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Optimal Fault-Tolerant Configurations of Thrusters

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Fault tolerance of spacecraft actuators significantly affects satellite reliability for mission success. One possible solution to enhance the fault tolerance against actuator failures is the use of underactuated control, because underactuated controller can control satellites with less number of inputs than the number of state variables. However, when some actuators have failed, the remained actuators are not necessarily able to generate control torques and/or translational forces along ideal directions. In other words, the directions of the control inputs are restricted. To overcome this difficulty for applying underactuated control, this study derives optimal fault-tolerant configurations of thrusters that maximize the controllability of the underactuated satellite. This study uses dual quaternions to express a thruster configuration, i.e., thruster position and attitude with respect to a body-fixed frame. The dual quaternion representation simplifies rotational torques and translational forces generated by the thrusters. Then a cost function in terms of the dual-quaternions is defined as the sum of the generated control forces and torques with respect to the body-fixed frame. The optimal configuration is derived by a method motivated by Thomson's problem. A numerical example shows the effectiveness of the proposed formulation and optimization method.

Key Words: Fault Tolerant Systems, Thrusters, Underactuated Systems

1. Introduction

Fault tolerance of spacecraft actuators significantly affects satellite reliability for mission success. Enhancement of the fault tolerance against actuator failures can be realized by underactuated control, because underactuated control enables driving satellite states to desired states with less number of inputs than the number of state variables. Thus underactuated control for satellites has been intensively studied and many control techniques also have been proposed.[1–3)]

However, one of the difficulties to use underactuated control is restricted input directions when some actuators have failed. In practical situation of actuator malfunctions, the remained actuators are not necessarily able to generate control torques and/or translational forces along ideal directions such as the directions along the principal axes of inertia. Such restriction on the input directions makes it harder for the underactuated control to be applicable. In other words, underactuated control can enhance the fault tolerance of the satellite when the controllability considering the underactuation of the satellite is satisfied even after some actuators have failed. In this context, this study derives optimal fault-tolerant configurations of thrusters that maximize the controllability of the underactuated satellite.

One thruster generates translational forces and coupled torques in one direction due to thruster mechanisms. Although a satellite position and attitude can be simultaneously controlled with thrusters, the restricted input direction complicates the proof of the controllability of the system, because controllability theorems with unidirectional inputs have not been proposed. The derivation of the controllability conditions is out of scope of this study, and this study assumes the controllability condition to generate translational forces and rotational torques in any directions.

This study expresses a thruster configuration, i.e., thruster position and attitude with respect to a body-fixed frame, using dual quaternions. The dual quaternion representation can simplify rotational torques and translational forces generated by the thrusters. Then a cost function in terms of the dual quaternions is defined as the sum of the generated control forces and torques with respect to the body-fixed frame. The optimal thruster configuration is derived using a solution to Thomsons problem as used in Ref. 4). In the optimization method, considering the geometric position and attitude of the thrusters as point charges, the arbitrary number of thrusters can be configured in an equal distance, which maximizes available control forces and torques in all directions. That is, the controllability of the position and attitude of the satellite under a few actuator failures is also maximized. A numerical example shows the effectiveness of the proposed formulation optimization method.

2. Problem Formulation

This paper considers controllability of a satellite position and attitude. The satellite position in this study means a free-floating state, and orbital motion is not considered. The equations of motion of the free-floating satellite is expressed with an affine system as:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where

$$\boldsymbol{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & q^T & \omega^T \end{bmatrix}^T \tag{2}$$

$$f(\boldsymbol{x}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \mathbf{0}_{3\times1} \\ 0.5[\tilde{\omega}]q \\ -I^{-1}(\omega \times I\omega) \end{bmatrix} \tag{3}$$

$$g(\boldsymbol{x}) = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ R_{b/i}^T & \mathbf{0}_{3\times3} \\ \mathbf{0}_{4\times3} & \mathbf{0}_{4\times3} \\ \mathbf{0}_{3\times3} & I^{-1} \end{bmatrix} \tag{4}$$

$$\boldsymbol{u} = \begin{bmatrix} F_x & F_y & F_z & T_x & T_y & T_z \end{bmatrix}^T \tag{5}$$

