

# Matching of Patched-Hyperbolas for Gravity-Assist Trajectory Design

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(Received June 21st, 2017)

In this paper, we propose several methods to find combinations of trajectories that are easy to match from noncontiguous trajectories around the lunar gravity assist without optimization calculation by defining a new trajectory called a virtual hyperbola. The two given perilune vectors are propagated in different ways in 2-body and N-body, and calculated  $\Delta v$  between the trajectory to tie and legs. The usefulness of the methods is confirmed from trade-off table about the correlation between the cost of  $\Delta v$  in the approach methods and the  $\Delta v$  of the optimization calculation, and computation time. As a result, it was possible to establish the methods with normal correlation and shortening calculation time.

Key Words: Matching, Gravity Assist, Lunar, Trajectory Design, Patched-Hyperbolas

## 1. Introduction

In the patched-conic approximation model, a multiple gravity-assist trajectory is divided into various planet-to-planet (or moon-to-moon) legs connected by conic arcs. To find valid trajectories for gravity assist, we search for incoming and outgoing legs which match their hyperbolic excessive velocities. The swing-by body is often assumed to have a zero sphere of influence and the effect of its gravity is regarded as an instantaneous change to the trajectory. However in the high fidelity model, we have to convert the patched conic orbits into a continuous, N-body integrated trajectory. In our test case, we found out that just by matching the V-infinity of the incoming and outgoing legs maybe not be sufficient or it might require a high additional maneuver. To save time in the conversion and to reduce the added  $\Delta V$  penalty cost (Optimization cost), the purpose of this paper is to find a way of selecting good matching trajectories for lunar gravity assists. Three approaches are applied to evaluate the two sets of incoming and outgoing legs. The relationship between  $\Delta v$  calculated in these approaches and  $\Delta v$  when optimizing is evaluated and if a strong correlation is found, it can be considered to be useful for the approach methods. The correlation is judged by whether it is a linear relation or not. In addition, when data that does not conform to the correlation is obtained, if the data is located below the correlation straight line, it means that the data generates negative correlation. Therefore, it is assumed that the case where it is located above is evaluated as good. A trade-off table of the approach methods and optimization calculation is made

using the above two and calculation time as evaluation items. The three methods are divided in the case of the 2-body problem and the N-body problem and the six pattern methods are evaluated. This results is applied to the future JAXA missions EQUULEUS, DESTINY<sup>+</sup> that use lunar gravity assists.

## 2. Model

"Virtual hyperbola" and three approach methods used in this paper are introduced.

### 2.1 Virtual Hyperbola

In this paper, virtual hyperbolic trajectory is defined. At first, the pre-given perilune vector of the incoming trajectory is transferred to the infinity backward and the perilune vector of the incoming trajectory is transferred to the infinity forward. The transferring to infinity is generated by setting the true anomaly to Equation (1).

$$T_{\infty} = \cos^{-1}\left(-\frac{1}{e}\right) \quad (1)$$

The normal vector of the orbital plane defined by the velocity vectors at the two infinities is calculated as shown in Equation (2).

$$\hat{\mathbf{h}} = \frac{\mathbf{V}_{in} \times \mathbf{V}_{out}}{|\mathbf{V}_{in} \times \mathbf{V}_{out}|} \quad (2)$$

The turn angle by the swing-by can be calculated as shown in Equation (3).

$$\varphi = \cos^{-1}\left(\frac{\mathbf{V}_{in} \cdot \mathbf{V}_{out}}{|\mathbf{V}_{in}| |\mathbf{V}_{out}|}\right) \quad (3)$$

Also, using the Equation (3), the radius of the moon at swing-by is obtained as shown in Equation (4).

$$r = \left( \frac{\mu}{|V_{in}| |V_{out}|} \right) \left( \frac{1}{\sin(\varphi/2)} - 1 \right) \quad (4)$$

Since the unit vector is obtained from Equation (5), the perilune position vector can be defined as Equation (6).

$$\hat{R}_p = \frac{V_{in} - V_{out}}{|V_{in} - V_{out}|} \quad (5)$$

$$R_p = r \hat{R}_p \quad (6)$$

The unit vector of the perilune velocity vector can be obtained from the Equation (2) and (5) as in Equation (7).

$$\hat{V}_p = \hat{h} \times \hat{R}_p \quad (7)$$

Since the orbital energy is preserved, the magnitude of the perilune velocity vector can be obtained as shown in Equation (8), so the moon velocity vector can be defined as Equation (9).

