Trajectory analysis for the Phobos proximity phase of the MMX mission

By Elisabet CANALIAS[1)], Laurence LORDA[1)], Thierry MARTIN[1)], Julien LAURENT-VARIN[1),2)], Jean Charles MARTY[1),2)], Yuya MIMASU[3)]

[1)] *CNES, French Space Agency, Toulouse, France.*
[2)] *GRGS Spatial Geodesy Research Group, Toulouse, France.*
[3)] *Institute of Space and Astronautical Science, JAXA, Sagamihara, Japan.*

(Received April 16th, 2017)

The work performed at CNES regarding the trajectory design in the vicinity of Phobos for a MMX-like mission is presented in this paper. The analysis concentrates on two different topics. Firstly, the design of the so-called QSO orbits, which are relative motions with respect to Phobos that have been identified as suitable trajectories for the observation of the moon during proximity phase operations. Special emphasis is put on the generation of 3D QSO, the ones including excursions out of the plane of the equator of Phobos. Secondly, we address the computation of the substitutes of the libration point orbits in the L_1 and L_2 regions of the Mars-Phobos system. If the operational feasibility of a scenario including libration point motions was proved, these orbits could be used for very low altitude observations or as an alternative to more classical approaches for descent operation. The main objective of the authors was to develop robust and flexible tools for mission design, based on dynamical systems theory, that can support the generation of operational scenarios for a Phobos exploration and sample return mission. We believe that this work represents a significant step towards the comprehension and utilization of sophisticated operational trajectories in the unique dynamical environment provided by Mars and its moons.

Key Words: elliptic three body problem, quasi satellite orbits, gravity harmonics, libration points

Nomenclature

BCBF	:	Body Centered Body Fixed
CRTBP	:	Circular Restricted Three Body Problem
ERTBP	:	Elliptic Restricted Three Body Problem
GH	:	Gravity Harmonics
ICRS	:	International Celestial Reference System
MMX	:	Mars Moon eXplorer
LPO	:	Libration Point Orbit
ODE	:	Ordinary Differential Equation
QSO	:	Quasi Satellite Orbit
S/C	:	spacecraft

1. Introduction

The design of trajectories for the exploration of the Martian moon Phobos has raised the interest of several space agencies and researchers in the past few years ([1], [2], [3]). Phobos is a tidally locked moon, whose equatorial plane roughly coincides with its orbital plane, and also with the equatorial plane of Mars. Any mission analyst working in the Phobos environment has to deal with unusual problems, derived from the mass-ratio and length-scale of this unique couple. To start with, the sphere of influence of Phobos is so close to its surface that the Keplerian motion of a S/C around this moon is not possible. Therefore, the gravitational attraction of Mars has to be included in any trajectory computation in the vicinity of Phobos. Furthermore, neither the eccentricity of the orbit of Phobos around Mars nor the non-uniformity of Phobos gravitational field can be neglected when computing representative motions in this environment. In the frame of three body dynamics, a kind of orbits called Quasi Satellite Orbits, inspired by the formation flying of two satellites around a central body, have been identified as possible observation orbits at distances of several tenths of km from the surface of Phobos ([4]). Besides, a simple computation in the CR3BP shows that libration points L_1 and L_2 are only a few km above the surface of the moon. Thus, orbits in these regions would be suitable for very close proximity observations and their associated invariant manifolds could be used for descent operations.

In the present paper, the work performed at CNES concerning the trajectory design in the vicinity of Phobos for a MMX-like mission will be presented. An introduction including the dynamical models that have been used is given in section 2. The methodology and some results relevant to QSO orbit design are presented in section 3. Furthermore, the discussion on how to find libration point orbits in the Mars-Phobos system, as well as some preliminary results are contained in section 4. The tools developed for this work are mainly aimed at mission design purposes and preliminary orbit selection, from a dynamical systems theory point of view. Consequently, particular effort has been put on making them robust, flexible and computationally efficient. Nonetheless, the final goal of the authors is to generate a complete operational scenario for a Phobos exploration mission. This is why our work should not be regarded exclusively as a theoretical exercise, but also as a practical approach to real mission design.

2. Dynamical models used

As already mentioned, pure keplerian motion around Phobos is not possible, as this moon has a collapsing sphere of influence. Therefore, if one wants to study the dynamics at the vicinity of Phobos, a three body problem taking into account the gravity of Mars and Phobos has to be considered. The simplest model describing the motion of a particle under the three body dynamics is the CR3BP (1).

$$\begin{aligned} \ddot{x}-2\dot{y} &= \frac{\partial\Omega}{\partial x} \\ \ddot{y}+2\dot{x} &= \frac{\partial\Omega}{\partial y} \\ \ddot{z} &= \frac{\partial\Omega}{\partial z} \end{aligned} \qquad (1)$$

where $\Omega(x,y,z)=\frac{x^2+y^2+z^2}{2}+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}$,

$r_1=\sqrt{(x-\mu)^2+y^2+z^2}$ and $r_2=\sqrt{(x+1-\mu)^2+y^2+z^2}$.

Equations (1) are expressed in the barycentric synodic frame, rotating with Mars and Phobos, with normalized mass, distance and time units (the interested reader can refer for instance to [5] for further details).

