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Orbital Stability Regions for Hypothetical Natural Satellites

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Previous work made conclusions on how Sun-Synchronous orbits, the modified Laplace plane, and the Kozai resonance can yield stable or unstable orbits for natural satellites less than 15 m in diameter around Bennu. This work expands upon that research to understand if asteroids with various obliquities have natural satellites and how these orbital phenomena may change. We found that natural satellites less than 1 meter will be stable in only Sun-Synchronous orbits regardless of the oblquity. However, the obliquity will change the range of inclination or semi-major axis that Sun-Synchronous orbits remain stable. Also, objects greater than a meter in size will more likely oscillate around a modified Laplace plane equilibria if the asteroid has a high obliquity as opposed to a low oblquity. Finally, the Kozai resonance instability region for large natural satellites with semi-major axes greater than 4 km only exists for an asteroid with an obliquity of $\phi = 176°$, but not for the other two cases.

Nomenclature

ϕ	:	oblquity
H_z	:	Angular momentum projected onto z-axis
e	:	eccentricity
i	:	inclination
Ω	:	longitude of the ascending node

1. Introduction

In previous research, we have investigated the possibility of natural satellites around Bennu, the target asteroid for OSIRIS-REx. It was of interest for the OSIRIS-REx team to determine whether Bennu might possess any natural satellites in long-term stable orbits that could interfere with spacecraft operations in Bennu's vicinity. The focus of this research is solely on the existence of stable orbits for a natural satellite and purposefully places how the natural satellite migrated to this orbit outside the bounds of this research. Numerical simulations modeling J_2, third body dynamics and solar radiation pressure is used on a large set of initial conditions that vary in semi-major axis, inclination, longitude of the ascending node and natural satellite diameter. To be considered stable, we require a set of initial conditions to remain in orbit for greater than 1000 years. Instability is defined as escape from the asteroid or impact. By constructing and executing an array of detailed simulations modeling the evolution of natural satellite orbits over thousand-year time scales, we assessed the possible sizes, distances from Bennu, and orbital orientations of long term stable orbits. We found three orbital phenomena that dictated the stability of natural satellites from a centimeter in diameter to 15 m in diameter that were stable around Bennu for 1000 years.[8)]

The first stable region will be due to the modified Laplace plane (MLP). The classical Laplace plane is normal to the axis about which the pole of a satellite's orbit precesses, where the perturbations on the satellite include a third-body and an oblate primary.[14)] The MLP also includes solar radiation pressure (SRP) in the perturbation model.[9)] We have already determined some characteristics of the MLP around Bennu.[7)] First is that the MLP becomes less stable as the distance between the satellite and primary increases. The MLP is also less stable as the diameter of the satellite decreases.

Next, the Kozai resonance may be responsible for stable and unstable regions for natural satellites. The Kozai resonance is caused by third body perturbations on a satellite. This resonance causes libration of the satellite's argument of periapsis. The libration results in an exchange between eccentricity and inclination in such a way that the satellite's angular momentum projection normal to the Sun-Bennu orbit plane is conserved.[3)] Using the averaged equations of motion for just third body dynamics, the angular momentum of the satellite's orbit projected normal to the Sun-Bennu orbit plane is conserved. This can be quantified as:

$$H_z = \sqrt{(1 - e^2)}\cos i \tag{1}$$

Therefore the eccentricity and inclination of the satellites orbit will vary, as inclination increases the eccentricity will decrease and vice versa. The Kozai resonance will most likely occur further out from the asteroid such that spherical harmonics will not significantly perturb the system. The satellite will also have to be massive enough to not be perturbed significantly from SRP. Also, it should be noted that at certain inclinations, particularly high inclinations, the Kozai resonance will cause instability since the eccentricity will be too large. Eccentricity will increase such that it will result in a collision with Bennu or escape outside of the Hill sphere.