$$V_p = \sqrt{|V_{in}|^2 + \frac{2\mu}{r}} \quad (8)$$

$$V_p = V_p \hat{V}_p \quad (9)$$

Also, the time of this perilune vector is defined by the Equation (10) from the time of the incoming orbit and the outgoing orbit.

$$t_p = \frac{t_f + t_b}{2} \quad (10)$$

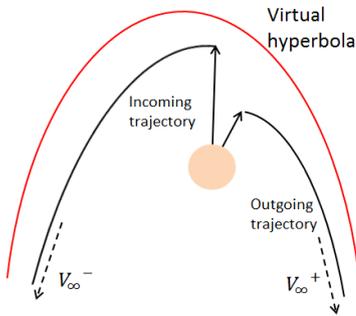


Fig. 1. Virtual Hyperbola

## 2.2 Approach Methods

The three approach methods for evaluating combination of trajectories that are easy to match are introduced.

### 2.2.1 Approach Method 1

The perilune vector of the virtual hyperbola is transferred to backward for N-days(N-days is optional.) and the perilune vector of the outgoing trajectory is transferred to backward for N-days. Computing the  $\Delta V$  transfer cost through the trajectory to tie between an outgoing leg and an opposite virtual hyperbola leg.

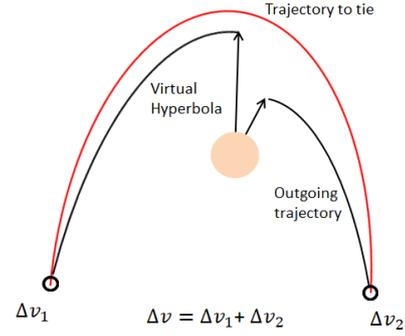


Fig. 2. Approach Method 1

### 2.2.2 Approach Method 2

The perilune vector of the incoming trajectory is transferred to backward for N-days and the perilune vector of the virtual hyperbola is transferred to backward for N-days. And like Approach Method 1, the perilune vector of the virtual hyperbola is transferred to backward for N-days(N-days is optional.) and the perilune vector of the outgoing trajectory is transferred to backward for N-days. Computing the total  $\Delta V$  transfer cost through the trajectory to tie between an incoming leg and an opposite virtual hyperbola leg, and between an outgoing leg and an opposite virtual hyperbola leg.

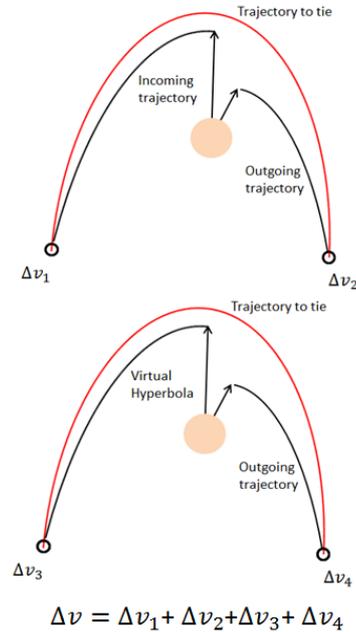


Fig. 3. Approach Method 2

### 2.2.3 Approach Method 3

The perilune vector of the virtual hyperbola is transferred to backward and forward for N-days. The perilune vector of the incoming trajectory is transferred to backward for n-days(n-days is optional.) and the perilune vector of the outgoing trajectory is transferred to forward for n-days. Computing the total  $\Delta V$  transfer cost through

the trajectory to tie between an incoming leg and a virtual hyperbola leg of the same side, and between an outgoing leg and a virtual hyperbola leg of the same side.

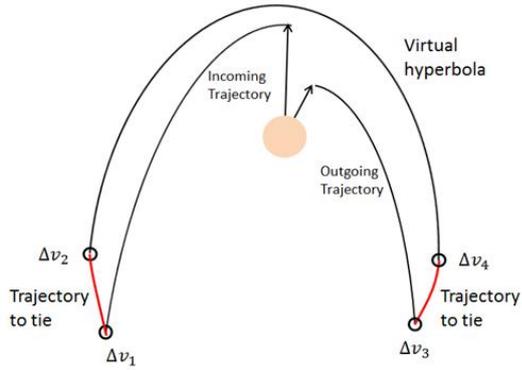


Fig. 4. Approach Method 3

### 2.3 Trajectory Propagation Methods

In this paper, a method based on 2-body and N-body is used. In case of 2-body, suppose that the spacecraft is affected only by the influence of gravity from the moon, and when tying the legs, use the Lambert problem and solve the boundary value problem. In case of N-body, suppose that the spacecraft is affected by the gravity of all the planets of the Sun, Solar System and the moon, and we use the Single Shooting method to solve the boundary value problem.

