LOW-ENERGY TRANSFER TO MULTI-CIRCLE ELLIPTIC HALO ORBIT IN EARTH-MOON ELLIPTIC RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

Since Farquhar firstly proposed halo orbit, it has been well studied both as a separatrix in phase space theoretically and as a perfect observation orbit practically. Recently, Parker and Anderson comprehensively researched the transfer to lunar halos in the circular restricted three-body problem (CRTBP)[1]. They claimed that the most substantial deviation of CRTBP is the nonzero eccentricity. So the motion of the Earth-Moon-spacecraft can be better described by elliptic restricted three-body problem (ERTBP) with $e \approx 0.0554$, where primaries revolve each other on elliptic orbits. However, the equations of motion in ERTBP implicitly depends on time (or true anomaly f), which makes it a non-autonomous system and therefore invalidates the concepts and methods developed based on CRTBP. For instance, the traditional halo orbits diverge quickly in ERTBP as shown in Fig.1. In our another paper[2], the stability of multi-circle elliptic halo (ME-Halo) orbits with various parameters in ERTBP is studied. The eigenvalues of monodromy matrix of the period orbit bifurcate when e becomes nonzero and there comes distinguished properties for ME-Halos. Since they are directly developed by using more nature dynamics in ERTBP, quick divergence caused by nonzero eccentricity is successfully suppressed. If we design an observation or communication mission along a ME-Halo, we can expect it to require orbit correction maneuver for longer intervals, and saving lots of fuel. Therefore we construct low-energy transfer orbit to such a ME-Halo in this paper, about which we find no study yet.

In this paper, we principally investigate the impulsive transfer to lunar ME-halo at L_1 point with $M=5, N=2$ and at L_2 point with $M=2, N=1$, where M and N are the revolution circles of primaries and third body respectively. Firstly we generate the lunar halo families at libration point by continuation. Then we choose halo orbits with period $T_E = M \cdot T_C = N \cdot 2\pi$, which can be continued to lunar ERTBP. After obtaining desired lunar ME-Halo orbit, we follow Parker and Anderson's primary scenario for low-energy transfer study in CRTBP. From different points identified by $\tau \in (0,1)$ along the ME-Halo, we generate time-dependent stable invariant manifolds approaching the Earth. Then we choose the perigee of the manifold as injection point P and use two impulsive maneuver ΔV_{LEO} and ΔV_{MI} to connect the parking orbit around the Earth and the manifold. We construct the perigee transfer orbits and then free the manifold injection point, seeking for a fuel-optimal transfer. The techniques in this study can be extended to other situations, but may be different as the properties of ME-Halos change with parameters.

In spite of the methodological resemblance, actually there are some significant different properties as listed here,

1. Manifolds are dependent on anomaly f now, therefore we need not only to target the injection point, but also to consider the phase angular of the Moon.
2. For certain parameters, like $M=2, N=5$ at lunar L_1, there comes two pairs of real reciprocal eigenvalues, which indicates that there are two stable/unstable directions associated with each point τ on ME-Halo. They span a two dimensional invariant manifold, as illustrated in Fig.2. This redundant dimension makes great arbitrariness and difficulties to designing.
3. Even for the same parameters, the situation varies depending on Halo's position among the whole family it belongs.
4. The coordinate frame is pulsating. Perigees should be detected with caution because we need to consider the pulsation of length unit. ΔV is actually $\mathrm{d}x/\mathrm{d}f$ and we must convert to $\mathrm{d}x/\mathrm{d}t$ before comparison.

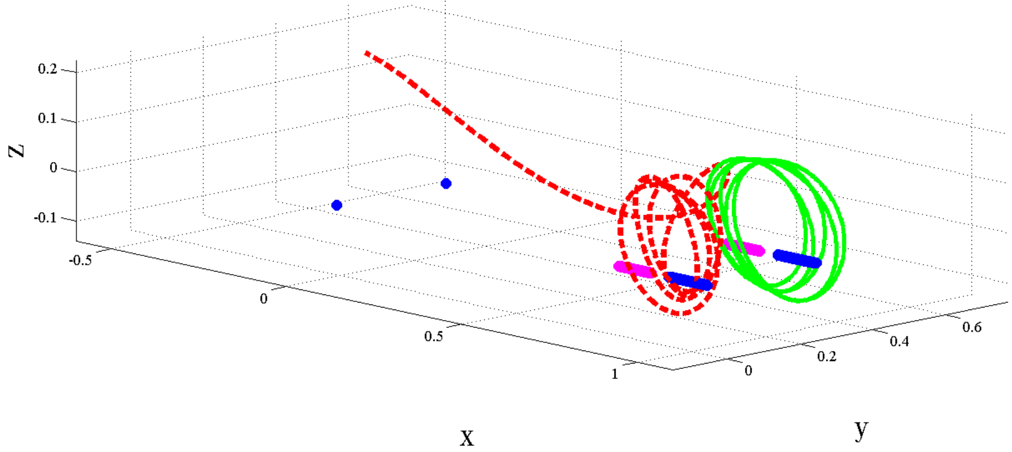

Figure In ERTBP, traditional halo (dashed red) diverges quickly, while ME-Halo (solid green) closed up as a periodic orbit. The orbit is plotted in synodic non-pulsating frame, and for clarity we displace ME-Halo a little. The magenta and blue lines is the pulsating traces of libration point and primaries. The blue dot indicates the Earth is almost fixed.

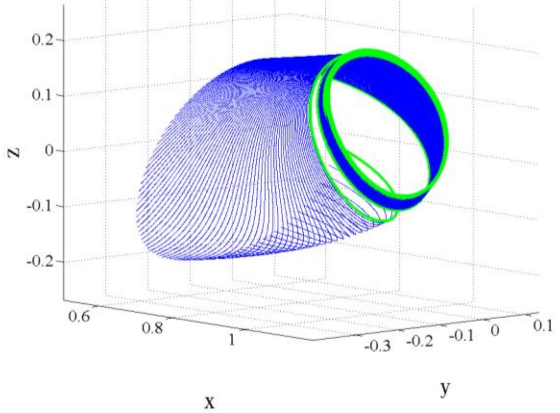

Figure These are manifolds generated from point $\tau=0$ on ME-Halo with $M=5, N=2$ by backward propagating until $x=0.7$. The orbit has two real eigenvalues smaller than 1, thus it has two stable directions spanning a two-dimensional stable surface. The blue (thin) manifolds fill up the gap between two green (thick) boundaries. Longer integration will cause self-intersection and the situation become complicated and obscure.

Reference

[1] J. S. Parker and R. L. Anderson, "*Low-Energy Lunar Trajectory Design*", July. 2013

[2] H. Peng and S. Xu, "*Numerical stability study of multi-circle elliptic halo orbit in the elliptic restricted three-body problem*", in preparing for *ISSFD*, 2014.