However, even if the eccentricity of Phobos orbit around Mars is not particularly high (e=0.0156), its effect is significant on the motions of satellites flying in this environment. Therefore, the first step towards the computation of realistic trajectories is the use of an elliptic three body model, whose ODEs can be found in the equations below.

$$\begin{aligned} \ddot{x}-2\dot{y} &= \frac{\partial\Omega_E}{\partial x} \\ \ddot{y}+2\dot{x} &= \frac{\partial\Omega_E}{\partial y} \\ \ddot{z} &= \frac{\partial\Omega_E}{\partial z} \end{aligned} \qquad (2)$$

where $\Omega_E(x,y,z)=\frac{1}{1+e\cos\nu}\left(\frac{x^2+y^2+z^2}{2}+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}\right)$. The true anomaly of Phobos around Mars, ν, is the independent variable of the dynamical equations (2), known as pulsating elliptic restricted three body problem. The mass unit is normalized as for the CR3BP. Note that the fact of using the true anomaly as independent variable leads to a time dependent normalized distance (actual orbital radius of Phobos for each value of ν) and time units (the angular velocity is not constant on an elliptic orbit).

Moreover, a complex gravity characterization of Phobos should be used instead of the point mass approximation. In the present paper, a spherical harmonics expansion of Phobos gravity field up to order 4 based on Chao Rubincam model has been used (see [6]). The GH model of Phobos is only used as an estimate, allowing us to realize in which way and up to which extent the three body motions are impacted by this kind of irregular field. Nevertheless, any mission spending a significant amount of time in the vicinity of Phobos is bound to produce scientific data which will allow for a better determination of its gravity field. The trajectory designer should bear this in mind, as the assessment of gravity differences between the model used for the computations and the actual accelerations observed around Phobos might be among the criteria for orbit selection (i.e. try to produce motions where the gravity field effect can be quantified). Furthermore, the design tools should be flexible enough to allow for updates of the gravity model at any time, as surface operations must take into account the most accurate model available.

Note that the GH expansions for the gravity of a rotating body allow us to compute its gravitational acceleration in a frame centered at the body and whose axes rotate with it (BCBF). Given that Phobos is a tidally locked moon, it is straightforward to prove that the synodic 3 body frame of the CR3BP can be transformed into Phobos BCBF by a simple translation of the origin. Nevertheless, the time dependence of the true anomaly in the pulsating ER3BP is different from the angular rotation velocity of Phobos around its own axis. Therefore, we would need different time-dependent scaling of the GH of each degree, $(1+e\cos\nu)^n$. There is another solution, however, which is to introduce a new set of dynamical equations, describing the dynamics of the ER3BP with the mean anomaly as independent variable. Equations (3) provide the full time-invariant ODEs of the Mars-Phobos ER3BP-GH in Phobos BCBF frame, with the fixed physical units of the CR3BP[1]. The terms $U_{G,i}$ stand for the gravitational acceleration due to body i, which for the case of Phobos includes the GH acceleration (see [7]).

$$\begin{aligned} \ddot{x}-2\dot{y} &= \frac{\partial\Omega_M}{\partial x} \\ \ddot{y}+2\dot{x} &= \frac{\partial\Omega_M}{\partial y} \\ \ddot{z} &= \frac{\partial\Omega_M}{\partial z} \\ \dot{\nu} &= \omega_z(\nu) \end{aligned} \qquad (3)$$

where,

$\Omega_M = U_{G,1}(q-\frac{1-e^2}{1+e\cos\nu}[1,0,0]^T)+U_{G,2}(q)-\frac{(1+e\cos\nu)^4}{(1-e^2)^3}\left(\frac{q^TPq}{2}+(1-\mu)\frac{1-e^2}{(1+e\cos\nu)^2}x\right)$, $q=(x,y,z)$, $\omega(\nu)=\frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}}[0,0,1]^T$, $\mathrm{W}(\nu)=\omega(\nu)\wedge$, $\mathrm{P}(\nu)=\mathrm{W}^2(\nu)$

[1]The introduction of ν as a new state variable, with ODE $\frac{\partial\nu}{\partial M}$ allows us to treat the non-autonomous systems as if it was autonomous

3. Quasi-Satellite Orbits

Since Keplerian bounded motions around Phobos do not exist, another type of motions has to be computed in order to explore this moon. It is possible to *orbit* Phobos in a kind of trajectories called Quasi Satellite Orbits. These relative motions with respect to Phobos are retrograde orbits, similar to the quasi-synchronous relative motion of two satellites in formation flying (chief and deputy). The difference between the QSO motions and the formation flying quasi-synchronous orbits comes from the fact that the mass of our chief satellite (Phobos) is not negligible, especially at distances of the order of tenths of km from the surface of the moon. This results in a variety of apparent motions of different relative periods, that tend to the quasi-synchronous formation flying epicycle when distances between the S/C and Phobos are of the order of 100 km or more. For the sake of complete surface mapping and precise gravity estimation, it may be interesting to make the S/C move in QSOs with non-zero relative inclination with respect to Phobos equator, in order to fly over high latitude zones. This is why the methodology implemented at CNES is not limited to planar motions, but can be extended to three dimensional QSO motions with free size and relative inclination as chosen by the user (within the limitations that result from the natural dynamics of the system, obviously). Finally, we would like to note here that the search for 3D *periodic* motions has not been performed in this work: the 3 dimensional orbits that we compute are quasi-periodic motions. Periodicity of the out-of-plane oscillations is a fragile feature, bound to disappear in realistic dynamics, and therefore not of essential interest for the purposes of operational trajectory design.

