Approximate Two-Point Boundary Value Problem Solutions to Low Thrust Trajectory by Superposition

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In celestial mechanics the ballistic two-point boundary value problem, i.e. the Lambert Problem, can be solved analytically for a given transfer time, initial and final positions. The solutions to the Lambert problem are often expressed as the initial and final velocities. When generalizing the problem to low-thrust trajectories, one needs to search a range of feasible velocities of instead of discrete point solutions with computationally intensive approaches such as grid searches.

Previously a linearized method has been suggested to quickly compute the ranges of feasible velocities for low-thrust trajectories [1]. However such method fails in long duration orbits. In this paper, we have developed a new method to approximate the feasible velocity ranges of a low-thrust trajectory with complex thrust profile using solutions of trajectories with simple thrust profile. Firstly the problem is formulated as follow: (1) The thrust profile $\vec{u}(t)$ is parametrized to a sum of constant pulse function, indicating the thrust direction on each segment of the trajectory. (2) The pulse on each segment can be turn on or off. Then the feasible velocity ranges can be spanned via grid search of all combination of thrust direction and magnitude.

However such method is very time consuming. To reduce the number of computations, we explore patterns in the solutions generated from grid search - the feasible range of trajectories with multiple thrusted segments can be approximated by sum of the feasible ranges of trajectories with single thrusted segment, i.e. a 'superposition' of solutions to simpler thrust profile yields the solution of more complex thrust profile. This approximation scheme has been applied to the test cases mentioned in Ref. [1], and successfully reproduces the feasible range in long orbit predicted by grid search. The analytical basis of this method will also be discussed in the full paper.

Solving the two-point boundary value problem of low-thrust trajectories efficiently can be very useful for the design of future low-thrust missions. Mapping the final velocity range of the current orbit with initial velocity range of the next orbit allows faster and more accurate prediction during a large scale tree search of multi-leg trajectories and can result in better solutions consuming less propellant. With the succeed of this approximation method, we can reduce number of computation from order $O(N^k)$ to $O(kN)$ and thus greatly improve the efficiency of low-thrust trajectory design.

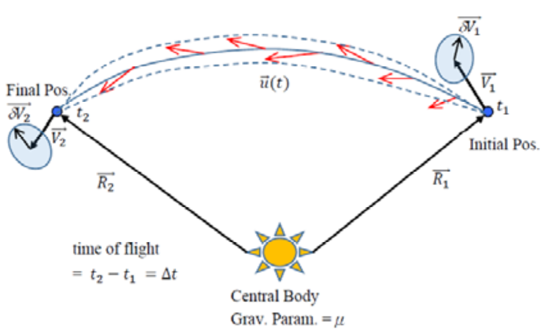

Fig.1 The low thrust Lambert problem. The blue regions represent the feasible sets of initial and final velocities.

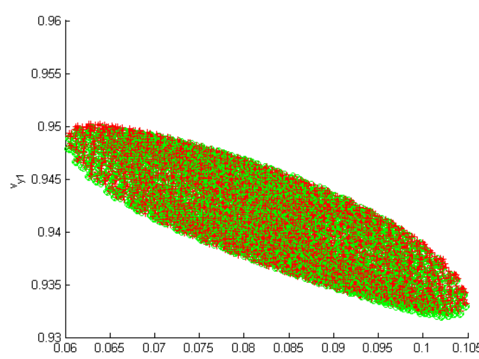

Figure 2: Example feasible range searched by grid search (green) and by superposition method (red).

References

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