Finally, an orbit will be stable if it is in a Sun-synchronous orbit. Sun-synchronous orbits are orbits that rely on a strong SRP perturbation to cause the node of the orbit to precess at the same rate as the asteroid travels about the sun. The orbits become more stable as the SRP perturbation grows, to the point where the object is stripped out of orbit.[12)] These orbits are responsible for natural satellites orbiting around Bennu with diameters ¡1 m and at inclinations between 36° and 146°.

A summary for these results can be seen in Figure 1. This figure represents the results for all simulations with varying semi-major axis, inclination, longitude of the ascending node, and natural satellite diameter. The figure shows the minimum natural satellite diameter that is stable for 1000 years at each semi-major axis and inclination. Larger diameter objects may exist at the same semi-major axis and inclination. As a satellite becomes smaller, the more likely that SRP will disturb its orbit and cause it to go unstable. Therefore the minimum natural satellite diameter gives the minimum diameter not impacted by SRP. Notice that inclinations between 36° and 144° at semi-major axis between 1 and 13 km have significantly smaller diameter natural satellites than the rest of the simulations. These smaller objects are stable due to Sun-Synchronous orbits. If natural satellites that are a stable due to Sun-synchronous orbits are taken away from Figure 1, we would see no stable orbits for objects with inclinations between 78° and 108° for radial distances greater than 4 km. This instability region is due to the Kozai resonance where higher inclinations correspond to a growth in eccentricity that is large enough to cause escape or an impact with Bennu.

The results discussed above and summarized with Figure 1, are for a particular asteroid, Bennu. We would like to investigate further into how natural satellite stability changes when we simulate an asteroid with different parameters. We investigate how the obliquity of the asteroid will effect the natural satellite stability. Therefore all parameters are kept the same as in previous work, but we look at an asteroid exactly the same as Bennu with obliquities of 45° and 135°.

2. Results

Above are the summary of results for various sized natural satillites with different initial conditons: semi-major axis, longitude of the ascending node, and inclination. The results for an asteroid with an obliquity of 45° can be viewed in Figure 2 and an asteroid with an obliquity of 135° can be viewed in Figure 3.

2.1. Sun-Synchronous Orbits

The first comparison between the varying obliquities, is the difference in their Sun-Synchronous orbits. The Sun-synchronous orbits are easy to determine in the figures, because they are any objects that had a diameter less than 1 meter. We also further verify that these are Sun-Synchronous orbits by observing the change in longitude of the ascneding node in the Sun-rotating frame. Sun-synchronous orbits precess in the inertial frame, but will remain bounded to $\Omega_R = 0°$ or $\Omega_R = 180°$ in the Sun-Bennu rotating frame. For the original, $\phi = 176°$, Sun-Synchronous orbits exist, for inclinations between 36° and 144° at semi-major axis between 1 and 13 km. For $\phi = 45°$, Sun Synchronous orbits exist for semi-major axis between 1 and 15 km, but exist for all inclinations. Finally, for $\phi = 135°$, the Sun-Synchronous orbits exist for all inclinations, but only for semi-major axes from 1 to 5 km. Therefore, it may be possible that objects with high obliquites will have Sun-Synchronous orbits at any inclinations, while low oblquity asteroids can only have possible natural satellites at higher inclinations. Finally, it also seems that the retrograde orbit will cause the Sun-Synchronous orbits to exist only at lower radii.

2.2. Modified Laplace Plane

In Figure 4a, many inital conditions are shown for an asteroid with $\phi = 176°$, and a natural satellite that is 3.75 m in diameter with a semi-major axis of 2 km. Each data point represents an inclination and a longitude of ascending node at a specific time in any given orbit. The sum of all the data yield information on which orbital regions are stable/unstable or if longitude of periapsis precesses 360° or less. Figure 4 have the inclination and longitude of the ascending node defined from the equator of the asteroid. In Figure 4a, their is one Laplace equilibria at $\Omega = 0°$ and $i = 90°$, there are some orbits that will oscillate around the equilibrium, but most precess a full $\Omega = 360°$. Figure 4b-c with $\phi = 45°$ and $\phi = 135°$ respectively have 3 modified Laplace equilibria, where the majority of possible orbits will oscillate around one of these three equilibria. The only difference between $\phi = 45°$ and $\phi = 135°$ is that the equilibria at a given inclination will shift the longitude of the ascending node 180°. It appears that higher obliquity asteroids, will have natural satellites larger than 1 m diameter mostly oscillating around the modified Laplace plane, and therefore will be bounded in the longitude of the ascending node, while the same asteroid at low obliquities will precess 360°.