3.1. Osculating elements

The work presented in this section is based on [8]. If we are interested in relative motions of the S/C with respect to Phobos, we can start by studying the unperturbed Hill's equations of relative motion for the elliptic case, assuming that the attraction of the second primary is negligible (μ=0). The corresponding equations of motion are called Tschauner-Hempel equations (4), and correspond to the well known Clohessy-Wiltshire equations for relative motion when e$\neq$0.

$$\begin{array}{rcl} \ddot{x}-2\dot{y}-\frac{3x}{1+e\cos\nu} & = & 0 \\ \ddot{y}+2\dot{x} & = & 0 \\ \ddot{z}+z & = & 0 \end{array} \quad (4)$$

The general form of the solutions of the Tschauner-Hempel equations can be written as a function of 6 parameters, C=$[\alpha,\ \phi,\ \delta_x,\ \delta_y,\ \gamma,\ \psi]\in R^6$, that we will call osculating elements.

$$\begin{array}{l} x=\alpha(1+e\cos\nu)\cos(\nu+\phi)+\delta_x \\ y=-\alpha(2+e\cos\nu)\sin(\nu+\phi)+\delta_y \\ z=\gamma\cos(\nu+\psi) \\ \dot{x}=-\alpha(\sin(\nu+\phi)+e\sin(2\nu+\phi)) \\ \dot{y}=-\alpha(2\cos(\nu+\phi)+e\cos(2\nu+\phi)) \\ \dot{z}=-\gamma\sin(\nu+\psi) \end{array} \quad (5)$$

Note that in the expressions in (5), α and γ represent the amplitudes of the motion in the in-plane and out-of-plane directions, while ϕ and ψ play the role of injection phases. Besides, non-zero values of (δ_x, δ_y) result in a displacement of the center of the motion in the orbital plane of Phobos ((0,0) being the center of mass of the moon). Hence, the parameters in C can be seen as intuitive design parameters of the solutions of the Tschauner-Hempel equations. Consider now the equations of the ER3BP (2) with two additional hypothesis: a) The mass of the second primary is much smaller than the mass of the first ($\mu \ll 1$) and b) the S/C is in the close vicinity region of the second primary ($r_2 \ll 1$). The second order terms in x,y,z, as well as the terms that are multiplied by μ and not divided by r_2 can then be neglected because they are much smaller than the remaining terms, giving as a result the following differential equations:

$$\begin{array}{rcl} \ddot{x}-2\dot{y}-\frac{3x}{1+e\cos\nu} & = & -\frac{1}{1+e\cos\nu}\left(\frac{\mu x}{r_2^3}\right) \\ \ddot{y}+2\dot{x} & = & -\frac{1}{1+e\cos\nu}\left(\frac{\mu y}{r_2^3}\right) \\ \ddot{z}+z & = & -\frac{1}{1+e\cos\nu}\left(\frac{\mu z}{r_2^3}\right) \end{array} \quad (6)$$

The method of variation of constants can then be applied to the solutions of the Tauschner-Hempel equations (5), in order to transform them into solutions of the equations (6). This is how a set of ODEs of the osculating elements with respect to time are obtained[2]. By solving the differential equations of the osculating elements and using the relationships in (5), one can easily compute the cartesian state vector of the S/C at each step of the integration. In this way, we get a preliminary approximation to the QSO motion, which is not yet a solution of the full ER3BP because some additional hypothesis have been used (see previous paragraph).

A multiple shooting method is applied to the preliminary QSO solution of equations (6) in order to transform it into a solution of the complete ER3BP. Finally, if we also want to include the effect of the irregular gravity of Phobos, a numerical continuation method is started on the QSO

[2]These differential equations are not included here. The interested reader should refer to [8]. Note however that some discrepancies exist between equations 3.66 of Cabral's document and the equations derived by the authors.

solutions of the ER3BP. The continuation parameter ε from 0 to 1 is multiplied to $U_{G,2}$ in equations (3), with the objective of controlling the percentage of high order terms of the gravity field of Phobos that are taken into account. Thus, at each step of the computation the differential equations governing the dynamics are slightly different, and we jump from one approximation to the next one by using a multiple shooting method.

This methodology presents several advantages for the mission analyst. First of all, it provides full control of the design parameters of the QSO (amplitudes, phases and displacement of the center). And secondly, there is no qualitative difference between applying it to the planar problem and applying it to the 6 dimensional space of the 3D case (no dramatic increase in computational time).

3.2. Two dimensional QSO orbits

When a QSO is contained in the orbital plane of Phobos around Mars ($z = \dot{z} = 0$) we call it 2 dimensional. In terms of the osculating elements, this means that the out-of-plane amplitude γ is set to 0. Periodic motions can be found in Phobos BCBF frame by choosing a Poincaré section (for instance $x = 0$) and looking for initial conditions $(0, y, 0, \dot{x}, \dot{y}, 0)$ which come back to the starting point $(0, y, 0)$ with exactly the same initial velocity. In figure 1, the relative period of the motion around Phobos of the planar QSOs is presented, as a function of their minimum distance to the surface. Moreover, some examples of the 2D QSOs that can be computed are shown in figure 2, as well as table 1. Note that when the S/C is far enough from Phobos (≥ 80 km) the gravity of this moon becomes negligible and the relative motion of the S/C tends to a keplerian epicycle[3] with period equal to the orbital period of Phobos and an y-amplitude which is twice the amplitude in x (in three body coordinates, x: coordinate along the axis going from Phobos to Mars, y: axis perpendicular to x-axis, in Phobos orbital plane, z: parallel to the angular momentum of Phobos around Mars).

