, change local configuration processing parameters, locally reprocess measurements, and locally generate LMC sets. These capabilities not only provide performance monitoring, but also provide insight about INR problems.

Figure 8 shows the main workstation display. Information on last received measurements, residual statistics over a sliding window, and event messages are updated nominally every 5 minutes. Figure 9 shows the filter residual plot display page. The analyst can select various combinations of star, landmark, and range residuals, either raw, normalized or RMS. The displayed data span can be either near real-time or retrieved from solution history. The analyst has options to change local filter configuration, edit measurements, and reprocess measurements in the filter.

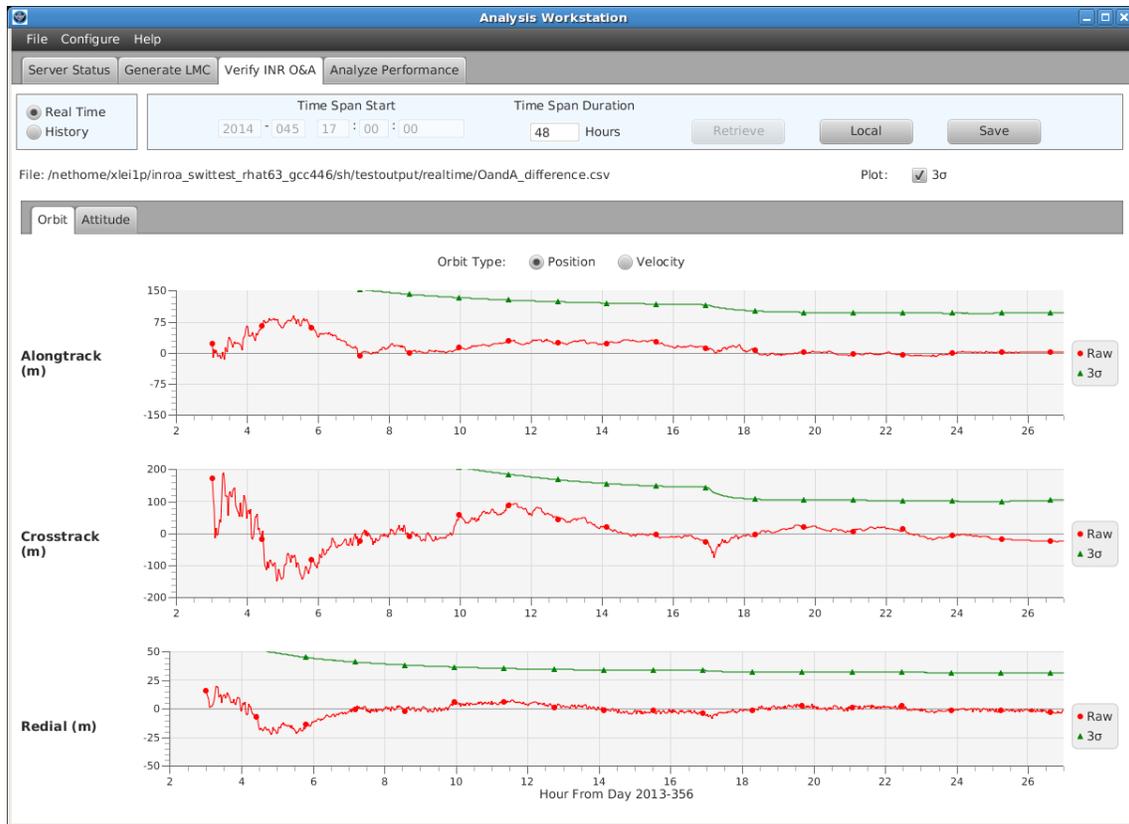


**Figure 8: INR O&A Workstation Server Status Display**



**Figure 9: INR O&A Workstation Filter Residual Display**

Figure 10 displays differences between onboard orbit and INR O&A-estimated orbit position, and 3-sigma uncertainty. The same page also allows display of attitude differences. Figure 11 displays the LMC variables over the last 7 days, which are averaged to compute LMC for the next 24 hours.