### 2.4 Evaluation Criteria

#### 2.4.1 Evaluation1

Correlation coefficients are used to evaluate whether there is a linear correlation. Assuming that the data of the cost of  $\Delta v$  by the approach method is  $x$ , the data of the cost of  $\Delta v$  by optimization is  $y$  and the average value of them is  $\bar{x}$ ,  $\bar{y}$ , then the correlation coefficient  $r$  is defined as in Equation (11).

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x}_i) (y_i - \bar{y}_i)}{\sqrt{\{\sum_{i=1}^n (x_i - \bar{x}_i)^2\} \{\sum_{i=1}^n (y_i - \bar{y}_i)^2\}}} \quad (11)$$

$n$  is the number of elements of data.  $r$  is calculated between  $-1 \leq r \leq 1$ , it can be determined that there is a positive correlation if it increases exponentially, a negative correlation if it increases negative, and there is no correlation as it is close to zero, In this paper we aim to have a positive correlation. Refer to Table 1. as the evaluation criteria of the values.

Table 1. Correlation evaluation

|                    |                    |
|--------------------|--------------------|
| $r \leq 0.2$       | No correlation     |
| $0.2 < r \leq 0.4$ | Weak correlation   |
| $0.4 < r \leq 0.7$ | Normal correlation |
| $0.7 < r \leq 1.0$ | Strong correlation |

### 2.4.2 Evaluation2

A linear approximation line is calculated from the data of the cost of  $\Delta v$  by optimization using the least squares method and evaluated from the line at the position of data outside the range of standard deviation  $\pm \sigma$  given by Equation (12).

$$\sigma = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y}_i)^2 \quad (12)$$

## 3. Results

The results on three approaches (2-body, N-body) are indicated. \* indicates the relationship between cost of  $\Delta v$  by the approach method and  $\Delta v$  by optimization. The solid line is the least squares line calculated from all the data, the dotted line at the top of the solid line is  $+\sigma$  from the least squares line, and the dotted line at the bottom is  $-\sigma$ .

### 3.1 Approach Method 1 (2-body)

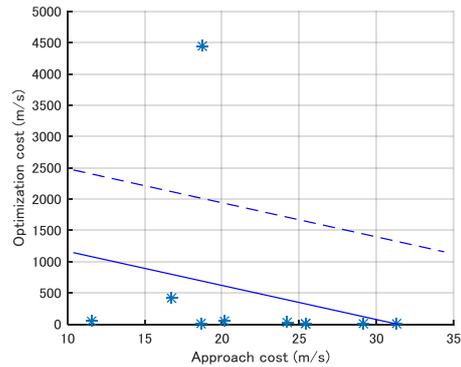


Fig. 5.  $\Delta v$  cost and least squares line

#### Approach Method 1 (2-body)

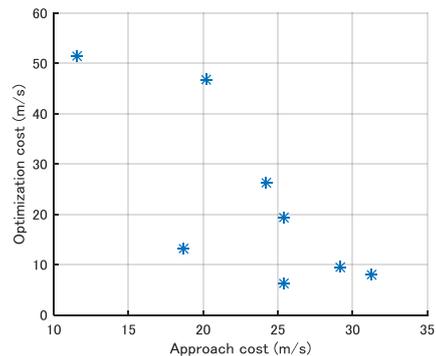


Fig. 6.  $\Delta v$  cost close up Approach Method 1 (2-body)

**Correlation coefficient :  $r = -0.2358$**

Since there is a negative correlation, there is no correlation between cost of  $\Delta v$  of the approach method and  $\Delta v$  of optimization.

### 3.2 Approach Method 2 (2-body)

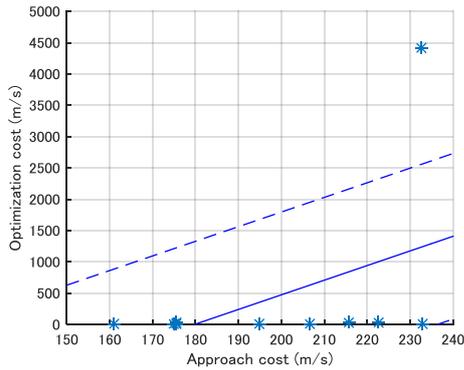


Fig. 7.  $\Delta v$  cost and least squares line Approach Method 2 (2-body)