2.3. Kozai Resonance

It should be briefly noted, that the Kozai resonance instability that existed for Bennu for possibly natural satellites at inclinations of 78° to 108°, do not exist for the two higher oblquity asteroids.

3. Results

Varying the obliquity of the asteroid has created several differences between where natural satellites will be stable and what orbital phenomena is causing stability or instabilty. The conclusions for these simulations are:

- Sun-synchronous orbits yield stable orbits for natural satellites less than 1 m in diameter for all asteroid obliquities. However, $\phi = 135°$ has Sun-Synchronous orbits stable for only semi-major axes less than 5 km, while $\phi = 45°$ and $\phi = 176°$ have stable Sun-synchronous orbits at 13 km or more. Also, $\phi = 176°$ had Sun-synchronous orbits only for a range of inclinations, while $\phi = 45°$ and $\phi = 135°$ have Sun-synchrnous orbits for all inclinations.

- The high oblquity cases, $\phi = 45°$ and $\phi = 135°$, have natural satellites greater than 1 meter oscillating around the Laplace plane, while the low oblquity case will have the longitude of the ascending node precess 360°.

- Kozai instability region only exists for the $\phi = 176°$ asteroid.

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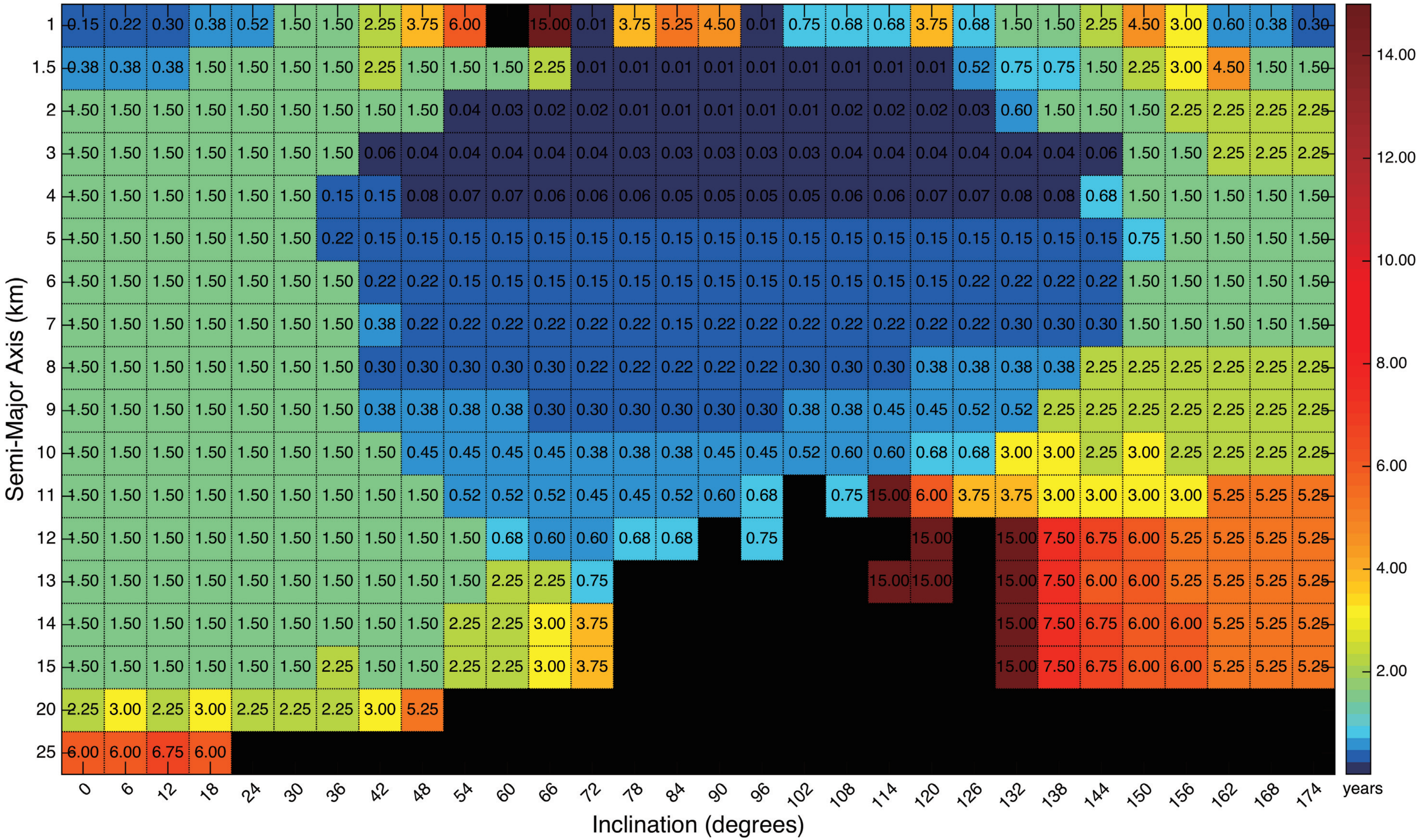