Table **1. Examples of low altitude 2D QSO.**

	Size (km) A_x, A_y	h_{min} (km)	h_{max} (km)	Period (hours)
1	14 x 16	1.5	5	2.64
2	19 x 25	6.6	14.45	3.83
3	24 x 35	10.8	26.9	4.75
4	30 x 48	17.5	36.6	5.74
5	35 x 60	23.1	51	5.99
6	42 x 75	29.5	65.4	6.5
7	48.5 x 90	36.1	81	6.92
8	51.5 x 100	39.2	88.2	7.1

[3]The epicycle is the typical formation flying relative motion, when the chief S/C has no gravity effect on the deputy

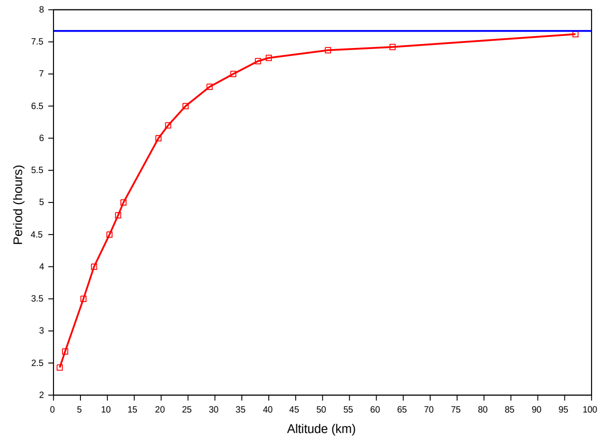

Fig. 1. Period of the 2D-QSO orbits (hours) as a function of its minimum distance to Phobos surface.

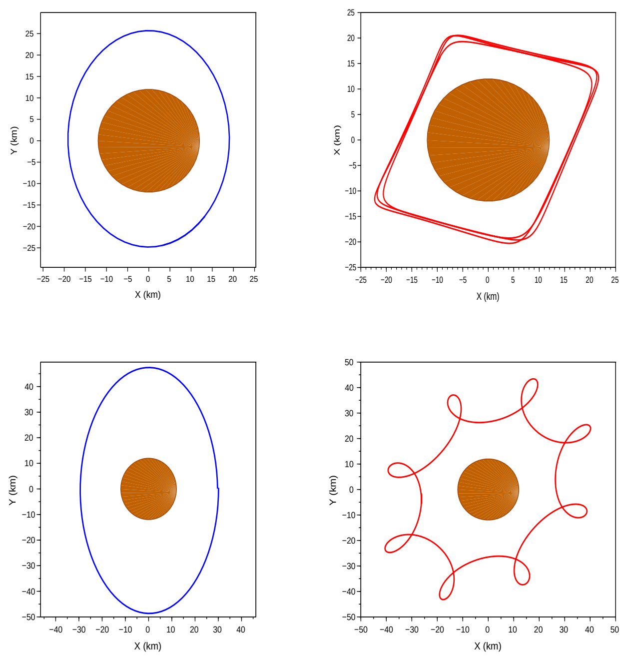

Fig. 2. Examples of 2D QSO orbits with different periods (spherical Phobos of radius 12 km, plotted to give an idea of the relative size of the orbits). In the first row: 2D QSO with period 3.83 hours [1:2 resonant orbit, orbit 2 in table 1] and in the second row: 2D QSO with period 5.7 hours [3:4 resonant QSO, orbit 4 in table 1]. On the left, the orbits are plotted in the equatorial plane of Phobos in BCBF rotating frame, while on the right the XY projection in Phobos centered ICRS frame is shown.

3.3. Three dimensional QSO orbits

For scientific purposes such as detailed imaging of the surface or gravity model characterization, the computation of 3 dimensional motions around Phobos is required. The closer to the surface and the more inclined with respect

Table **2. Examples of 3D QSO.**

	Size (km) A_x, A_y, A_z	h_{min} (km)	h_{max} (km)	Latitude max (deg)
1	22 x 30 x 3	9.5	20	8.7
2	25 x 40 x 8	12.5	32	18.7
3	28 x 45 x 10	15.5	35.5	22.8
4	30 x 50 x 18	17.8	42.3	33.7
5	55 x 110 x 45	44	107.8	42.3
6	75 x 145 x 60	72.7	153.5	42.6

to Phobos equator that the S/C can fly the better, because in this way areas at high latitudes are accessible with a higher resolution. The problem of trying to decrease the altitude while increasing the apparent orbital inclination is that QSO motions quickly become unstable. It is beyond the scope of the present paper to perform mathematical stability analysis of the QSOs. Our goal is to use the methodology that has been explained in the previous sections for generating 3 dimensional QSOs that allow both the fulfillment of the scientific requirements and the operational stability of the associated scenarios (practical notion of stability, meaning survival without ground control for several days if needed).

The only difference of the 3D case with respect to the 2D QSO generation of section 3.2. is that now the methodology has to be applied with an out-of-plane amplitude $\gamma \neq 0$. We know from the literature that any QSO with $\gamma > 0.9\alpha$ will quickly become highly unstable and we have experienced it in our simulations. Bearing this inequality in mind, we have successfully computed 3D QSOs for several couples of γ and α respecting this *stability* condition, while trying to maximize the reachable latitude above Phobos equator for each orbital size (see table 2 for a summary of the characteristics of several 3D QSO computed in this way).

When we allow the distance to Phobos surface to increase, it is obviously easier to reach high latitudes of the surface while respecting the stability relation between in-plane and out-of-plane amplitudes. Nevertheless, distant orbits may be unusable for particular scientific objectives, as large distances to the target imply low resolution of the observations. On the contrary, if either planar QSOs or 3D QSO with small out-of-plane amplitude are used around Phobos, the minimum distance to the surface can be drastically reduced down to only a few km. The drawback is that for the low altitude QSOs the observation of high latitude regions becomes difficult and it may require large depointing angles, which can seriously harm the quality of the images. This is why motions like examples 3 and 4 of table 2 are interesting, because they exhibit a good trade-off between reachable latitude and distances to the surface (see figure 3).

