An Analytical Solution for Multi-revolution Transfer Trajectory with Periodic Thrust and Non-Singular Elements

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Transfer trajectories from low to high altitude orbits include many revolutions, thus take longer, with low-thrust propulsion systems. The amount of acceleration provided by the propulsion system is small enough to be regarded as a perturbation. The differential equations expressing the trajectory dynamics can be formulated as differential algebraic equations, one form is to be known as the planetary equations of Gauss's form [1]. Solving those equations with full numerical method has a substantial computational cost because of dividing one revolution into tiny arcs of integration. To reduce computational cost, analytical formulae have been developed previously by many researchers. However, these formulae are restricted to particular cases or low-flexibility thrust control models. In order to achieve the desired flexibility, thrust control can be assumed as Fourier series, whose period synchronizes one orbit period. Additionally, by averaging it over the course of one revolution, high frequency terms were found to be reduced to a finite number of coefficients of low-frequency term by the orthogonal condition. These finite number of coefficients are called Thrust Fourier Coefficients (TFCs) [2]. Consequently, the averaged planetary equations with TFCs are analytically formulated, and its variations are on the track of secular variations with full numerically method. This provides a powerful method to reduce computational cost while bring flexibility for thrust control. However, the classical orbital elements adopted in this theory are susceptible to suffer from uncertainties. Such uncertainties are caused due to right ascension of ascending node (Ω) and argument of perigee (ω) values, when a trajectory is close to a circular orbit ($e \sim 0°$) or inclined by a certain angle ($i \sim 0$ or $180°$). Therefore, the classical orbital elements are not sufficient to analyse various spiralling transfers, e.g. orbital plane change maneuvers for piggyback satellites, or LEO to GEO transfers for all-electric propulsion satellites.

This research offers a new analytical method to overcome above-mentioned issue. The new method avoids uncertainties by employing non-singular, so-called the equinoctial elements in the planetary equations [1]. The relevant formulation for which is provided in [1]. Here, TFCs are substituted in thrust model in perturbation terms of the planetary equations, and averaged out for each revolution to reformulate it with equinoctial elements analytically. This new analytical approach then chases only secular variations of exact solutions, thus can be extended to more general manoeuvre cases. Fig. 1 and 2 show a simple perigee-kick case as an example. In Fig. 1 the thrust profile is seen: the blue line depicts the acceleration with higher Fourier coefficients and the red curve depicts the same with only TFCs. Fig. 2 shows a comparison of the full numerical method and the analytical method for 10 revolutions. In the latter, the computational cost is observed to be reduced down to about 1/1000 of the numerically method. More complicated cases are also investigated.

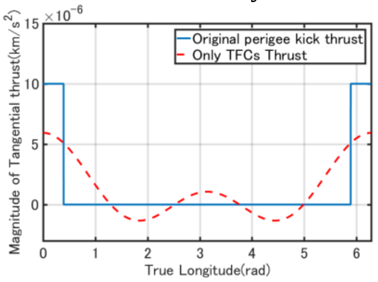

Fig 1. The original and the only TFCs perigee-kick control profile

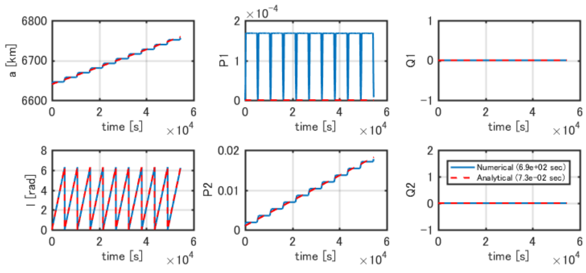

Fig 2. Equinoctial elements propagated by numerical and analytical method with periodic control thrust

References

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