**Figure 10: INR O&A Workstation Orbit Difference Display**



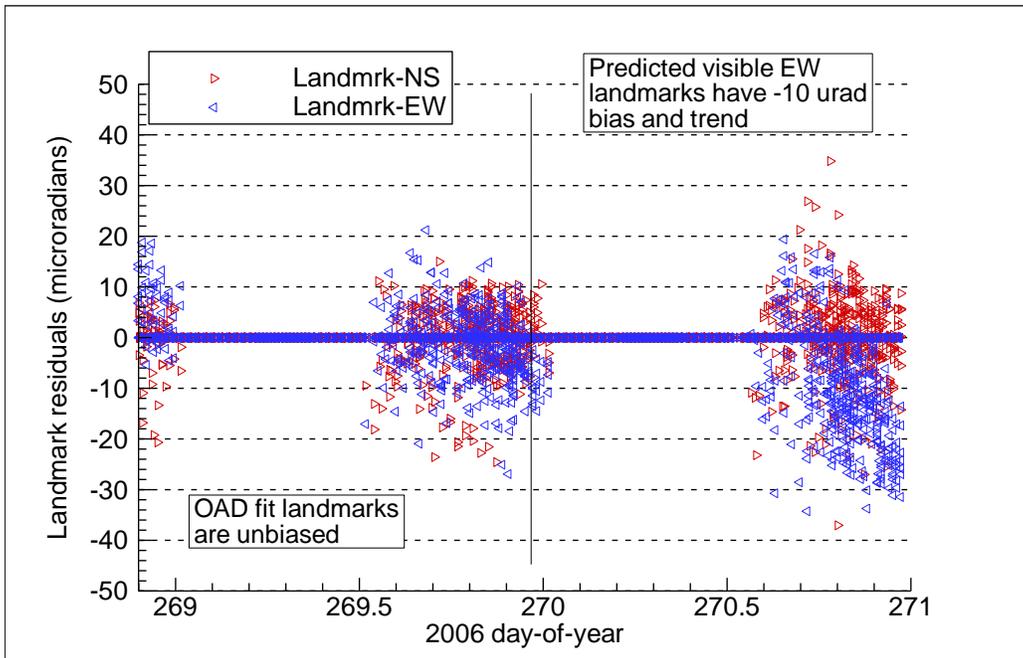
**Figure 11: INR O&A Workstation LMC Generation Display**

## Anomaly Resolution in Post-Launch Testing

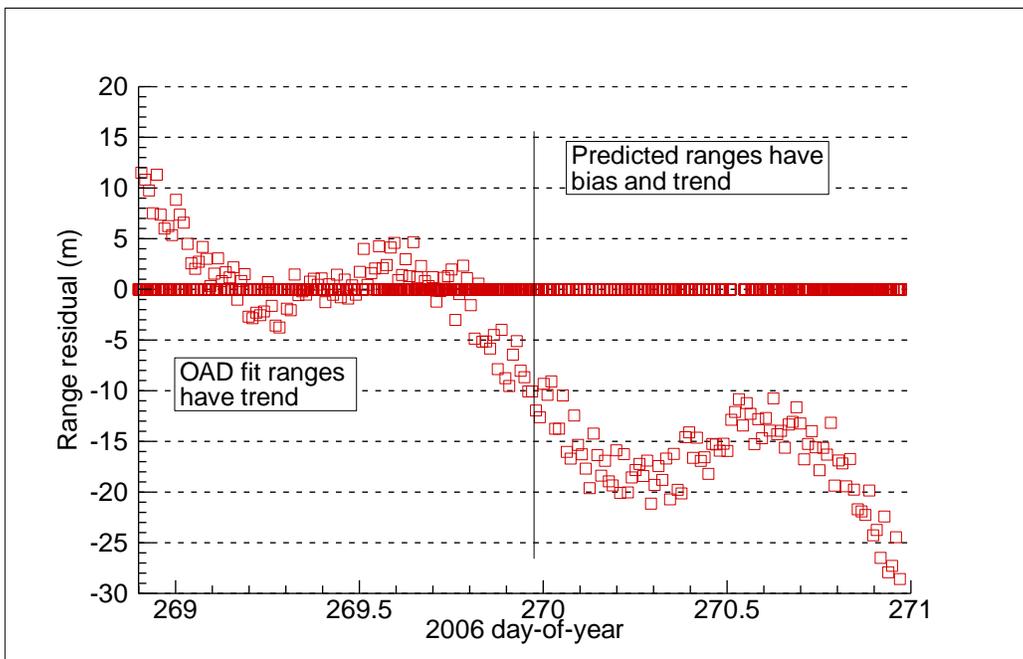
The ground and flight components of GOES systems are technologically sophisticated and complex. The first-launched satellites of both the GOES I-M and NOP series have had significant INR problems, and INR problems also occurred with some later launches. The problems included incorrect database parameters, unknown systematic errors, difficulty in modeling maneuvers, errors in modeling instrument behavior, and algorithm approximations.