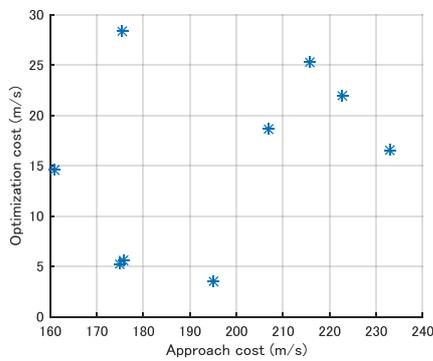


Fig. 8.  $\Delta v$  cost close up Approach Method 2 (2-body)

**Correlation coefficient :  $r = 0.4443$**

It can be seen that there is an ordinary correlation from the correlation coefficient. There is one piece of data apart from the standard deviation, but it is thought that it is good as a tendency because it is located at the upper part and has a positive correlation.

### 3.3 Approach Method 3 (2-body)

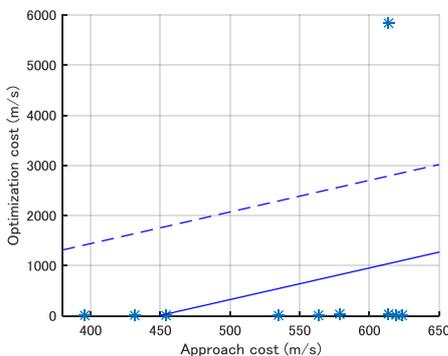


Fig. 9.  $\Delta v$  cost and least squares line Approach Method 3 (2-body)

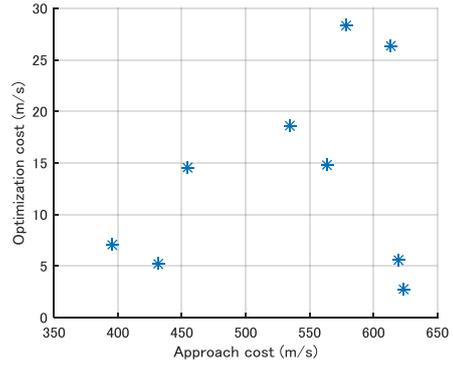


Fig. 10.  $\Delta v$  cost close up(Method3 2-body)

**Correlation coefficient :  $r = 0.2933$**

It can be read that there is a weak correlation from the correlation coefficient. There is also one point apart from the standard deviation here, but it is located at the upper part and is thought to be a tendency because it has a positive correlation.

### 3.4 Approach Method 1 (N-body)

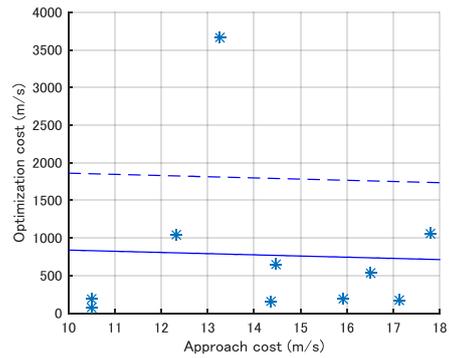


Fig. 11.  $\Delta v$  cost and least squares line

Approach Method 1 (N-body)

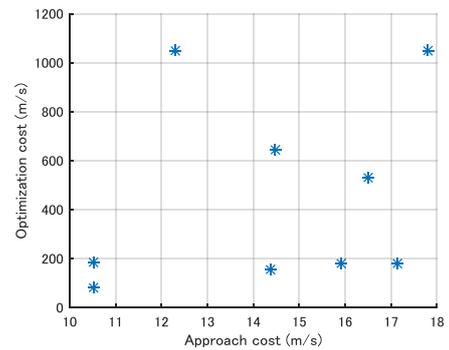


Fig. 12.  $\Delta v$  cost close up Approach Method 1 (N-body)

**Correlation coefficient :  $r = -0.0384$**

Since there is a negative correlation, there is no correlation between cost of  $\Delta v$  of the approach method and  $\Delta v$  of optimization.