The satellite position are expressed with x, y, and z. The attitude angle is formulated with quaternion $q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^T$ and the angular velocity is $\boldsymbol{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$. The quaternions in this study define q_0 as a scalar part and the others as a vector part. The matrix I represents the moment of inertia of the satellite and $R_{b/i}$ means a directional cosine matrix from an inertial frame to the body-fixed frame. In Eq. (3), $[\tilde{\omega}]$ is defined as follows.

$$[\tilde{\omega}] = \begin{bmatrix} 0 & \omega_x & -\omega_y & \omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_x & \omega_y & -\omega_x & 0 \end{bmatrix} \tag{6}$$

Assuming that N thrusters are fixed with the satellite body and they generate translational forces in one directions, the translational forces $\begin{bmatrix} F_x & F_y & F_z \end{bmatrix}$ and torques $\begin{bmatrix} T_x & T_y & T_z \end{bmatrix}$ with respect to the body-fixed frame are written as

$$\begin{aligned} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} &= \begin{bmatrix} \boldsymbol{d}_1 & \cdots & \boldsymbol{d}_N \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix} \\ &= D\boldsymbol{f} \end{aligned} \tag{7}$$

$$\begin{aligned} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} &= \begin{bmatrix} \boldsymbol{r}_1 \times \boldsymbol{d}_1 & \cdots & \boldsymbol{r}_N \times \boldsymbol{d}_N \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix} \\ &= A\boldsymbol{f} \end{aligned} \tag{8}$$

Each thruster position and orientation (attitude) are represented with position vectors $\boldsymbol{r}_i$ and thrust directional vectors $\boldsymbol{d}_i$ ($i = 1, \ldots, N$), respectively. Note that the thrust magnitudes should be positive i.e., $f_i \geq 0$ ($i = 1, \ldots, N$), due to thruster mechanisms.

2.1. Controllability

Although controllability of affine systems with unidirectional inputs is shown by Goodwine,[5] this theorem requires that the system can generate control forces/torques in any directions. That is, underactuated control cannot be explicitly considered. Pena et al.[6] also show thruster configuration conditions that can generate control forces and torques in all directions. In these studies, the controllability of a satellite position and attitude is assumed that thrusters can generate translational forces and rotational torques in any directions.

Matsuno et al.[7] show geometric conditions of thrusters for position and attitude control of a satellite. In Ref. 7), it is shown that three thrusters can control position and attitude of a free-floating satellite. The controllability with the three thruster configuration has not been proved, but verified by deriving a control method using three thrusters. It is noted that such three-thruster configuration can control the satellite position and attitude, even though the thruster configuration does not satisfy the Goodwine theorem.

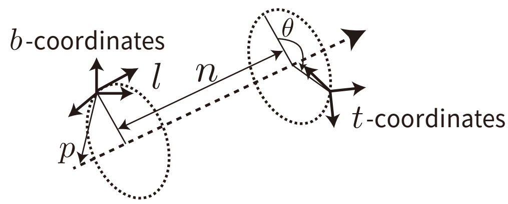

Fig. 1. Screw motion.

2.2. Dual Quaternions

Thruster configurations can be defined as position and attitude of thrusters with respect to the satellite body-fixed frame. In this study, the position and attitude of each thruster is represented with dual quaternions to simplify the formulation.

A dual quaternion $\boldsymbol{q}_{t/b}$ is defined as

$$\boldsymbol{q}_{t/b} = q_{t/b,r} + \epsilon q_{t/b,d} \tag{9}$$

where $q_{t/b,r}$ and $q_{t/b,d}$ are quaternions called a real part and dual part, respectively. This study deals with quaternions as a vector in four dimensions, which are described as nonbold variables. Their subscripts indicate the quaternions written in thruster-fixed coordinates (t-coordinates) with respect to the satellite body-fixed coordinates (b-coordinates). Dual quaternions satisfy the relations $q_{t/b,r}^T q_{t/b,r} = 1$ and $q_{t/b,r}^T q_{t/b,d} = 0$, and ϵ is dual number, which satisfies $\epsilon^2 = 0$.