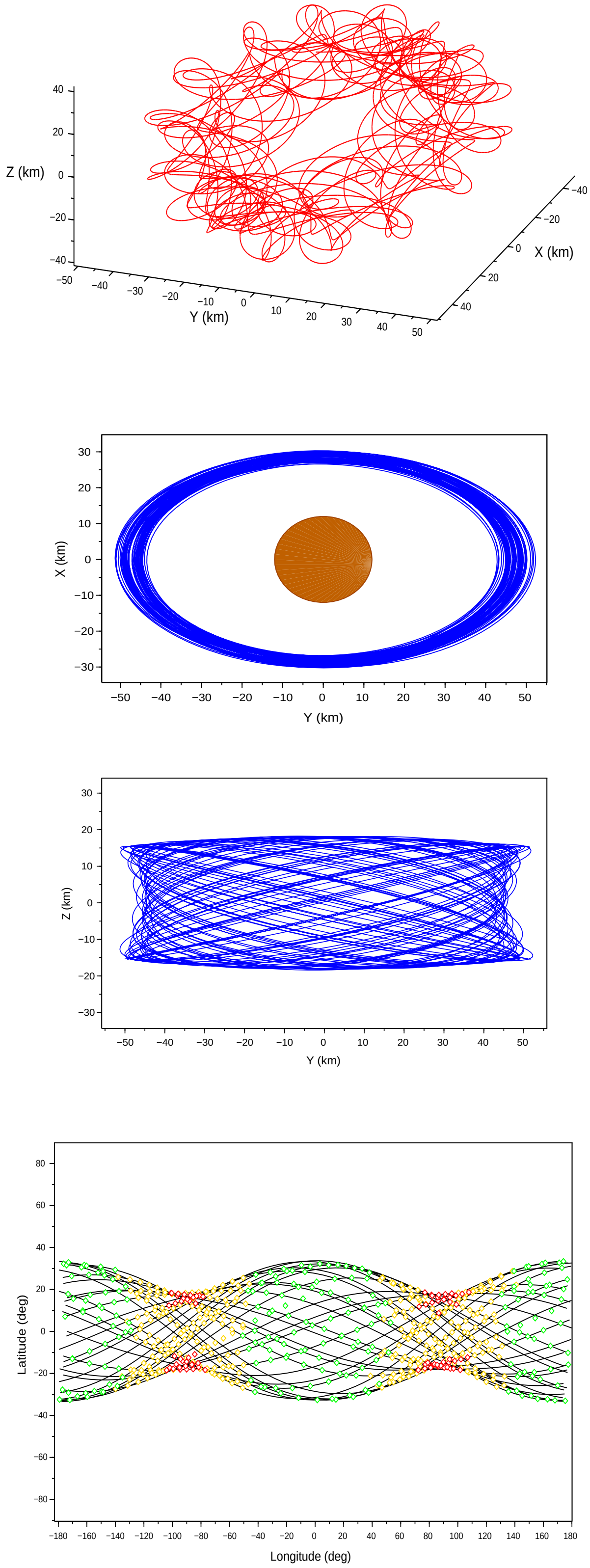

Fig. 3. Example of 3D QSO [orbit 4 in table 2]. 14 days integration under the dynamics of ERTBP-GH. From top to bottom: Phobos centered trajectory in ICRS frame, YX and YZ projection in Phobos BCBF (km), Ground-track of the S/C on Phobos surface with altitude information (red: 35 to 42 km, yellow: 25 to 35 km, green: 17 to 25 km).

3.4. QSO orbits in realistic dynamical models

In the previous sections, a methodology for the computation of QSO trajectories in the ERTBP including the gravity harmonics of Phobos has been presented. We consider this methodology to be satisfactory for preliminary mission design purposes, as it allows for an assessment of the types of QSO orbits, their sizes and orbital velocity, the ground-track path on the surface of Phobos.... Furthermore, the tools that have been developed are fast (computational time of the order of 1 minute for obtaining a 14 days integration of a 3D QSO orbit in the ER3BP-GH model, with an Intel core i3 at 2.40 GHz) and flexible (as the design parameters can be chosen by the user, and the simulations can be easily run with different orbital parameters of Phobos around Mars or with a different spherical harmonics model for the complex gravity of the moon). Despite all these assets, our tools would be useless if they provided as a final output an artificial 3D trajectory that only made sense when integrated under simplified dynamical equations. In other words, we need to check that we are able to easily turn our preliminary approximations to QSOs into operational trajectories, through a procedure that implies as few changes in significant orbital characteristics as possible.

Together with our colleagues at the GRGS, we have performed the first step towards the transformation of our QSO trajectories for use in a real mission, using a realistic dynamical model which takes into account the real ephemeris of Mars and Phobos, the complex gravity model of this moon, the J_2 attraction of Mars, Mars tidal effects, the effect of the Sun as third body and the solar radiation pressure, in addition to some almost negligible relativistic accelerations. The procedure that has been used is the following:

a) Select an interval of time of several hours in the ephemeris file obtained using the methodology explained above under the ERTBP-GH equations,
b) Apply a least squares method to adjust the state vector at the initial date in such a way that the numerical integration of the orbit under the realistic dynamics fits the preliminary QSO in the best possible way, during the interval selected at step a),
c) Integrate the modified state vector from the previous step under the realistic equations of motion for one week.