Fig. 1. This figure summarizes the overall results of the simulations for an asteroid with obliquity of 176°. For each semi-major axis and inclination simulated, the minimum natural satellite diameter is determined and shown with a corresponding color, where blue is the smallest diameter to red being the largest. Larger diameter objects may exist at the same semi-major axis and inclination. Notice that higher inclinations are capable of much smaller diameter object's being stable than more equatorial orbits.

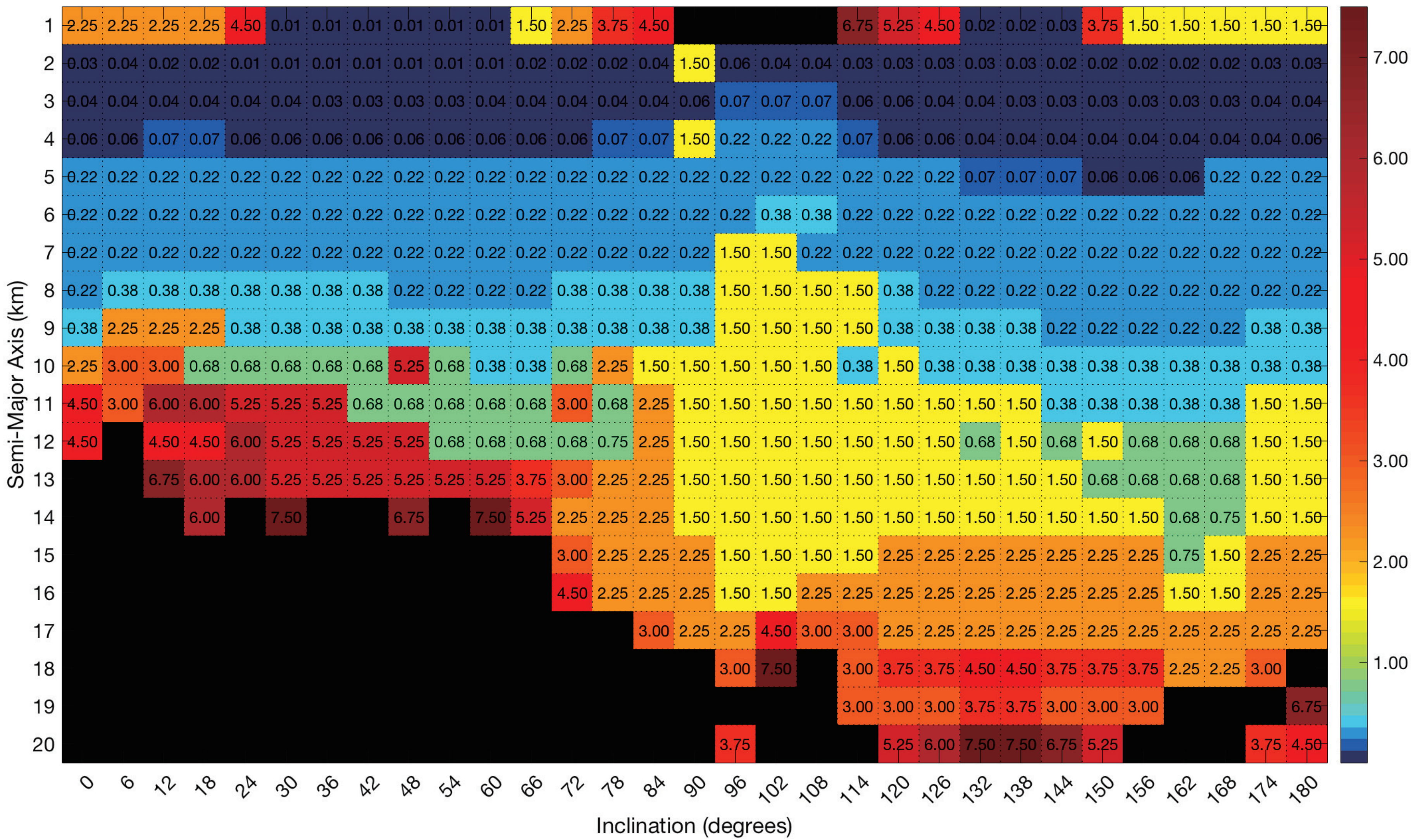


Fig. 2. This figure summarizes the overall results of the simulations for an asteroid with obliquity of 45°. For each semi-major axis and inclination simulated, the minimum natural satellite diameter is determined and shown with a corresponding color, where blue is the smallest diameter to red being the largest. Larger diameter objects may exist at the same semi-major axis and inclination.