A dual quaternion is expressed with

$$\boldsymbol{q}_{t/b} = \begin{bmatrix} \hat{\boldsymbol{l}} \sin\frac{\hat{\theta}}{2} & \cos\frac{\hat{\theta}}{2} \end{bmatrix} \tag{10}$$

where $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{l}}$ are a dual angle and a dual vector, respectively. These variables define a screw motion as shown in Fig. 1, and they are formulated as follows.

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta} + \epsilon \boldsymbol{n} \tag{11}$$

$$\hat{\boldsymbol{l}} = \boldsymbol{l} + \epsilon \boldsymbol{p} \times \boldsymbol{l} \tag{12}$$

2.3. Thruster configuration with dual quaternions

The i-th thruster's position and attitude can be expressed with a dual quaternion using the position vector of the thruster r^i as

$$\boldsymbol{q}_{t/b,i} = q_{t/b,i} + \frac{1}{2}\epsilon r^i \odot q_{t/b,i} \tag{13}$$

where $r^i = [0, \quad \boldsymbol{r}_i^T]^T$ and $q_{t/b,i}$ is the quaternion that represents the i-th thruster attitude with respect to the body-fixed frame, and the position vector of the thruster is also written with respect to the body-fixed frame. The multiplication of two quaternions, p and q, is defined as

$$p \odot q = \begin{bmatrix} p_0 q_0 - \bar{\boldsymbol{p}}^T \bar{\boldsymbol{q}} \\ q_0 \bar{\boldsymbol{p}} + p_0 \bar{\boldsymbol{q}} + \bar{\boldsymbol{p}} \times \bar{\boldsymbol{q}} \end{bmatrix} \tag{14}$$

In Eq. (14), $\bar{*}$ indicates the vector part of a quaternion.

As shown in Eq. (13), the real part of the dual quaternion shows attitude of the thruster, whereas the dual part includes both the position vector and attitude quaternion. This means that the real part can represent translational force direction and the dual part can express torque direction, because the translational force directions of thrusters depend on their position vectors with respect to the satellite body, whereas torque directions depend both position and thruster orientation. Thus, the dual quaternion can simply represent the translational force directions and torques.

3. Optimal Thruster Configurations

A thruster configuration is optimized in terms of position and attitude controllability of a satellite. In this study, the controllability that requires translational forces and rotational torques in any directions is considered. It should be noted that this controllability is obtained by applying Goodwine's theorem[5] to position and attitude control of a satellite with thrusters shown in Eq. (1).

The dual part of the i-th thruster is written as

$$\frac{1}{2}r^i \odot q_{t/b,i} = \frac{1}{2}\begin{bmatrix} -\boldsymbol{r}_i{}^T\bar{\boldsymbol{q}}_i \\ q_0\boldsymbol{r}_i + \boldsymbol{r}_i \times \bar{\boldsymbol{q}}_i \end{bmatrix} \tag{15}$$

The vector part in Eq. (15) can represent a torque direction if $q_0 = 0$ and $\bar{\boldsymbol{q}}$ is translational force direction. Such definition is realized when the thrust direction coincides with the Euler axis and the rotation angle of the quaternion becomes 180 deg. This definition indicates that a thruster orientation is defined so that the Euler axis lies at the thrust direction, and this can be realized because the thruster direction has two degrees of freedom, that is, an elevation angle and an azimuth angle of the thrust direction.

3.1. Optimization

As stated above, the controllability condition that can generate translational forces and torques in any directions is considered, and the cost function to be minimized is defined as

$$J = \sum_{i \neq j}^{N}\left(\boldsymbol{q}_i \cdot \boldsymbol{q}_j\right) + \sum_{i \neq j}^{N}\left(q_{i,d} \cdot q_{j,d}\right) \tag{16}$$

where $i \neq j$ means all possible combinations of N thrusters. The first term of the right-hand side means the maximization of the force vectors, and the second term indicates the maximization of the torque vectors. Furthermore, this cost function geometrically indicates that all combinations of the vector parts of the quaternions should be orthogonal, which minimize the cost function J. This study solves this problem as Thomson's problem,[8] which is the problem to find particle positions equally distributed on a unit sphere. For the optimization of the cost function J, considering the vector parts of the quaternions lead the same formulation as Thomson's problem.

