Dynamical Modelling for Flat-Spin Recovery Applications
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The recovery from a flat-spin motion is a remarkable practical application of spinning-satellite dynamics.1 2 Flat-spin refers to a satellite that initially spins about its minimum axis of inertia but under energy-dissipation effects, ends up spinning about its maximum axis of inertia (since this is the minimum-energy state). A flat-spin recovery re-establishes the intended minimum-inertia spin.

If a body-fixed torque acts about a principal axis of inertia there exists a linear combination of energy \(E\) and angular momentum-squared \(H\) that is a first integral of motion. For a torque about the minimum axis of inertia axis \((x)\), this integral is denoted by \(\Delta E_{\text{max}}\), which is the deficit between the current value of the energy \(E(t)\) and its maximum possible value \(E_{\text{max}}\) for a given value of \(H(t)\):

\[
\Delta E_{\text{max}}(t) = \frac{H^2(t)}{2A} - E(t) - \frac{BC}{2A} \left\{ k_x \omega_x^2(t) + k_z \omega_z^2(t) \right\} \geq 0 \tag{1}
\]

\(A < B < C\) are the moments of inertia along \(x, y, z\) and \(\omega_j\) \((j = 1, 2, 3)\) are the rates about \(x, y, z\), and:

\[
k_x = (C - B) / A; \quad k_z = (C - A) / B; \quad k_y = (B - A) / C \tag{2a-c}
\]

Eq. (1) shows that \(\omega_x, \omega_z\) move on an ellipse with varying semi-major and minor-axes \(a(t)\) and \(b(t)\):

\[
\omega_x(t) = a(t) \sin \phi(t); \quad \omega_z(t) = b(t) \cos \phi(t) \tag{3}
\]

with:

\[
a(t) = \frac{2A \Delta E_{\text{max}}}{BC k_x} > b(t) = \frac{2A \Delta E_{\text{max}}}{BC k_2} \Rightarrow b = \kappa a \quad \text{with:} \quad \kappa = \sqrt{\frac{k_1}{k_2}} \tag{4a-d}
\]

The angle \(\phi(t)\) in Eq. (3) is the eccentric anomaly of the osculating ellipse.

Finally, we propose a new formulation using the variables \(a(t)\) and \(\phi(t)\) instead of \(\omega_x\) and \(\omega_z\):

\[
\dot{\omega}_x(t) = m_x - \kappa k_x a^2 \sin \phi \cos \phi
\]

\[
\dot{\phi} = k_x \omega_x + \left\{ m_x \cos \phi - (m_x / \kappa) \sin \phi \right\} / a \quad \text{with:} \quad k_x = \sqrt{k_1 k_2} \tag{5a-d}
\]

After the recovery is achieved, the spin rate \(\dot{\omega}_x(t)\) oscillates about a straight line, see Fig. 1. Eq. (5b) leads to compact analytical solutions in terms of Fresnel Integrals. Their known asymptotic properties enable us to predict the asymptotic values of \(\Delta E_{\text{max}}\) and the nutation angle. Fig. 2 shows the asymptotic results for \(\Delta E_{\text{max}}\) for a torque with a fixed \(T_1 = 10\) Nm and a range of \(T_2\) values.

Fig. 1. Example of Fast Flat-Spin Recovery.
Fig. 2. \(\Delta E_{\text{max}}(T_2)\) with Fixed \(T_1 = 20\) Nm.

References