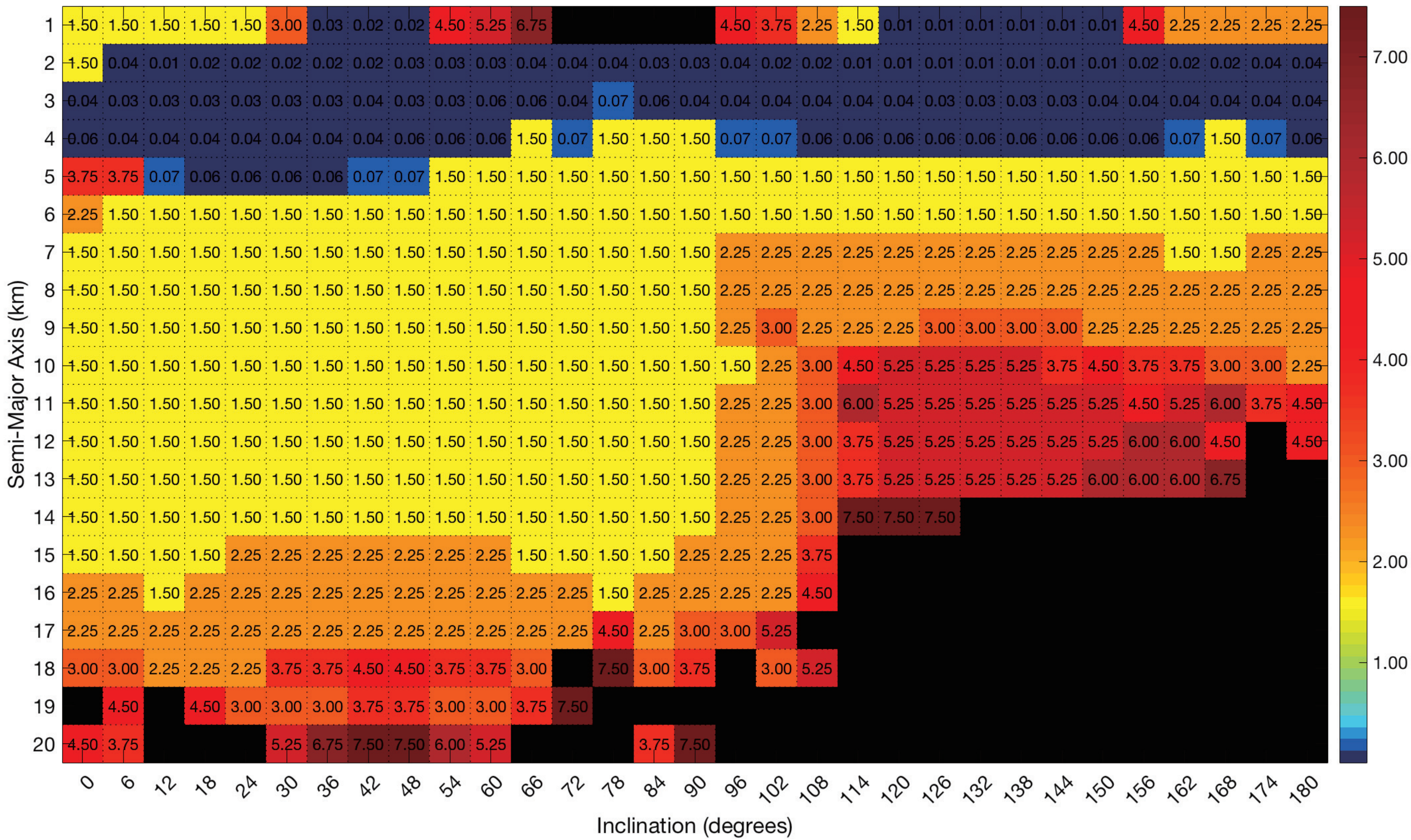
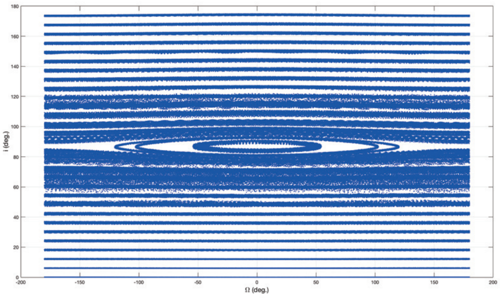
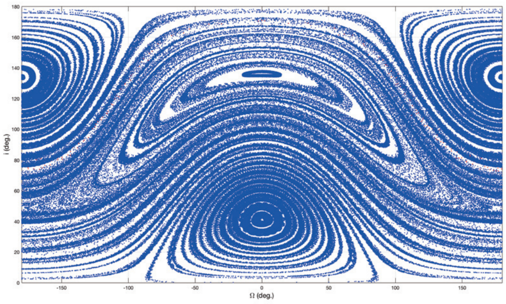


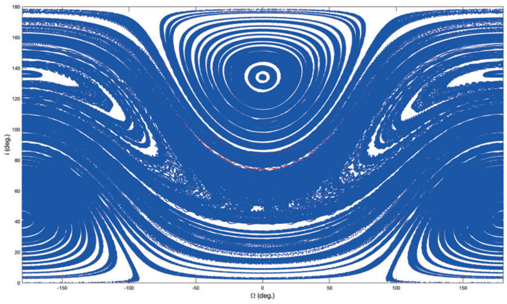
Fig. 3. This figure summarizes the overall results of the simulations for an asteroid with obliquity of 135°. For each semi-major axis and inclination simulated, the minimum natural satellite diameter is determined and shown with a corresponding color, where blue is the smallest diameter to red being the largest. Larger diameter objects may exist at the same semi-major axis and inclination.



(a) $\phi = 176°$



(b) $\phi = 45°$



(c) $\phi = 135°$

Fig. 4. Inclination vs. longitude of the ascending node for $a = 2\ km$, $D = 3.75\ m$ for 1000 years. The higher obliquity asteroids, have the motion of the natural satellite orbits oscillating around the Laplace plane, and therefore their longitude of the ascending node is bounded. Figure (a), has a small retrograde obliquity, therefore the longitude of the ascending node is not bounded to the Laplace plane.

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