**Assessment of the Gaussian Covariance Approximation over an Earth-Asteroid Encounter Period**

Daniel W. Mattern[1]

[1]*Omitron,Inc., Beltsville, MD, 20705;* [1]*NASA GSFC, Flight Dynamics Facilty, Greenbelt, MD, 20771*

**Keyword:** Covariance Analysis, Planetary Defense, Astrodynamics

Previous analysis examined methods of determining the statistical likelihood of an asteroid impact with the Earth. This work focused on assessing the efficacy of deflecting the hypothetical asteroid, 2015 PDC. To do so, different deflection velocities and representative covariance matrices were applied to the asteroid at different locations along its trajectory, and the covariance matrix was then propagated in time – using a state transition matrix (STM) – throughout the Earth encounter period. This propagated covariance was then used to compute a Mahalanobis distance which provided the minimum position standard deviation that the Earth's surface "contacted" over the encounter period [1].

Propagating a covariance matrix via a STM and the subsequent calculation of the Mahalanobis distance, however, both require an analytical, Gaussian approximation of the asteroid's covariance matrix. In other words, the uncertainty surrounding the 6-parameter state must be represented by a 6x6 matrix in Cartesian space. If one accounts for position and velocity uncertainty in the vis-viva equation, solves for the position uncertainty as a function of the velocity uncertainty, and accounts for the change in position error as a function of time, $\Delta r(t) \propto \Delta v * t$, it can be seen that the growth of the position uncertainty is related both to the initial velocity and the initial velocity error (see Figure 1, below).

$$(v+\Delta v)^2 = \mu\left[\frac{2}{(r+\Delta r)} - \frac{1}{a}\right] \rightarrow \quad \Delta r(t) = \frac{2a\mu}{\mu + a[v^2 + 2v(\Delta v * t) + (\Delta v * t)^2]} - r$$

For planetary defense, an asteroid's state often needs to be predicted for many years, sometimes decades. Depending on where in the asteroid's orbit the prediction begins and the initial velocity uncertainty, this long-term prediction can cause the position uncertainty to grow to scales many times larger than the Earth's radii, sometimes larger than the Moon's orbit radius. For potentially hazardous asteroids, propagating large uncertainty distributions through an "encounter period" with the Earth presents a significant gravitational gradient as the distribution passes the Earth-Moon system. This gradient, in turn, challenges the assumption that the uncertainty distribution can be approximated as Gaussian in Cartesian space.

This paper aims to examine the strength of the analytical covariance assumption over the Earth-encounter period. To do so, the covariance matrix will be propagated via STM to where the uncertainty distribution's leading edge encounters the Earth's sphere of influence: approximately 1.5 million km. At this point, the covariance matrix will be sampled using a Monte Carlo approach. Next, both the Monte Carlo samples and the analytical covariance will be propagated through the Earth encounter period.

In this scenario, the Monte Carlo sampled covariance is considered to represent the "true" evolution of the asteroid's state uncertainty. As such, to evaluate the validity of the analytical approximation, a volume will be applied to each Monte Carlo sample based on the number of samples used and the covariance matrix's determinant. Next, those samples will be integrated over the analytical covariance to identify the percent volume subsumed by the Monte Carlo samples. Repeating this methodology over a number of nominal miss distances and a number of uncertainty scales, this study aims to identify a bounds on where the Gaussian covariance matrix approximation holds.

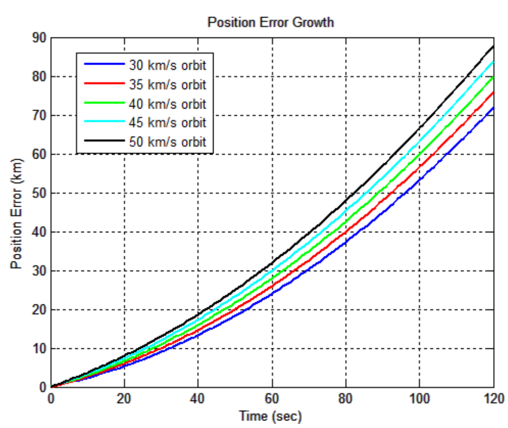


Fig 1. Position error growth as a function of initial velocity error

## References

[1] Mattern, D., *Deflection of an Earth-bound Asteroid using a Gravity Assisted Kinetic Impactor*, AIAA/AAS Astrodynamics Specialist Conference, SPACE Conferences and Exposition, **AIAA 2016-5377** (2016)