If the final integrated trajectory of step c) maintains the characteristics of the original QSO in the ERTBP-GH for several days, we consider that our method fulfills the expectations of preliminary mission design and that our approximations to this kind of motion are usable for the estimation of interesting parameters for Phobos explo-

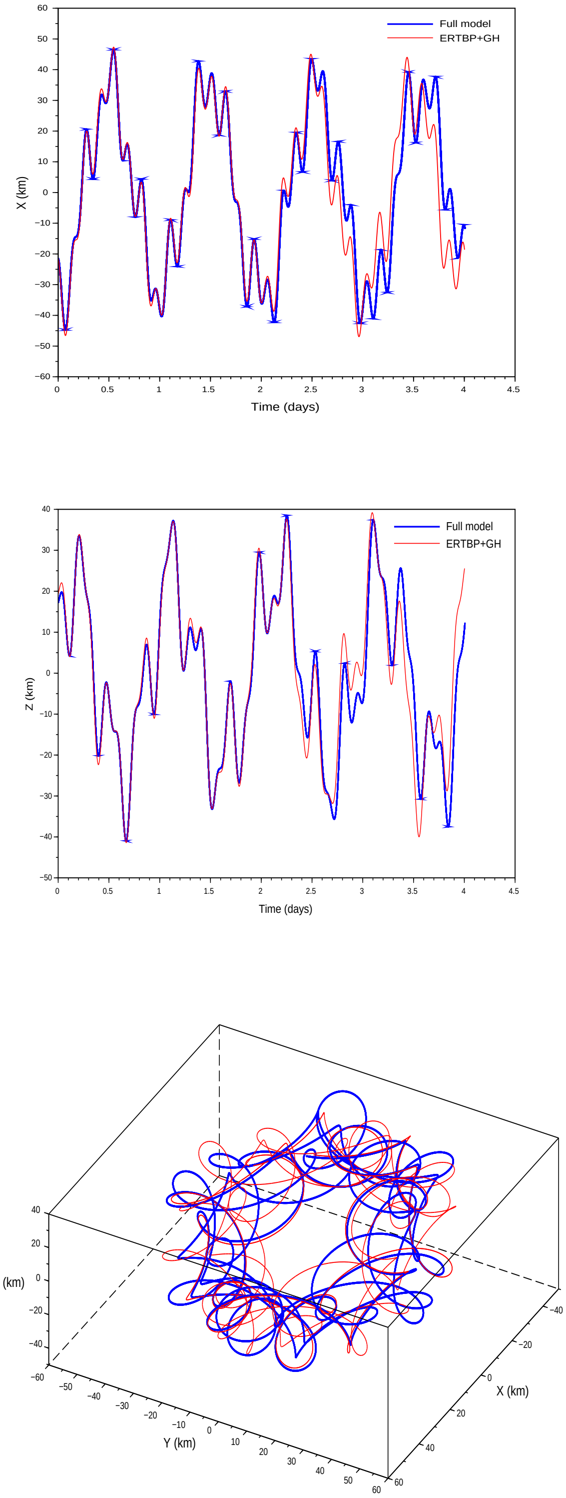

Fig. 4. Integration of a 30 x 50 x 18 km QSO using realistic dynamics. From top to bottom: Comparison of the X coordinate in ICRS Phobos centered frame vs time, idem for Z coordinate and XYZ view of the two trajectories (red=ER3BP-GH, blue=full model) for a total integration time of 7 days.

ration such as the distances to the surface, orbital velocity, eclipse duration.... This is the case for large and middle size QSO orbits (from x amplitudes starting at 30 km), which match the preliminary estimation of the trajectory for around 2 to 3 days almost perfectly, and whose deviations from it in a week are not dramatic and do not imply qualitative changes in the aforementioned parameters. An example of these results is shown in figure 4. On the contrary, for low altitude QSOs ($\leq$15 km from the surface), large deviations from the original QSO estimation were detected at very early stages (just after day 1) when trying to adjust the motion to make it follow a realistic model of the dynamics. If the propagation was continued, for some low altitude cases collisions or escape from Phobos vicinity occurred before the end of the first week. We expect to improve these results in the near future, simply by using a more sophisticated adjustment of the initial conditions. Nonetheless, the use of a spherical harmonics expansions is probably not the best approach for low altitude orbit computations. These results confirm the obvious fact that the operations of the S/C in low altitude QSOs can entail risks and that the fuel budget and frequency of the maneuvers can be significantly high, especially if the modeling of the gravity of Phobos is not accurate enough or is not taken into account in the appropriate way when generating preliminary approximations to these orbits.

4. Libration Point Orbits

The so-called libration points of a three body system are equilibrium points of the dynamical equations in (1). In particular, L_1 and L_2 stand for the libration points at both sides of the small primary of the system. The dynamics around these points in the CR3BP has been exhaustively studied in the past (see for instance the series of books [9]). Periodic and quasi-periodic bounded orbits around L_1 and L_2, belonging to the so-called center manifold, can be computed by using numerical procedures. Moreover, the hyperbolic character of these equilibrium points yields the existence of asymptotic manifolds associated to the LPOs, which can be exploited for mission design purposes. For the Mars-Phobos system, libration points are found only a few km above the surface of Phobos (roughly 2 to 3 km). This means that the irregularities of the moon's gravity field will have a significant effect on the libration point orbits and their manifolds. Consequently, any model considering Phobos only as a point mass is not valid for assessing the feasibility of the use of LPOs for Phobos proximity operations. Furthermore, the eccentricity of the orbit of Phobos around Mars also has an effect on the motion of the S/C. The final goal of the work presented in this section is to compute the dynamical substitutes of the libration point orbits in a three body model taking into account the irregular gravity of Phobos and the orbital eccentricity of the moon (ER3BP-GH). Impressive advances in this direction have been done in [7], and this PhD. dissertation has inspired most of the work explained in the following paragraphs.