The cost function J is further written as follows.

$$J = \sum_{i \neq j}^{N}\left(q_{i,r} \cdot q_{j,r}\right) + \sum_{i \neq j}^{N}\epsilon\left(q_{i,r} \cdot q_{d,j} + q_{i,d} \cdot q_{r,j}\right) + \sum_{i \neq j}^{N}\left(q_{i,d} \cdot q_{j,d}\right) \tag{17}$$

The second term of the right-hand side in Eq. (17) is transformed as

$$\sum_{i \neq j}^{N}\epsilon\left(q_{i,r} \cdot q_{d,j} + q_{i,d} \cdot q_{r,j}\right) =$$

$$\sum_{i \neq j}^{N}\left[\bar{\boldsymbol{q}}_i \cdot \left(\frac{1}{2}\boldsymbol{r}_i \times \bar{\boldsymbol{q}}_j\right) + \left(\frac{1}{2}\boldsymbol{r}_i \times \bar{\boldsymbol{q}}_i\right) \cdot \bar{\boldsymbol{q}}_j\right] \tag{18}$$

$$= \sum_{i \neq j}^{N}\left[\boldsymbol{r}_j \cdot \left(\frac{1}{2}\bar{\boldsymbol{q}}_j \times \bar{\boldsymbol{q}}_i\right) + \boldsymbol{r}_i \cdot \left(\frac{1}{2}\bar{\boldsymbol{q}}_i \times \bar{\boldsymbol{q}}_j\right)\right] \tag{19}$$

The cost function J is minimized by finding optimal combinations of $q_{i,r}$ and $q_{j,r}$ as shown in Eq. (17). Furthermore such optimal combinations of the quaternions indicate that the cross products among them are maximized. Thus the cross products in Eq. (19) are maximized, and the optimal thruster configuration can be found by minimizing the inner products among the position vectors $\boldsymbol{r}_i$ and the quaternions.

As stated above, this study uses a solution to solve Thomson's problem for the optimization of the thruster configurations. One of techniques for Thomson's problem is an energy potential method of point charges, and this method is also used to optimize configurations of control moment gyros.[4] This paper thus uses a similar method shown in Ref. 4) to derive the optimal thruster configurations.

The energy potential method considers endpoints of thrust position vectors and vector parts of the quaternions as point charges on a unit sphere. They are interacted one another and moved on the sphere, resulting in equally distributed positions on the sphere surface. In this study, the following potential energy of point charges is used.

$$\phi\left(\boldsymbol{r}_i\right) = \frac{1}{|\boldsymbol{r}_i|} \tag{20}$$

This potential energy represents coulomb potential for the interaction, and the following force acts between the i-th and the j-th charges.

$$\boldsymbol{f}_{ij} = A_i A_j \frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|^2} \tag{21}$$

where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, and A_i and A_j are the magnitudes of the charges. Furthermore, the force is projected to confine the particles on the surface and virtual damping effect is added as:

$$\boldsymbol{F}_{ij} = -\left(\boldsymbol{f}_{ij} \times \boldsymbol{r}_i\right) \times \boldsymbol{r}_i - c_i\dot{\boldsymbol{r}}_i \tag{22}$$

where c_i is a damping coefficient.

The optimization of N-thruster configuration is calculated as follows:

1. N point charges that describe the thruster attitude are randomly distributed on a unit sphere.
2. The interacting forces shown in Eq. (22) are used to move the point charges on the sphere.
3. The position of the point charges are normalized to confine the charges to the surface on the sphere.
4. The preceding steps are repeated until position changes become sufficiently small.

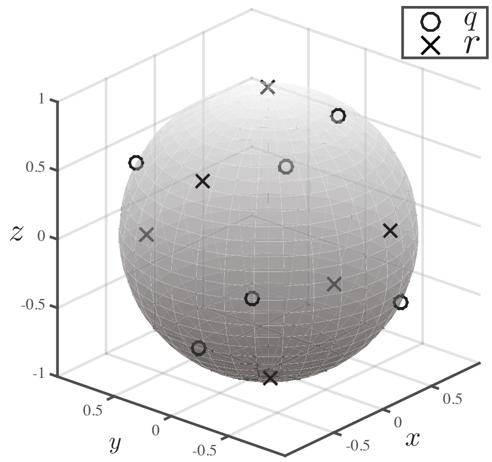

Fig. 2. Optimal azimuth and elevation angles on a unit sphere.