### 3.5 Approach Method 2 (N-body)

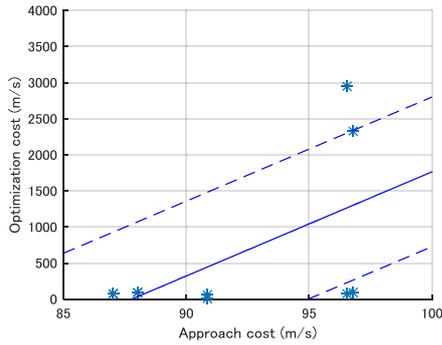


Fig. 13.  $\Delta v$  cost and least squares line

Approach Method 2 (N-body)

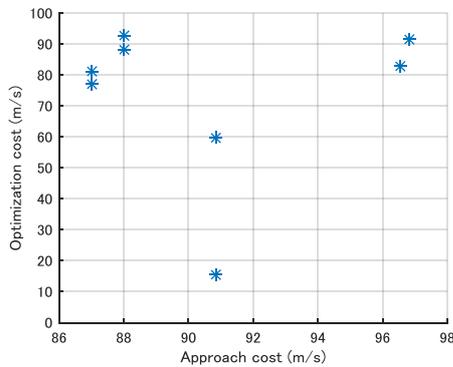


Fig. 14.  $\Delta v$  cost close up Approach Method 2 (N-body)

**Correlation coefficient** :  $r = 0.5765$

It can be seen that there is an ordinary correlation from the correlation coefficient. There are four points apart from the standard deviation, including those on the line, but there are both the one located at the upper part and the one located at the lower part, and it is difficult to estimate the influence on the correlation.

### 3.6 Approach Method 3 (N-body)

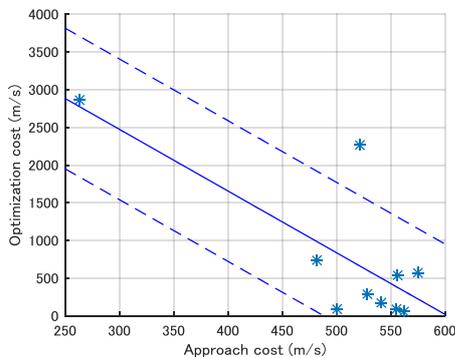


Fig. 14.  $\Delta v$  cost and least squares line

Approach Method 3 (N-body)

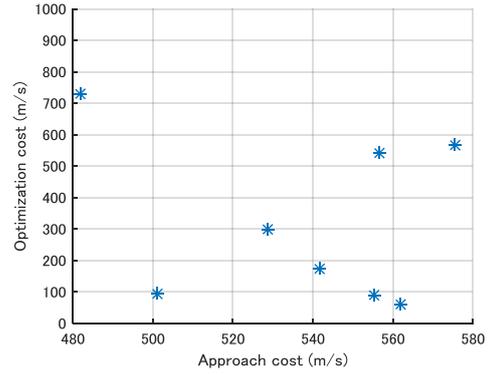


Fig. 15.  $\Delta v$  cost close up Approach Method 3 (N-body)

**Correlation coefficient** :  $r = -0.7565$

Since there is a strong negative correlation, there is no correlation between cost of  $\Delta v$  of the approach method and  $\Delta v$  of optimization.

### 3.7 Trade-off table

The results 3.1-3.6 are summarized in Table 2 as a trade-off table. Evaluation items are Evaluation1 indicating correlation, Evaluation2 indicating data deviating from correlation and calculation time. Especially focusing on Evaluation 1 and calculation time, evaluate comprehensively.

Table 2(a). Trade-off table

|                             | Approach1<br>2-body | Approach2<br>2-body | Approach3<br>2-body |
|-----------------------------|---------------------|---------------------|---------------------|
| <b>Evaluation1</b>          | ×                   | △                   | △                   |
| Evaluation2                 | —                   | ◎                   | ◎                   |
| <b>Computation<br/>time</b> | ◎                   | ◎                   | ◎                   |
| Comprehensive<br>judgment   | ×                   | ○                   | △                   |

Table 2(b). Trade-off table

| Approach1<br>N-body | Approach2<br>N-body | Approach3<br>N-body | Optimization |
|---------------------|---------------------|---------------------|--------------|
| ×                   | ○                   | ×                   | ◎            |
| —                   | ○                   | —                   | —            |
| ○                   | △                   | ○                   | ×            |
| ×                   | ○                   | ×                   | ○            |

#### 4. Conclusion

In this paper, we propose a new approach and analyze it to find combinations of trajectories that are easy to match without optimization. In the approach methods, we focused on that there is a strong correlation between the  $\Delta v$  when optimized and the cost of  $\Delta v$  in the approach methods, and how much calculation time can be shortened, and made a trade-off table including optimization calculation. As a result, it turned out that the Approach Method 2 (2-body, N-body) is good. However, it also shows that the correlation is insufficient. In the future, we will confirm the correlation further by changing the transferring time and increasing the number of data points from 10 pieces.

#### References

- [1] Gislaïne de Felipe, Classification of Out-of-Plane Swing-By Trajectories, National Space Research Institute, 12227-010 São José dos Campos, Brazil, 1999