The methodology we plan to apply to fulfill the objective of computing the dynamical substitutes of LPOs in the Mars-Phobos environment can be summarized in the following steps:

1. Computation of a LPO with the desired design parameters (size, period...) or an associated manifold in the CR3BP, using the methods found in the literature (like the Lindstedt-Poincaré procedures [10]).
2. Computation of the dynamical substitute of the object computed in step 1) in a model taking into account the GH of Phobos, by means of a numerical continuation method. The continuation parameter ε varying from 0 to 1 allows for a progressive inclusion of the acceleration caused by the GH of Phobos.
3. Assessment of the effect of the eccentricity of Phobos and obtention of the corresponding object in the ER3BP-GH (equations (3)), by using the eccentricity as continuation parameter (from 0 to e=0.0156).

The final trajectories after step 3) are not fully realistic orbits yet. Nevertheless we believe they are a good starting point for the estimation of representative quantities such as the size and orientation of the LPOs, their periods and station keeping costs, as well as for the assessment of the feasibility of the use of the associated invariant manifolds for operational issues such as the transfer from low altitude QSO to the LPO region, or for descent and ascent operations in the sample return phase. Unfortunately, the development of our analysis tools is still on-going at the moment of writing this paper and therefore, concluding results cannot be presented by the authors yet. However, it will be shown in what follows that the characteristics of the LPOs in the Mars-Phobos environment change in a qualitatively significant way when computed in models that are not the plain CR3BP, contrarily to what happens in other systems, classicaly modeled by using the CR3BP (such as the Sun-Earth). Therefore, we strongly recommend discarding the use of this simplified model for the study of LPOs in the environment of the Martian moons.

4.1. Periodic orbits in the CR3BP-GH

For the sake of simplicity and as a first application of the methodology, only periodic orbits around the libration points L_1 and L_2 have been taken as initial orbits in the CR3BP and transformed into periodic motions in the CR3BP-GH. Planar Lyapunov orbits have been computed in the Mars-Phobos CR3BP, with sizes going from a few hundreds of meters to several kms (in the direction of the axis joining Mars and Phobos). Moreover, 3D halo orbits have also been computed, with Y amplitudes that can reach several km. The inclusion of Phobos spherical harmonics expansion in the equations of motion has a qualitative effect on these families of periodic orbits: they are progressively tilted and their shapes and periods can be affected. An example of a planar Lyapunov orbit with initial amplitude of 500 m in the x direction is presented here (see figure 5). For this example, the gravity harmonics have been introduced thanks to a 10 step continuation. The objective at each step was to find a periodic orbit, with a period as close as possible to the initial one. In the future, we plan to introduce more elaborated continuation methods, such as the energetic approach explained in section 4.2 of [7]. Moreover, other ways of expressing the irregularities of Phobos gravity field will have to be explored, in order to assess its impact on the evolution of the periodic orbit families. For instance, the use of a gravity model derived from a high fidelity shape model of the moon (assuming constant density) can cast some light on the effects of the non-sphericity of the moon for very low altitude operations around features such as the Stickney crater[4]. We acknowledge that these effects cannot be observed when using spherical harmonics expansions.

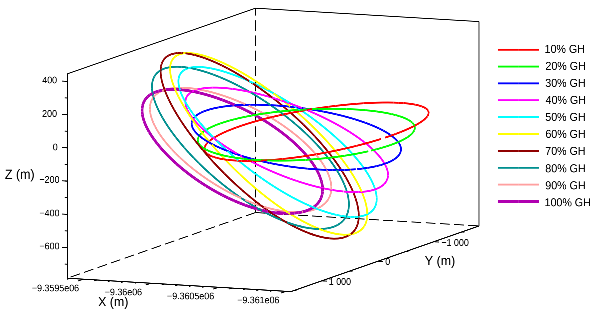

Fig. 5. Continuation of planar Lyapunov orbit, CR3BP to CR3BP-GH.

4.2. Effect of Phobos eccentricity

Additional simulations have been done using the final orbits computed in the previous section (i.e the dynamical substitutes of the LPOs resulting from the inclusion in the equations of motion of the CR3BP of the acceleration computed by the spherical harmonics expansions of Phobos gravity field) and using a continuation method on the value of the eccentricity to transform them into solutions of the ER3BP-GH. These analyses are still not finished at the moment of writing the paper. However, we have already been able to observe some features of the behavior of the dynamical substitutes of LPOs in the ER3BP-GH. Contrarily to what happened when we introduced the effect of the GH, switching on the eccentricity does not result in a significant modification of the qualitative characteristics of the LPOs. The orbits keep their size and relative tilting or inclination with respect to the axes of the Mars-Phobos synodic frame (see figure 6). However, we have not been able to find periodic motions similar to the original LPOs in the ER3BP-GH yet. This can be due to an implicit problem of the methodology that is applied, which may not be able to catch the libration effects derived from the eccentricity by simple numerical continuation. We want to investigate this issue in the near future, before moving on in terms of stability assessment, station keeping and navigation issues and the computation of the manifolds associated to the new objects in the ER3BP including the irregular gravity of Phobos.