Table 1. Angles among the position vectors and the vector parts of the quaternions [deg].

	$\boldsymbol{r}_1$	$\boldsymbol{r}_2$	$\boldsymbol{r}_3$	$\boldsymbol{r}_4$	$\boldsymbol{r}_5$	$\boldsymbol{r}_6$
$\bar{q}_1$	128.8	51.2	115.1	120.8	64.9	49.2
$\bar{q}_2$		129.5	129.2	116.4	50.8	63.6
$\bar{q}_3$			64.9	49.2	115.1	130.8
$\bar{q}_4$				49.2	129.2	116.4
$\bar{q}_5$					130.4	50.6
$\bar{q}_6$						129.4

5. After the distribution of the point charges, fix their positions, and then other N point charges that describe the thruster position vectors are randomly distributed on the unit sphere.
6. The preceding steps 2–4 are repeated.

4. Numerical Examples

This section shows an optimal thruster configuration to verify the proposed optimization method for thruster configurations. The number of thrusters are set to 6, and the magnitudes of the charges and the damping coefficients are set to $A_i = 50.0$ and $c_i = 3.0$ ($i = 1, \ldots, 6$), respectively.

Figure 2 shows the position of the distributed point charges, in which circle symbols and cross ones represent the charges describe thruster attitude and position, respectively. The corresponding azimuth and elevation angles are shown in Fig. 3. These figures show that the distance among all point charges are successfully distributed on the sphere. The corresponding force directions and torque directions of this thruster configuration is illustrated in Fig. 4. Both directions are equally distributed in any directions, which indicates that the thruster configuration maximizes the controllability of position and attitude of the satellite. Table 1 shows the angles among the position vectors and the vector parts of the quaternions. Most of the angles take about 90 ± 40 deg and they indicate that the point charges are successfully distributed on the sphere with equal distance.

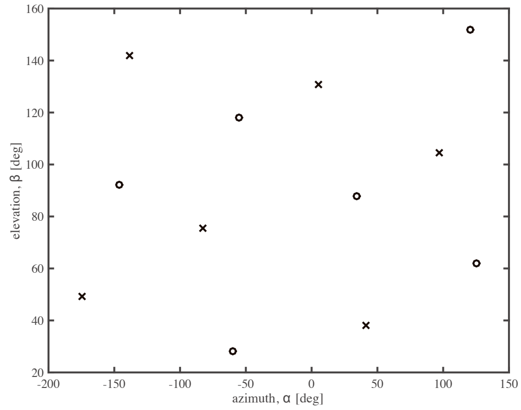

Fig. 3. Azimuth and elevation angles.

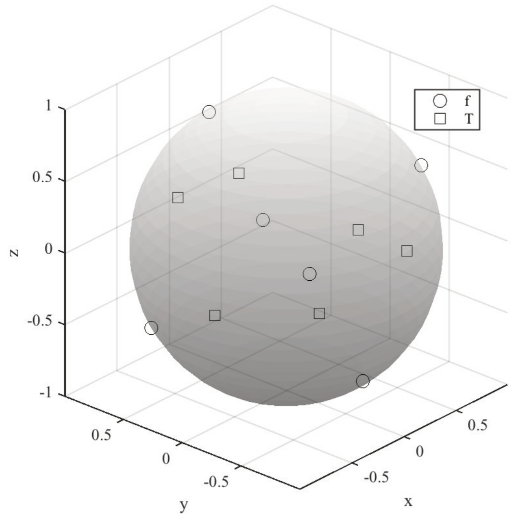

Fig. 4. Optimal force and torques directions.

5. Conclusions

This study presents optimal fault-tolerant configurations of thrusters that maximize position and attitude controllability of a spacecraft. The controllability takes into account underactuated control by a few thrusters, and the optimal thruster configurations are fault-tolerant even when a thruster malfunction occurs. The thruster position and attitude is expressed with dual quaternions to simplify the formulation. The optimization of the thruster configuration is reduced to a similar problem to Thomson's problem. Then, the optimal thruster configuration has been derived by successively using the energy potential method. An numerical example verifies the effectiveness of the proposed method. Applying region constraints on specific directions and different weightings will be one of future topics.

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