Optimal Trajectory Design of Formation Flying based on Attractive Sets

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The Linear Quadratic Regulator (LQR) theory is often used for optimal trajectory design of leader-follower formation flying [1]. Once the initial and final states are specified, the optimal trajectory can be obtained based on the LQR theory. However, by this method, it is necessary to repeat the calculation to find the optimal initial state.

We present a new method of optimal trajectory design for leader-follower formation flying in low-Earth orbit. Assuming the equations of motion is described by Hill's linearized equations, and using a linear quadratic performance index, we can derive based on the LQR theory the attractive set that is a set of all initial states to reach desired state with a given cost. Using the attractive set for optimal trajectory design, we can obtain optimal initial condition uniquely.

As a preliminary analysis, a fixed final-state finite-time problem and an infinite-time problem are formulated, and attractive sets are investigated. The results reveal the relation of the optimal control solutions obtained for different boundary conditions. Then the attractive set is applied to find the solution of the optimal rendezvous problem. Figure 1 shows the attractive sets and controlled trajectories for the infinite-time problem and for the fixed final-state finite-time problem. The black, blue and red lines represent initial orbits, contour of the attractive sets, and the controlled trajectories starting from x-axis, respectively. Table1 shows the summary of results and it is found that the rendezvous is completed in shorter time for the finite-time, fixed final-state problem although ΔV is as small as that of the infinite problem. It is interesting to note that the contours of attractive set tend to overlap with periodic orbits of Hill's equation when the weight of control input becomes large. This result suggests that the amount of cost needed to achieve rendezvous does not depend on initial state on the periodic orbit. Thus our method can provide new insights into trajectory design of formation flying.

Table 1. The terminal time and ΔV between the case the terminal time is infinite and finite.

Condition of the terminal time	Infinite	Finite-time, fixed-state
Rendezvous completed time [s]	59,697	10,000
ΔV[m/s]	35.995	36.315

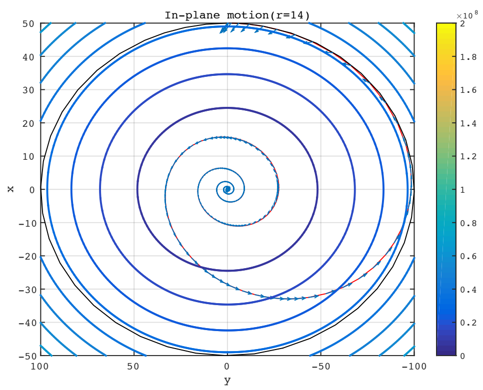

Fig.1(a). Infinite-time problem.

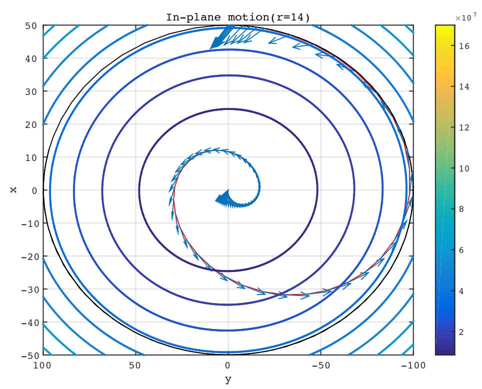

Fig.1(b). Fixed final-state, finite-time

Fig.1. Attractive set for different boundary conditions of the terminal time.

References

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