The GOES-R series is significantly more complex than the previous series, and utilizes new technology not previously used in this application. While much has been learned from previous GOES series, it will not be surprising if INR problems appear in early GOES-R images. Thus the INR O&A analysis capability is expected to be very useful during GOES-R Post Launch Testing (PLT).

To understand how filter residuals and O&A differences can provide insight on INR problems, we briefly describe how they were used for GOES-13. Recall that the GOES NOP system processes the last day's star, landmark and range measurements to compute O&A parameters that are used onboard to make INR corrections for the next 24 hours. During GOES-13 PLT, growing trends were observed between measured and predicted landmark and range measurements, although GOES-13 met INR requirements. Figure 12 is a sample of GOES-13 landmark residuals during the least-squares fit period and during the predicted day. Figure 13 is a similar plot for range residuals. Strong patterns in the residuals suggested systematic errors in the processing, but initial attempts to discover the problem source were unsuccessful. The range residuals of Figure 13 could have been caused by a range bias or a star-landmark EW bias, but the apparent bias changed with spacecraft yaw-flip and longitude. This would not happen solely with range bias. Analysis over about 8 months identified several problems, but none could explain the largest patterns in range residuals. This issue was partially resolved three years later when NASA used dedicated ground tracking during GOES-14 PLT. The independent orbit solution confirmed that the primary problem was due to a range bias, which was eventually traced as an incorrect satellite transponder delay in ground software. However, the fundamental cause of some GOES-13 behavior is still not fully understood. Further details of the GOES-13 investigations are provided in other references<sup>26, 27, 28</sup>.



**Figure 12: OAD Fit/Prediction Visible Landmark Residuals for Upright GOES-13**



**Figure 13: OAD Fit/Prediction Range Residuals for Upright GOES-13**

The GOES-13 analysis described above was primarily based on measurement residuals from a batch least squares O&A solution, with predicted residuals computed from the O&A solution. The GOES-R INR O&A system uses a Kalman filter rather than batch least squares. A Kalman filter updates the state estimate as each measurement is processed, and uses the update to predict the next measurement residuals, which are then used to update the state, etc. Hence, Kalman filter measurement residuals will behave quite differently than the residuals of Figures 12 and 13.

When Kalman filter models are a reasonably accurate description of the real system, the measurement residuals will be white (uncorrelated). Thus systematic patterns in residuals suggest modeling error. When patterns appear, the analyst must determine what type of modeling error can cause the observed pattern. This pattern matching problem can either be solved by trial and error introduction of errors in simulated measurements, or covariance error analysis tools can be used. Further information on the options may be found in Gibbs<sup>29</sup>.

## Summary

Operational GOES-R Product Generation INR computes ABI attitude parameters using only ABI star measurements and high-rate spacecraft angular rate telemetry. Orbit information is obtained onboard the satellite using GPS signals. The Kalman filter in the GOES-R INR O&A and LMC Support system uses ABI star and landmark measurements and ground-spacecraft range measurements to compute a coupled orbit and ABI attitude solution that is partially independent of the operational INR. This redundant solution provides:

1. A solution for spacecraft orbit that is independent of the GPS-derived onboard orbit solution. The accuracy is expected to be comparable to that of the onboard solution, so it can be used to validate the onboard solution, or if necessary, it can be used as a backup for operational INR purposes.
2. A solution for ABI attitude that is based on both star and landmark measurements, so it should be more accurate than the operational star-based attitude. The INR O&A attitude can be used to validate the operational attitude.
3. Insight on anomalies based on differences between orbit solutions or attitude solutions.
4. The coupled orbit and attitude solution using three data types provides some observational redundancy that will allow insight into operational problems caused by anomalous behavior of components. Furthermore, measurement residuals of the Kalman filter have well-defined statistical characteristics determined from the underlying models. Discrepancies between actual and expected residuals provide much insight on problem sources.
5. An interactive graphical workstation application allowing what-if analyses.
6. A 7-day predicted ephemeris needed by the ground ABI Star Selection process.
7. Computation of a Line-of-sight Motion Compensation (LMC) coefficient table used by the ABI to minimize pointing errors.

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