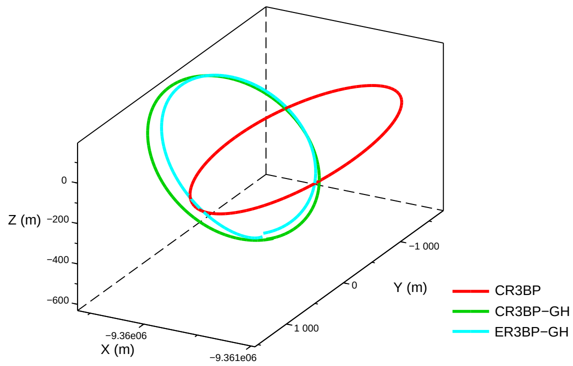

Fig. 6. Evolution of an example LPO, from CR3BP to ER3BP-GH.

5. Conclusions

In this paper, the work performed at CNES devoted to the trajectory design for the proximity operations of a Phobos exploration mission has been presented.

On the one hand, for Phobos observation at altitudes of several tenths of km from the surface, we have developed a tool for QSO computation which uses a robust trajectory generation method, with application oriented results. It allows for a fast generation of preliminary QSO trajectories using intuitive design parameters known as osculating elements. The characteristics of this method, using several dynamical systems methods for preliminary approximation to QSOs, as well as multiple shooting and numerical continuation techniques for dynamical equations *upgrade*, represent a remarkable improvement with respect to the brute-force numerical search for solutions. It is worth noting that our methodology uses a single representation of

[4]The polyhedron gravity model has been used by the authors for the mission analysis of other space probes ([11], [12]) but it is yet to be implemented for Phobos, even though its use would probably help the design of the descent and sample return operations

both the planar and the 3D QSOs, with similar computational effort despite the difference in the phase space dimension of these two cases. Furthermore, the trajectories that are obtained are suitable for preliminary assessment of mission design parameters, as it has been proved that they can be easily converted into realistic motions. Finally, our methodology has provided notable results for the generation of QSO trajectories that are particularly demanding in terms of the trade-off between altitude from the surface and relative inclination with respect to Phobos equator. This type of QSOs may be the most interesting for scientific purposes, such as the global mapping of the surface of Phobos or the measurements aimed at improving the current knowledge of this moon's gravity field.

On the other hand, we have also developed tools for the computation of dynamical substitutes of libration point orbits using the specific dynamics of the Mars-Phobos system. As an example, the dynamical substitute of a planar Lyapunov orbit around the L_1 point has been presented. The goal of our on-going studies is the improvement of the numerical methods used for the computation of LPOs in the ER3BP taking into account Phobos irregular gravity field, in addition to the computation of invariant manifolds associated to the dynamical substitutes of these LPOs. We have good reasons to believe that even if the aforementioned orbits and manifolds may not completely solve the problem of descending to the surface for sample recollection, they can provide an alternative scenario for observations at very low altitude, as well as help solving transfer problems from the QSO to the surface and backwards (see [7], [3]). In the near future, we plan to use our findings in this topic to propose an operational scenario using the LPOs for an MMX-like mission.

6. References

[1] Kawakatsu, Y. "Martian Moons eXplorer mission overview and current status." Misasa VI: Frontiers in Earth and planetary materials research, origin, evolution and dynamics, 2016.

[2] Cacciatore, F. and Martín, J. "Mission analysis and trajectory GNC for Phobos proximity phase of Phootprint mission." Advances in the Astronautical Sciences, Vol. 153, Second IAA DyCoss'2014.

[3] Wallace, M., Parker, J., Strange, N., and Grebow, D. "Orbital Operations for Phobos and Deimos Exploration." AIAA/AAS Astrodynamics Specialist Conference, Minneapolis, Minnesota, Jet Propulsion Laboratory, Pasadena, 2012.

[4] Gil, P. and Schwartz, J. "Simulations of Quasi-Satellite Orbits Around Phobos." Journal of Guidance, Control and Dynamics, Vol. 33, No. 3, pp. 901–914, 2010.

[5] Moulton, F. An introduction to celestial mechanics. Dover, 2 edn., 1970.

[6] Chao, B. and Rubincam, D. "The gravitational field of Phobos." Geophysical research letters, Vol. 16, No. 8, pp. 859–862, 1989.

[7] Zamaro, M. "Natural and artificial orbits around the Martian moon Phobos." Department of Mechanical and Aerospace Engineering, University of Strathclyde, 2014-2015.

[8] Cabral, F. and Gil, P. "On the Stability of Quasi-Satellite Orbits in the Elliptic Restricted Three-Body Problem." Master Thesis at the Universidade Técnica de Lisboa, 2011.

[9] Gómez, G., Llibre, J., Martínez, R., Simó, C., Masdemont, J., and Jorba, A. Dynamics and mission design near libration point orbits, Vol. Volumes 1-3. World Scientific, 2 edn., 2000.

[10] Masdemont, J. "High-order expansions of invariant manifolds of libration point orbits with applications to mission design." Dynamical systems, an International Journal, Vol. 20, pp. 59–113, 2005.

[11] Lorda, L., Canalias, E., Martin, T., Garmier, R., and Biele, J. "Mascot: Analyses of the Descent and Bouncing Trajectories to Support the Landing Site Selection." 26th International Space Symposium in Flight Dynamics, Matsuyama, Japan, 2017.

[12] Jurado, E., Martin, T., Canalias, E., Blazquez, A., Garmier, R., Ceolin, T., Gaudon, P., Delmas, C., Biele, J., Ulamec, S., Remetean, E., Torres, A., Laurent-Varin, J., Dolives, B., Herique, A., Rogez, Y., Kofman, W., Jorda, L., Zakharov, V., Crifo, J.-F., and Rodionov, A. "Rosetta lander Philae: Flight Dynamics analyses for landing site selection and post-landing operations." Acta Astronautica, Vol. 125, pp. 65–79, 2016.